Soft Budget Constraints, Bank Capital, and the Monetary Transmission Mechanism

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Abstract

This paper investigates the effect of monetary policy in a situation where the bank misallocates its credit because of soft budget constraint problems and the bank's asset–liability management is subject to a capital requirement. Under these circumstances, an expansionary monetary policy does not always increase the amount of bank lending. Moreover, even if bank lending does increase, it may not stimulate economic activity when soft budget constraint problems are present. Therefore, in order to stimulate the economy, solving the soft budget constraint problems in the banking sector and thus improving the *quality* of bank lending is more important than simply increasing the *volume* of bank lending.

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1 Introduction

Since the collapse of the stock and real estate markets at the beginning of the 1990s, the Japanese economy has been stagnating[1]. To counter this, the Bank of Japan (BOJ) has expanded the monetary base to an unprecedented extent. In particular, in March 1999, the BOJ introduced a zero interest rate policy, under which the call rate was lowered to close to zero[2]. Despite such measures, the Japanese economy did not begin to recover, which implies that the traditional monetary policy has lost its effectiveness as an instrument for enhancing economic activities in Japan.

Why has this policy been so ineffective during the recent recession? To answer this question, two important factors that were widely observed in Japan in this period should be taken into account. The first factor is the banks' incentives to allocate credit. In the early and mid-1990s, many Japanese banks provided additional funds to old unprofitable borrowers rather than financing new profitable firms. In this period, the amount of bank credit provided to real estate and construction firms increased despite the fact that the profitability of such firms was seriously damaged by the collapse of asset bubbles. The second factor is the role of capital requirements. Since the implementation of the Basel Accord in 1988, bank capital has been a significant determinant of bank credit. In particular, because Japanese banks suffered from a shortage of risk capital because of the collapse of stock and asset bubbles, the amount of bank lending was strictly constrained by the capital requirement.

However, the misallocation of bank credit and the role of capital requirement have not been taken into consideration in traditional monetary theory. In the literature on monetary theory,

[1]From 1991 to 2005, the average annual growth rate of real GDP was just above 1%, compared to a 4% growth rate in the previous 20 years.

[2]In February 1999, the BOJ decided to "flexibly provide ample funds and encourage the uncollateralized overnight call rate to move as low as possible" in order not only to stimulate the economy but also to prevent deflation from worsening. For more details, see Ito and Mishkin (2004).

much importance has been attached to how much the banks lend (i.e., the *volume* of bank lending), but to whom the banks lend (i.e., the *quality* of bank lending) has not been taken into account. Moreover, because changes in bank lending has been considered to be induced by changes in bank reserves, the role of capital requirements has not been paid much attention compared to that of reserve requirements[3].

To derive the optimal monetary policy under these circumstances, we develop a model that incorporates a bank's incentives to misallocate credit and the capital requirement into the existing framework of monetary theory. In the model, the misallocation of bank credit is induced by soft budget constraint problems, as in Berglof and Roland (1997). That is, when the bank cannot commit to terminating nonperforming projects ex ante and the bank has incurred sunk costs in old projects, the bank may refinance old poorly performing projects rather than financing new profitable projects. Extending Berglof and Roland's model such that many firms with varying levels of net worth demand bank lending, we analyze how the soft budget constraint problems in the banking sector affect the macroeconomy. Moreover, in this paper, the amount of bank lending is constrained by capital requirements. Under these settings, we analyze the effect of monetary policy in the presence of soft budget constraint problems and a capital requirement.

This paper derives several new results in terms of the effect of monetary policy and bank recapitalization. First, in the presence of soft budget constraint problems and the capital requirement, whether an expansionary monetary policy increases the amount of bank lending depends on the proportion of good projects in the economy. When the proportion of good projects is large, an increase of bank credit induced by a monetary expansion today increases the bank's capital base tomorrow, which, in turn, increases the amount of bank lending tomorrow. On the other hand, when the proportion of good projects is small, the increase of bank credit induced by a monetary

[3]For more details on the monetary transmission mechanism induced by changes in reserves, see Kashyap and Stein (1994) and Freixas and Rochet (1997).

expansion today may decrease the bank's capital base tomorrow, which decreases the amount of bank lending tomorrow under the capital requirement.

Second, even if an expansionary monetary policy increases the amount of bank lending, an increased amount of bank lending will be allocated to refinancing unprofitable projects in the presence of soft budget constraint problems. This type of lending does not promote an upturn in the economy. Therefore, a monetary expansion may not be effective in stimulating economic activity when firms face soft budget constraints.

Finally, in a situation where the bank is allowed to engage in monitoring activity to improve the profitability of new projects, bank recapitalization by an injection of public funds becomes efficient. This is because recapitalization induces the bank to finance and monitor more new profitable projects, which contributes to solving the soft budget constraint problems.

These results imply that in order to stimulate the economy, it is more important to solve the soft budget constraint problems and thus to improve the *quality* of bank lending, rather than simply increasing the *volume* of bank lending. From this viewpoint, restoring efficiency in the banking sector is indispensable for the effective operation of monetary policy.

The model relates to three areas of literature. First, Bolton and Freixas (2004) and Van den Heuvel (2005) analyzed the monetary transmission mechanism in the presence of a capital requirement. In these papers, in order to analyze the response of bank lending to monetary policy under capital constraint, capital constraint is always binding, and the bank's capital base is endogenously determined[4]. These two factors are also taken into account in this paper. In addition, we introduce moral hazard behavior by the bank, such as misallocating its credit, into the model. Then, we show that a monetary expansion may not be an effective instrument for stimulating the economy.

Second, Holmstrom and Tirole (1997) and Repullo and Suarez (2000) analyzed the effect of

[4]Thakor (1996) considered the same issue in a situation where the bank's equity base is given exogenously.

monetary policy when entrepreneurs may engage in moral hazard behaviors such as diverting project sources toward private uses. However, neither paper considered moral hazard behavior in the form of the bank misallocating its credit.

Third, as noted above, Berglof and Roland (1997) and Dewatripont and Maskin (1995) analyzed the misallocation of bank credit because of soft budget constraint problems. Incorporating several important factors such as the net worth of firms and the endogenously determined bank capital into their models, we derive the optimal monetary policy when the soft budget constraint problems prevail in the economy. In terms of the misallocation of bank credit in Japan during the recent recession, another view has been explored. Sakuragawa (2002) and Peek and Rosengren (2003) argued that the historical cost accounting system induced Japanese banks to misallocate their credit[5]. However, in this paper, the misallocation of bank credit is induced not by the historical cost accounting system but by the soft budget constraint problems. This is because the soft budget constraint problems were observed not only in Japan but also in many transition economies such as Central and Eastern European countries. Therefore, we believe that the framework that we present here is applicable not only to the Japanese economy but also to these transition economies.

This paper is organized as follows. Section 2 provides the basic model. Section 3 characterizes the optimal asset–liability management of the bank. Section 4 examines how monetary policy affects the amount of bank lending and the aggregate income. Section 5 introduces monitoring activity by the bank and examines the effect of a capital injection. In section 6, the bank is allowed to issue its capital. Section 7 concludes the paper.

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[5]Under the historical cost accounting system, banks' balance sheets appear healthier if they refinance their troubled borrowers, thus keeping these loan accounts.

2 The model

Consider the following model where there are three dates, $t = 0, t = 1, t = 2$, and three types of agent: a bank, firms, and a government[6]. All agents are risk neutral. We use r_t ($r_t \geq 1$) to denote the risk-free interest rate at $t = 0, 1$.

There is a continuum of firms with varying levels of initial wealth, w_0. w_0 is distributed on the interval $[\underline{w}, 1)$ at the beginning of $t = 0$[7]. All firms have access to the same technology. At $t = 0$, each firm has one project requiring an investment of one unit of capital. As $1 > w_0$, a firm must raise $1 - w_0$ to undertake the project. To analyze the bank's behavior explicitly, we assume that a firm does not have any assets other than its own project and that it relies entirely on bank financing. Therefore, the amount $1 - w_0$ is financed by the bank.

Although neither the firms nor the bank know the quality of the project (good or bad) at $t = 0$, they know the distribution of it: a proportion α ($0 < \alpha < 1$) of projects become good at $t = 1^-$, and a proportion $1 - \alpha$ of projects become bad at $t = 1^-$. A good project yields $(\pi_G, 0)$ ($\pi_G > 0$) at $t = 1$ and (π_G, B) ($B > 0$) at $t = 2$, where π_G represents a verifiable return and B represents a nonverifiable private benefit for a firm. A bad project yields $(\pi_G, 0)$ at $t = 1$ and (π_G, B_L) ($0 < B_L < B$) at $t = 2$ if the firm exerts effort at $t = 1^-$. On the other hand, if the firm exerts no effort, a bad project yields $(0, 0)$ at $t = 1$. In this case, the bank decides whether to terminate or refinance this project. If terminated, a bad project yields $(L, 0)$ at $t = 1$, where L represents the liquidation value of the project. For simplicity, L is assumed to be zero. If refinanced at the cost of one additional unit of capital, a bad project yields (π_R, B) at $t = 2$. Assume $r_1 < \pi_R < \pi_G$ so that refinancing is ex post profitable (the first inequality) but the

[6]The model is very similar to that of Berglof and Roland (1997), except that firms are characterized as heterogeneous (initial net worth differs among firms), and the amount of bank lending at $t = 1$ is constrained by the bank's equity base. This extension enables the effect of monetary policy to be analyzed when the soft budget constraint problems prevail in the economy.

[7]$\underline{w}$ can be negative.

return is lower than that yielded if a firm exerts effort (the second inequality).

At $t = 1$, new firms enter the market. The initial wealth w_1 across the firms is uniformly distributed on the interval $[\underline{w}, 1)$. In addition, assume that new firms rely entirely on bank financing. Although neither the new firm nor the bank knows the quality of the project (good or bad) at $t = 1$, they know the distribution of it: a proportion β $(0 < \beta < 1)$ of new projects become good, yielding (π_G, B) at $t = 2$ and a proportion $1 - \beta$ of them become bad, yielding $(0, 0)$ at $t = 2$. Furthermore, assume that $\beta\pi_G > r_1$, i.e., financing a new project generates a positive net present value. The timing is illustrated in Figure 1.

The bank has initial equity capital of $\bar{E}(> 0)$ at $t = 0$ and issues deposit accounts D_0 to fund a total amount of loans, L_0, at $t = 0$. In addition, the amount of loans L_1 at $t = 1$ is financed by bank capital E_1 at $t = 1$ and by deposit accounts D_1 issued at $t = 1$. The bank must satisfy the minimum capital requirement γ at $t = 0, 1$. The bank bears a monitoring cost m_0 for a project it finances at $t = 0$[8]. For simplicity, the bank has all the bargaining power and thus it can appropriate all the verifiable returns of firms. Therefore, the bank's payoff is the sum of the verifiable returns of the projects π, whereas the firm's payoff is the sum of the unverifiable returns of the projects B.

Finally, the government is concerned with the level of aggregate income. The government manipulates interest rates to achieve an efficient level of aggregate income. In this model, aggregate income is calculated as the sum of the verifiable income of firms.

3 Bank lending

This section investigates how the amounts of bank lending at $t = 0$ and $t = 1$ are determined. Subsections 3.1 and 3.2 examine bank lending at $t = 1$ and $t = 0$, respectively.

[8] The bank's monitoring activity for a project it finances at $t = 1$ is considered in section 5.

3.1 Bank lending at $t = 1$

This subsection examines the bank lending at $t = 1$. To highlight the relation between bank capital and bank lending, this paper supposes that the capital requirement is binding at $t = 1$[9]. Then, the amount of bank lending, L_1, at $t = 1$ is given by:

$$L_1 = E_1/\gamma, \tag{1}$$

where E_1 denotes the amount of bank capital at $t = 1$. In addition, as L_1 is financed by equity E_1 and deposits D_1, then $L_1 = D_1 + E_1$.

At $t = 1$, the bank decides whether to invest in new projects (the amount of which are denoted by L_1^N) or to refinance firms with bad projects (the amount of which are denoted by L_1^R). Lemma 1 describes the optimal lending behavior of the bank at $t = 1$.

Lemma 1

At $t = 1$, if the net worth of a new firm satisfies $w_1 > \hat{w}_1$, where $\hat{w}_1 \equiv 1 - \beta\pi_G/\pi_R$, the bank prefers to lend to a new firm rather than refinancing a firm with a bad project. In addition, $\partial\hat{w}_1/\partial r_t = 0(t = 0, 1)$.

Proof See Appendix 1.

Lemma 1 indicates that L_1 is allocated first to new, well-capitalized firms, and next to firms with bad projects. From this lemma, the amount of new lending L_1^N and the amount of refinancing L_1^R can be expressed by:

$$L_1^N = \int_{max\{\hat{w}_1, \hat{w}_1'\}}^{1} (1 - w_1) f(w_1) dw_1 \tag{2}$$

$$L_1^R = max\{0, L_1 - L_1^N\} = \Delta W_1^R, \tag{3}$$

[9]This assumption is accepted in recent works that emphasize the interaction between bank capital and monetary policies. For more details, see Bolton and Freixas (2004) and Van den Heuvel (2005).

where $\hat{w}_1^{'}$ satisfies:

$$L_1 = \int_{\hat{w}_1^{'}}^{1} (1 - w_1) f(w_1) dw_1,$$

and ΔW_1^R represents the number of firms refinanced at $t = 1$. From (2) and (3), if L_1 is so small that $\hat{w}_1 < \hat{w}_1^{'}$ is satisfied, the bank does not refinance firms with bad projects (i.e., $L_1^R = 0$). On the other hand, if L_1 is so large that $\hat{w}_1 > \hat{w}_1^{'}$ is satisfied, the amount $L_1^R = L_1 - L_1^N$ is allocated to refinancing bad projects.

Let p be the probability that a firm with a bad project can be refinanced. Then, p becomes:

$$p = \frac{L_1^R}{(1 - \alpha)\Delta W_0}. \tag{4}$$

The denominator denotes the number of firms that demand refinancing, and ΔW_0 represents the number of firms financed at $t = 0$. From (4), the probability that a firm with a bad project is refinanced increases as the bank allocates more funds to refinance such projects (i.e., L_1^R increases) or when the proportion of good projects in the economy becomes larger (i.e., α becomes larger)[10]. Here, to make the following arguments simple, we focus on the situation where L_1 is not so large that the total amount of funds allocated to the firms with bad projects L_1^R is less than the aggregate demand for refinancing (i.e., $p < 1$)[11]. Under this setting, although refinancing the firms with bad projects is ex post efficient, credit rationing occurs in the market for refinancing because of the short supply of bank lending at $t = 1$.

In Berglof and Roland (1997), the bank is not capital constrained. Therefore, the extent to which the bank refinances bad projects depends only on the relative profitabilities of financing new projects and refinancing bad projects. However, in this paper, as (4) shows, p is determined endogenously. That is, the supply and the demand for refinancing are dynamically influenced by the banking behavior at $t = 0$[12]. This formulation of p is fundamental to the whole analysis of

[10] The relation between p and ΔW_0 will be investigated in section 4.

[11] Indeed, the following results hold even if we consider the situation in which L_1^R is so large that all bad projects are refinanced and, thus, credit rationing does not occur (i.e., $p = 1$).

[12] The level of bank capital at $t = 1$ will be given later in (9) and (10).

the monetary transmission mechanism in this paper.

3.2 Bank lending at $t = 0$

This subsection examines bank lending at $t = 0$. At $t = 0$, the bank must decide what firms to invest in and must raise funds by issuing deposit accounts to depositors (D_0).

Before examining the amount of bank lending at $t = 0$, we consider the amount of bank capital and the expected revenue of bank lending at $t = 0$. First, we examine the amount of bank capital E_0 at $t = 0$. At the beginning of $t = 0$, the bank has initial equity capital $\bar{E}$. As the bank should pay a monitoring cost $m_0 (> 0)$ per project, E_0 becomes:

$$E_0 = \bar{E} - m_0 \Delta W_0, \tag{5}$$

where ΔW_0 represents the number of firms financed at $t = 0$.

Next, we consider the expected payoff of the bank. From firms with good projects, the bank can obtain π_G at $t = 1, 2$. On the other hand, the amount the bank can obtain from firms with bad projects depends on the probability that the bank will refinance a bad project at $t = 1$. If L_1^R is so large that p is sufficiently large, a firm with a bad project—knowing that it has a high probability of being refinanced at $t = 1$—prefers not to exert effort and to obtain private benefit B. However, if p becomes small, a firm with a bad project is less likely to be refinanced at $t = 1$, and thus it chooses to exert effort and to obtain B_L. Therefore, the condition under which a firm with a bad project does not exert effort is given by:

$$\begin{aligned} pB &\geq B_L \\ p &\geq B_L / B. \end{aligned} \tag{6}$$

When p is so large that (6) is satisfied, firms face soft budget constraints. Under soft budget constraints, firms with bad projects do not exert effort, and the bank refinances many bad projects. We call this situation that soft budget constraint problems prevail in this economy. In this case, the expected revenue of bank lending at $t = 0$ is $\alpha\pi_G + (1-\alpha)p{\pi_R}$[13]. On the other hand, when p is so small that (6) is not satisfied, firms face hard budget constraints. In this case, the expected revenue of bank lending at $t = 0$ is π_G. Obviously, the bank can obtain a higher revenue per project under hard budget constraints.

Now, we turn to characterizing the bank's lending behavior at $t = 0$. First, consider the amount of bank lending L_0 at $t = 0$. Similar to the procedure we used to derive L_1^N in (2), L_0 is given by:

$$L_0 = \int_{\hat{w}_0}^{1} (1 - w_0) f(w_0) dw_0 = (\Delta W_0)^2/2, \tag{7}$$

where $\hat{w}_0$ represents the initial wealth level, below which a firm cannot be financed at $t = 0$ and $\Delta W_0 = 1 - \hat{w}_0$. For simplicity, suppose that the loan account is the only asset and the deposit account is the only liability for the bank. Thus, $L_0 = D_0 + E_0$ is satisfied. In addition, assume a situation where the capital requirement is not binding at $t = 0$[14].

As the softness of the firms' budget constraints affects the expected payoff of bank lending, the amount that the bank lends at $t = 0$ differs depending on whether the firms' budget constraints are hard or soft. As is shown here, this difference is fully reflected in the bank's choice of the marginal firm ($\hat{w}_0$) financed at $t = 0$. First, consider the bank's lending behavior at $t = 0$ when (6) does not hold, so that firms face hard budget constraints. Under hard budget constraints, the firms exert effort at $t = 1^-$ and yield π_G at $t = 1, 2$. Then, the following lemma can be derived.

Lemma 2

Under hard budget constraints, the bank has an incentive to lend to a firm with $w_0 \in [\hat{w}_0^H, 1]$ *at* $t = 0$, *where* $\hat{w}_0^H \equiv 1 + m_0 - \frac{(1+r_1)\pi_G}{r_0 r_1}$. *In addition,* $\partial \hat{w}_0^H / \partial r_t > 0 (t = 0, 1)$.

Proof See Appendix 2.

[13] Recall that the liquidity value of the project is 0.

[14] This assumption is made to avoid too many cases. However, the following results hold even if the capital constraint is binding at $t = 0$.

Using ΔW_0^H to denote the number of firms financed at $t=0$ under hard budget constraints, $\Delta W_0^H = 1 - \hat{w}_0^H$. Lemma 2 suggests that, under hard budget constraints, the decrease in r_t induces the bank to lend to more firms. This result coincides with those of traditional monetary theory.

Next, consider the bank's lending behavior at $t=0$ when firms face soft budget constraints. Under soft budget constraints, the firms do not exert effort, and the bank refinances them. Then, the following lemma can be derived.

Lemma 3

Under soft budget constraints, the bank has an incentive to lend to a firm with $w_0 \in [\hat{w}_0^S, 1]$ *at* $t=0$, *where* $\hat{w}_0^S = 1 + m_0 - \frac{(1+r_1)\alpha\pi_G}{r_0 r_1} - \frac{(1-\alpha)p(\pi_R - r_1)}{r_0 r_1}$. *In addition, the sign of* $\partial\hat{w}_0^S/\partial r_t (t=0,1)$ *depends on the sign of* $\partial p/\partial r_t (t=0,1)$.

Proof See Appendix 3.

Using ΔW_0^S to denote the number of firms financed at $t=0$ under soft budget constraints, $\Delta W_0^S = 1 - \hat{w}_0^S$. In contrast to the previous case, the decrease in r_t does not always induce the bank to lend to more firms under soft budget constraints. That is, in some situations, decreasing the interest rate decreases the probability that firms with bad projects are refinanced at $t=1$ (i.e., p). This in turn reduces the profitability of bank lending and leads to a decrease in the amount of bank lending at $t=0$. In addition, from the definitions of $\hat{w}_0^H$ and $\hat{w}_0^S$, it is obvious that $\hat{w}_0^H < \hat{w}_0^S$. That is, the bank has an incentive to lend to more firms under hard budget constraints than under soft budget constraints because the profitability of bank lending is higher under hard budget constraints.

4 Bank lending, aggregate income, and monetary policy

This section explores how monetary policy is transmitted to economic agents and how it affects the amount of bank lending and aggregate income when soft budget constraint problems prevail in the economy. To investigate these problems, we focus on a situation where firms with bad projects face soft budget constraints at the equilibrium interest rate. That is, at $r_0 = r_1 = \hat{r}$, p satisfies (6). For simplicity, a change in an interest rate induced by monetary policy is considered to be exogenous.

Subsection 4.1 investigates the relation between monetary policy and bank lending and shows that an expansionary monetary policy does not always increase the amount of bank lending under a capital requirement. Subsection 4.2 investigates the relation between monetary policy and aggregate income and shows that the increase of bank lending induced by a monetary expansion is not effective when such lending is allocated to refinancing bad projects. Consequently, in order to stimulate economic activity by implementing monetary policy, it is essential to solve the soft budget constraint problems in the banking sector and to restore the efficiency of bank lending.

4.1 Monetary policy and bank lending

This subsection examines how monetary policy affects the amount of bank lending. For the present, we focus on the effect of a monetary expansion implemented at $t = 0$. The effect of monetary policy implemented at $t = 1$ will be investigated at the end of this subsection.

First, consider the relation between L_0 and r_0. As (7) indicates, to derive this we have only to check the relation between ΔW_0^S and r_0. In the light of Lemma 3, ΔW_0^S is $1 - \hat{w}_0^S$, which can be rewritten as:

$$\Delta W_0^S = \frac{(1 + r_1)\alpha\pi_G}{r_0 r_1} - m_0 + \frac{(1 - \alpha)(\pi_R - r_1)}{r_0 r_1} p. \tag{8}$$

A decrease in r_0 affects the size of ΔW_0^S in two ways. First, as the first term in (8) indicates,

ΔW_0^S becomes larger as r_0 becomes smaller because the smaller the opportunity cost of bank lending is, the more the bank lends at $t = 0$. Second, whether the third term representing the expected profitability of refinancing becomes larger is uncertain because, in the light of Lemma 3, the sign of the derivative of p with respect to r_0 is uncertain. Thus, if p becomes smaller in some situations at $t = 1$, the profitability of bank lending under soft budget constraints decreases, which induces the bank to lend less to firms at $t = 0$.

Next, consider the relation between L_1 and r_0. It is supposed that at an equilibrium interest rate, firms face soft budget constraints at $t = 1$[15]. Note, as Lemma 1 indicates, that the change in r_0 does not affect the amount of bank lending to new firms $L_1^N(\partial\hat{w}_1/\partial r_0 = 0)$. In other words, if the amount of bank lending at $t = 1$ is altered by a monetary expansion, it is fully reflected in the change of the amount for refinancing L_1^R. That is, $\partial L_1/\partial r_0 = \partial L_1^R/\partial r_0$ holds. In addition, given (1), $\partial L_1/\partial r_0 = 1/\gamma(\partial E_1/\partial r_0)$ holds.

Now, the bank capital E_1 at $t = 1$ is given as follows. Under hard budget constraints, E_1^H is represented by:

$$E_1^H = \pi_G \Delta W_0^H - r_0 D_0, \tag{9}$$

where ΔW_0^H represents the number of firms financed at $t = 0$ under hard budget constraints. On the other hand, under soft budget constraints, E_1^S is represented by:

$$E_1^S = \alpha\pi_G \Delta W_0^S - r_0 D_0. \tag{10}$$

In (9) and (10), the first terms denote retained earnings yielded from firms financed at $t = 0$, while the second terms denote the amount paid out to depositors. The first term differs between (9) and (10) because only firms with good projects yield π_G at $t = 1$ under soft budget constraints, whereas under hard budget constraints, firms with bad projects exert efforts and can yield π_G at $t = 1$. From these formulations, the size of E_1 is determined endogenously[16]. Thus, a monetary

[15] That is, $max\{\hat{w}_1, \hat{w}_1'\} = \hat{w}_1$ in (2), $L_1^R = L_1 - L_1^N$ in (3), and (6) is satisfied.

[16] The possibility that the bank issues its equity will be investigated in section 6.

expansion at $t = 0$ affects banking behavior at $t = 1$ by changing the size of E_1.

Concerning the relation between E_1^H and ΔW_0^H under hard budget constraints, we can derive that an increase in ΔW_0^H increases E_1^H. Concerning the relation between E_1 and ΔW_0^S under soft budget constraints, on the other hand, we can derive the following lemma.

Lemma 4

When the proportion of good firms (α) is small, an increase in the number of firms financed at $t = 0$ (ΔW_0^S) decreases the bank's equity capital at $t = 1$.

Proof See Appendix 4.

The increase in ΔW_0^S has two effects on E_1^S. First, the increase in ΔW_0^S increases retained earnings at $t = 1$ by increasing the credit to firms with good projects, which increases the first term in (10). Second, as the increase in ΔW_0^S is financed by issuing more deposit accounts at $t = 0$, it increases the amount paid out to depositors. This change increases the second term in (10). Therefore, if α is so small that the latter effect dominates the former one, an increase in ΔW_0^S decreases E_1^S.

Substituting (5), (7), and (10) into (1), L_1 can be rewritten as a function of ΔW_0^S, as follows:

$$L_1 = \frac{1}{\gamma}\{(\alpha\pi_G - r_0 m_0)\Delta W_0^S - \frac{r_0(\Delta W_0^S)^2}{2} + r_0\bar{E}\}. \tag{11}$$

Here, D_1^R denotes the demand for refinancing and L_1^R denotes the supply of refinancing. Then, we obtain:

$$D_1^R = (1-\alpha)\Delta W_0^S \tag{12}$$

$$L_1^R = \frac{1}{\gamma}\{(\alpha\pi_G - r_0 m_0)\Delta W_0^S - \frac{r_0(\Delta W_0^S)^2}{2} + r_0\bar{E}\} - L_1^N. \tag{13}$$

As a firm with a bad project requires one unit of capital to be refinanced, the right-hand side (RHS) of (12) represents the number of firms with bad projects at $t = 1$. The RHS in (13) represents the amount allocated to refinancing bad projects $L_1 - L_1^N$ as a function of ΔW_0^S.

Using (12) and (13), we can derive the relation between monetary policy and bank lending under soft budget constraints.

Proposition 1

Suppose there is a situation where firms face soft budget constraints at the equilibrium interest rate $r_0 = r_1 = \hat{r}$*. In this situation, the relation between a monetary expansion and bank lending at* $t = 0, 1$ *is classified as follows.*

(a). When α *is small, a monetary expansion increases the amount of bank lending at* $t = 0$ *but decreases the amount of bank lending at* $t = 1$ *(*$\partial L_0/\partial r_t < 0$*,* $\partial L_1/\partial r_t > 0$*). In addition, a monetary expansion alleviates the soft budget constraint problems (*$\partial p/\partial r_t > 0$*).*

(b). When α *is large, a monetary expansion increases the amount of bank lending at both* $t = 0$ *and* $t = 1$ *(*$\partial L_0/\partial r_t < 0$*,* $\partial L_1/\partial r_t < 0$*). In addition, a monetary expansion worsens the soft budget constraint problems (*$\partial p/\partial r_t < 0$*).*

Proof See Appendix 5.

In Figure 2, $D_1^R = D_1^R(\Delta W_0^S)$ represents the firms' demand function for refinancing, and $L_1^R = L_1^R(\Delta W_0^S)$ represents the bank's supply function for refinancing. Figure 2(a) depicts the case where α is so small that Lemma 4 is satisfied. Hereafter, this case is referred to as case (a). Figure 2(b) depicts the case where α is so large that Lemma 4 is not satisfied. Hereafter, this is referred to as case (b).

In Figures 2(a) and 2(b), D_1^R is increasing in ΔW_0^S. This is because the more firms that are financed at $t = 0$, the more projects become bad and, thus, the greater is the demand for funds for refinancing at $t = 1$. On the other hand, L_1^R is decreasing in ΔW_0^S in Figure 2(a) because, for low values of α, the increase in ΔW_0^S decreases E_1 in the light of Lemma 4 and thus decreases L_1^R. In contrast, L_1^R is increasing in ΔW_0^S in Figure 2(b) because, for high values of α, the increase in ΔW_0^S increases E_1 and L_1^R.

Now, we consider the effect of a monetary expansion at $t = 0$ and explain the rational behind Proposition 1. First, a decrease in r_0 shifts D_1^R rightward. This is because it lowers the opportunity cost of bank credit and induces the bank to lend to more firms at $t = 0$, which leads to an increase in the number of firms with bad projects at $t = 1$. On the other hand, a decrease in r_0 shifts L_1^R upward. This is because it lowers repayments to depositors, which increases E_1 in (10) and thus increases the supply of funds for bad projects.

As is shown in Figure 2(a)(b), the effect of monetary expansion varies with respect to the value of α.

First, consider case (a). Note that a monetary expansion affects ΔW_0^S through two channels. First, as we have already noted, a monetary expansion at $t = 0$ shifts the D_1^R curve rightward and L_1^R upward. These shifts lead to an increase in ΔW_0^S. Movement 1 in Figure 2(a) represents this first channel. As a result, the equilibrium in Figure 2(a) shifts from E_a to $E_a^{'}$. Second, this shift in ΔW_0^S further affects the size of L_1^R and D_1^R through the change in p. Obviously, an increase in ΔW_0^S further increases D_1^R. On the other hand, the increase in ΔW_0^S decreases L_1^R because, for low values of α, it decreases E_1 in the light of Lemma 4 and thus decreases L_1. As these shifts in D_1^R and L_1^R increase the denominator but decrease the numerator in (4), p decreases. In turn, a decrease in p decreases ΔW_0^S because, as (8) shows, a decrease in p lowers the profitability of bank lending at $t = 0$ under soft budget constraints. This is the second channel. Movement 2 in Figure 2(a) represents this channel. Then, the equilibrium in Figure 2(a) shifts from $E_a^{'}$ to $E_a^{''}$. In terms of results, if the first channel is dominant, a monetary expansion increases ΔW_0^S. This leads to an increase in L_0 and a decrease in L_1. Moreover, as p becomes smaller in this case, a monetary expansion alleviates the soft budget constraint problems. On the other hand, if the second channel is dominant, it decreases ΔW_0^S. However, in order for the second channel to be dominant, α should be sufficiently small. As this situation is considered to be unrealistic, we omit this situation in the following arguments.

Next, consider case (b). In this case, α is so large that L_1^R is an upward sloping function of ΔW_0^S. Here, the first channel operates as it did in case (a). That is, a decrease in r_0 increases ΔW_0^S. In terms of the second channel, on the other hand, the increase in ΔW_0^S increases E_1, which leads to an increase in L_1^R. As both D_1^R and L_1^R increase in this case, the effect on p is ambiguous. However, when α is large or γ is small, an increase in ΔW_0^S increases E_1 and L_1 sufficiently. Therefore, under these parameters, a monetary expansion at $t = 0$ increases both L_0 and L_1[17]. Moreover, under these parameters, L_1^R increases more than D_1^R, which leads to an increase in p. Therefore, a monetary expansion worsens the soft budget constraint problems in this case.

The effect of a monetary expansion at $t = 1$ is very similar to that of a monetary expansion at $t = 0$ except that the upward shift of L_1^R does not occur. This is because, in (10), a decrease in r_1 does not affect the amount paid out to depositors at $t = 1$. Therefore, when r_1 decreases, the D_1^R curve shifts rightward, but the L_1^R curve is stable. As these shifts in D_1^R and L_1^R lead to an increase in ΔW_0^S, with the other effects being the same as in the previous case, a decrease in r_1 affects L_0 and L_1 through the first and second channels.

Proposition 1(a) states that, under a capital requirement, a monetary expansion does not always increase the amount of bank lending when the proportion of good firms (α) is small. In addition, Proposition 1(a) states that a monetary expansion decreases p in case (a) but increases p in case (b). In other words, a monetary expansion hardens the firms' budget constraints in case (a), but softens them in case (b). This difference arises because a monetary expansion decreases the bank's capital base at $t = 1$ in case (a). Thus, the amount allocated to refinancing bad projects decreases in case (a), but increases in case (b). This difference is important in

[17] We can consider the situation where α is sufficiently large or γ is sufficiently small so that the slope of L_1^R is steeper than that of D_1^R. However, the equilibrium in this case is not stable. Under this situation, we cannot derive meaningful results even if we examine the comparative statics. Therefore, this case is omitted.

examining the relation between monetary policy and aggregate income, discussed in the next subsection.

4.2 Monetary policy and aggregate income

This subsection investigates the relation between monetary policy and aggregate income and shows that an increase of bank credit induced by a monetary expansion does not always enhance economic activity efficiently. In deriving this result, the soft budget constraint problems play a key role. As discussed in subsection 4.1, we can focus on the effect of monetary expansion at $t = 0$ without loss of generality.

To begin with, the aggregate income in this economy should be defined. Recall that the verifiable returns of firms are counted as income in this economy, but the private benefits of firms are not. Therefore, the aggregate income is defined by the sum of firms' verifiable returns π_i.

Obviously, the aggregate income is affected by whether the soft budget constraint problems prevail in this economy. First, we calculate the aggregate income when firms face hard budget constraints. In this case, firms with good projects yield π_G at $t = 1, 2$, and firms with bad projects exert effort at $t = 1^-$ and also attain π_G at $t = 1, 2$. Therefore, the aggregate income at $t = 0$ and $t = 1$ is represented by:

$$Y_0^H = (1 + r_1)\pi_G \Delta W_0^H \tag{14}$$

$$Y_1^H = \beta \pi_G \Delta W_1^H. \tag{15}$$

Y_0^H represents the sum of verifiable returns yielded by firms financed at $t = 0$. Y_0^H is given by (14) because all the firms financed at $t = 0$ yield π_G at both periods under hard budget constraints. On the other hand, Y_1^H represents the sum of verifiable returns yielded by new firms financed at $t = 1$. Y_1^H is represented by (15) because, under hard budget constraints, the bank lends only to new firms yielding π_G with probability β. $Y^H = Y_0^H + Y_1^H$ denotes the sum of aggregate income at $t = 0$ and $t = 1$ when firms face hard budget constraints.

In terms of the relation between r_0 and Y_0^H, as Lemma 2 indicates, a decrease in r_0 increases the amount of bank lending at $t = 0$ (i.e., ΔW_0^H), which leads to an increase in aggregate income Y_0^H ($\partial Y_0^H/\partial r_0 < 0$). Next, the relation between r_0 and Y_1^H can be derived as follows. Under hard budget constraints, the bank capital at $t = 1$ is given by (9). In addition, as the bank does not refinance bad projects, $max\{\hat{w}_1, \hat{w}_1^{'}\} = \hat{w}_1^{'}$ in (2). Thus, we can derive $\partial\hat{w}_1^{'}/\partial r_0 > 0$ because a decrease in r_0 increases ΔW_0^H and decreases the repayment to depositors, which increases the bank's capital base at $t = 1$ and thus increases the amount of bank lending at $t = 1$. Therefore, a decrease in r_0 increases the amount of bank lending at $t = 1$, which immediately increases the aggregate income Y_1^H ($\partial Y_1^H/\partial r_0 < 0$). In sum, a decrease in r_0 increases the sum of aggregate income at $t = 0$ and $t = 1$ ($\partial Y^H/\partial r_0 < 0$).

On the other hand, when firms face soft budget constraints, firms with bad projects exert no effort at $t = 1$. Therefore, the aggregate income at $t = 0$ and $t = 1$ is represented by:

$$Y_0^S = (1 + r_1)\alpha\pi_G\Delta W_0^S$$

$$Y_1^S = \beta\pi_G\Delta W_1^S + \pi_R\Delta W_1^R. \tag{16}$$

Unlike Y_0^H, Y_0^S is represented by the sum of verifiable returns yielded only by firms with good projects. In addition, unlike Y_1^H, Y_1^S includes returns yielded by firms with bad projects refinanced at $t = 1$ (the second term in (16)). $Y^S = Y_0^S + Y_1^S$ denotes the sum of aggregate income at $t = 0$ and $t = 1$ when firms face soft budget constraints. In terms of the relation between r_0 and Y^S, as Lemma 3 and Proposition 1 indicate, the sign of $\partial Y^S/\partial r_0$ is ambiguous.

In order to derive the relation between aggregate income and the interest rate under soft budget constraint, suppose that firms face soft budget constraints at the equilibrium interest rate $r_0 = r_1 = \hat{r}$. Then, we can derive the following proposition.

Proposition 2

Suppose firms face soft budget constraints at the equilibrium interest rate $r_0 = r_1 = \hat{r}$. *Then,*

(a). when α *is small, there exists* $\tilde{r}_a$ *such that the aggregate income is* Y_H *when* $1 \leq r_0 \leq \tilde{r}_a$ *and* Y_S *when* $\tilde{r}_a < r_0$. *In addition,* $\partial Y^H / \partial r_0 < 0$ *when* $1 \leq r \leq \tilde{r}_a$ *and* $\partial Y^S / \partial r_0$ *is ambiguous when* $\tilde{r}_a < r_0$.

(b). when α *is large, there exists* $\tilde{r}_b$ *such that the aggregate income is* Y_S *when* $1 \leq r_0 \leq \tilde{r}_b$ *and* Y_H *when* $\tilde{r}_b < r_0$. *In addition,* $\partial Y^S / \partial r_0 < 0$ *when* $1 \leq r_0 \leq \tilde{r}_b$ *and* $\partial Y^H / \partial r_0 < 0$ *when* $\tilde{r}_b < r_0$.

Proof See Appendix 6.

The rational behind Proposition 2 can be clarified as follows. Note that it is assumed that firms face soft budget constraints at the equilibrium interest rate $r_0 = r_1 = \hat{r}$. Then, the aggregate income Y at $r = \hat{r}$ equals $Y^S(\hat{r})$. The effect of monetary policy on aggregate income is illustrated in Figures 3(a) and 3(b) below.

Figure 3(a) depicts the relation between Y and r_0 when α is small. Proposition 1(a) suggests that a decrease in r_0 increases L_0 but decreases L_1 under soft budget constraints. Thus, a decrease in r_0 increases the aggregate income Y^S if an increase in Y_0^S outweighs a decrease in Y_1^S. In addition, Proposition 1(a) suggests that a decrease in r_0 hardens the budget constraints of firms ($\partial p / \partial r_0 > 0$). Thus, if r_0 is decreased further, firms will face hard budget constraints at some point. Figure 3(a) shows that Y jumps from Y^S to Y^H at $\tilde{r}_a$. Therefore, in this case, by decreasing r_0 sufficiently, the government can solve the soft budget constraints problems and increase aggregate income.

Figure 3(b) depicts the relation between Y and r_0 when α is large. Proposition 1(b) suggests that both L_0 and L_1 increase as r_0 decreases. Thus, a decrease in r_0 increases the aggregate income ($\partial Y^S / \partial r_0 < 0$). However, in this case, as the slope of Y^S in Figure 3(b) is gentle, a monetary expansion cannot increase the aggregate income sufficiently. This is because, under soft budget constraints, the increased amount of bank lending at $t = 1$ is allocated to refinancing old unprofitable projects, with the result that Y_1^S does not increase sufficiently. On the other hand, in order to harden the budget constraints of firms, r_0 should be increased in case (b). Thus, Y jumps from Y^S to Y^H at $\tilde{r}_b$. Therefore, when α is large, the government may be able to achieve a higher aggregate income by increasing r_0 to $\tilde{r}_b$ to harden the budget constraints of firms than it can achieve by decreasing r_0 to one.

Having characterized the relation between the interest rate and aggregate income in Proposition 2, next we consider whether monetary policy is an effective tool for stimulating economic activity when firms face soft budget constraints at the equilibrium interest rate $r_0 = r_1 = \hat{r}$. Suppose that there exists a desirable level of aggregate income Y^*[18]. Under these settings, the problem here is to examine whether the government, starting from $\hat{Y}^S$, can achieve Y^* by implementing monetary policy. We consider two cases separately.

First, consider case (a). As a decrease in r_0 hardens the budget constraints in this case, aggregate income jumps from $Y^S(\tilde{r}_a)$ to $Y^H(\tilde{r}_a)$ at $\tilde{r}_a$. In this case, as Figure 3(a) shows, Y^* can be achieved if r_0 is decreased further to r^*.

Next, consider case (b). A decrease in r_0 increases aggregate income in this case. However, as is shown in Figure 4(b), the maximum aggregate income that can be achieved by a monetary expansion is $Y^S(1)$, which is less than Y^*. Therefore, Y^* cannot be achieved by a monetary expansion. On the other hand, if r_0 is increased to $\tilde{r}_b$, aggregate income jumps from $Y^S(\tilde{r}_b)$ to $Y^H(\tilde{r}_b)$. However, again, $Y^H(\tilde{r}_b)$ is less than Y^*. Therefore, in case (b), neither monetary expansion nor monetary tightening can achieve Y^*.

Proposition 2(b) shows that in the presence of the soft budget constraint problems, even

[18] Although it is important to consider how Y^* is determined in this economy, addressing this issue is beyond the scope of this paper. Therefore, it is assumed that Y^* is given exogenously.

if a monetary expansion increases the volume of bank lending, it does not stimulate economic activity sufficiently. Therefore, as Proposition 2(a) suggests, from the point of view of increasing aggregate income, solving the soft budget constraint problems is essential for monetary policy to operate efficiently. In other words, in order to stimulate economic activity, improving the *quality* of bank lending is more crucial than simply increasing the *volume* of bank lending. These results imply that solving the bank's moral hazard problem is important not only in the banking sector but also in the macroeconomy.

5 Bank monitoring at $t = 1$

The previous section showed that to stimulate the economy effectively, the soft budget constraint problems in the banking sector should be solved. However, under the framework developed thus far, solving these problems requires the bank to *decrease* the amount of bank capital at $t = 1$. This is because the smaller E_1 is, the smaller p will be. Obviously, this result is not desirable from the viewpoint of the prudential regulation of banks. To fill this gap, this section introduces the bank's incentive to monitor new firms at $t = 1$, and then shows that the soft budget constraint problems are solved not only in the case where bank equity at $t = 1$ is sufficiently small, but also where it is sufficiently large.

Suppose that monitoring new firms at $t = 1$ (which costs $m_1 > 0$ per project) improves the proportion of good projects from β (without monitoring) to $\beta^{'}$ (with monitoring). $\beta^{'} - \beta$ is denoted by $\Delta\beta$. The bank's choice as to whether it will monitor new firms is assumed to be observable but unverifiable. Under these settings, the bank decides two things at $t = 1$: how to allocate its funds, and whether to monitor new firms.

Before examining how the introduction of monitoring at $t = 1$ influences the effect of monetary policy examined in the previous section, we should describe the amount of bank capital, the amount of bank lending, and the aggregate income in the case where the bank decides to monitor new firms at $t = 1$. In addition, we will describe the expected profits of the bank. First, consider the amount of bank equity at $t = 1$. Suppose the monitoring cost is paid out from retained earnings at $t = 1$. Then, when firms face hard budget constraints and the bank monitors new firms at $t = 1$, the bank capital at $t = 1$ E_1^{HM} is given by:

$$E_1^{HM} = \pi_G \Delta W_0^M - r_0 D_0 - m_1 \Delta W_1^M, \tag{17}$$

where W_t^M represents the number of firms financed at $t = 0, 1$ if the bank monitors new firms at $t = 1$. The third term in (17) denotes the total monitoring cost at $t = 1$. Similarly, when firms face soft budget constraints and the bank monitors new firms at $t = 1$, the bank capital at $t = 1$ E_1^{SM} is given by:

$$E_1^{SM} = \alpha \pi_G \Delta W_0^M - r_0 D_0 - m_1 \Delta W_1^M. \tag{18}$$

As in (1), the amount of bank lending at $t = 1$ is given by:

$$L_1^{iM} = E_1^{iM} / \gamma, \tag{19}$$

where $i = S, H$. Comparing (17) and (18) with (9) and (10), $E_1^{iM} < E_1^i$ if m_1 is high. If this inequality holds, immediately $L_1^{iM} < L_1^i$. That is, when the monitoring cost is high, the amount of bank lending is smaller if the bank monitors at $t = 1$ than if the bank does not monitor at $t = 1$.

Now, we proceed to examine the bank's problem about how to allocate its funds at $t = 1$. In terms of this issue, we can derive the following lemma.

Lemma 5

If the bank monitors new firms at $t = 1$, it prefers to lend to a new firm with $w_1 \in [\hat{w}_1^M, 1]$ ($\hat{w}_1^M \equiv 1 + m_1 - \beta' \pi_G / \pi_R$) rather than to refinance a firm with a bad project. In addition, $\partial \hat{w}_1^M / \partial r_t = 0 (t = 0, 1)$.

Proof Very similar to the proof of Lemma 1.

Suppose $\Delta\beta\pi_G/\pi_R - m_1 > 0$ is large enough, $\hat{w}_1^M < \hat{w}_1$ holds. That is, if monitoring at $t = 1$ generates a positive net present value, the bank has an incentive to lend to more new firms if it monitors at $t = 1$ than if it does not. This is because monitoring improves the expected profitability of new lending. Here, to make the argument simple, it is assumed that although m_1 is large enough, $\Delta\beta\pi_G/\pi_R - m_1 > 0$ holds. These assumptions mean that although monitoring is very costly, it improves the profitability of new lending sufficiently.

Under these settings, the bank is more likely to lend to new firms at $t = 1$. However, as the relation between (17), (18), and (19) shows, monitoring at $t = 1$ decreases the amount of bank capital, which induces the bank to lend less to new firms at $t = 1$. This means that the bank faces a trade-off in terms of monitoring at $t = 1$. Figure 4 illustrates this incentive of the bank. In Figure 4, $\hat{w}_1^M$ is located on the left of $\hat{w}_1$, which means that the bank has an incentive to lend more to new firms if it monitors at $t = 1$ than if it does not monitor. However, as E_1^M becomes smaller because of monitoring cost payments, L_1^M becomes smaller if the bank monitors at $t = 1$. These two points suggest that if the bank monitors at $t = 1$, firms with bad projects are less likely to be refinanced and, thus, hard budget constraints will prevail in this economy.

Next, consider aggregate income if the bank monitors at $t = 1$. As described above, monitoring may harden the budget constraints of firms. For the later explanation, we focus on the case where firms face hard budget constraints if the bank monitors at $t = 1$. Under hard budget constraints, all firms financed at $t = 0$ yield π_G at $t = 1, 2$. Thus, aggregate income is written as:

$$Y_0^M = \Delta W_0^M (1 + r_1)\pi_G,$$

where $\Delta W_0^M = \Delta W_0^H = 1 - \hat{w}_0^H$. In addition, as the bank does not refinance firms with bad projects at $t = 1$, aggregate income at $t = 1$ is written as:

$$Y_1^M = \Delta W_1^M \beta' \pi_G,$$

where $\Delta W_1^M = 1 - \hat{w}_1^{M'}$ and $\hat{w}_1^{M'}$ satisfies:

$$\int_{\hat{w}_1^{M'}}^1 (1 - w_1) f(w_1) dw_1 = L_1^M.$$

$Y^M = Y_0^M + Y_1^M$ denotes the sum of aggregate income at $t = 0$ and $t = 1$ if the bank monitors new firms at $t = 1$.

Now, in order to investigate the bank's incentive to monitor at $t = 1$, we compare the expected profit from bank lending when the bank monitors at $t = 1$ with this profit when the bank does not monitor. In addition, we focus on the situation where firms face hard budget constraints if the bank monitors at $t = 1$.

Under these settings, the bank lends only to new firms at $t = 1$ if the bank monitors at $t = 1$. Then, the expected profit from bank lending at $t = 1$ $E\pi_1^M$ can be written as:

$$E\pi_1^M = \int_{\hat{w}_1^{M'}}^1 \beta^{'} \pi_G f(w_1) dw_1 - r_1 D_1^M - r_1 E_1^{HM}.$$

The first, second, and third terms denote the expected profits from new lending, the repayments to depositors, and the opportunity cost of the bank, respectively. Similarly, if the bank monitors at $t = 1$, the expected profits from bank lending at $t = 0$ $E\pi_0^M$ are given by:

$$E\pi_0^M = \left(\int_{\hat{w}_0^H}^1 \pi_G f(w_0) dw_0 - r_0 D_0 - m_1 \Delta W_1^M \right) r_1 + \int_{\hat{w}_0^H}^1 \pi_G f(w_0) dw_0 - r_0 r_1 \bar{E}.$$

The first and second terms denote the returns yielded by firms at $t = 1, 2$. Note that, using (17), the first term can be rewritten as $r_1 E_1^{HM}$. Using $E\pi^M$ to denote the sum of the bank's expected profit if it monitors at $t = 1$, we have $E\pi^M = E\pi_0^M + E\pi_1^M$.

On the other hand, the expected profit of the bank if it does *not* monitor at $t = 1$ is computed as follows. For the later explanation, we focus on the situation where the soft budget constraint problems prevail if the bank does not monitor at $t = 1$. Under these circumstances, the expected payoff to bank lending at $t = 1$ $E\pi_1^S$ becomes:

$$E\pi_1^S = \int_{\hat{w}_1}^1 \beta \pi_G f(w_1) dw_1 + \pi_R L_1^R - r_1 D_1 - r_1 E_1^S.$$

$E\pi_1^S$ differs from $E\pi_1^M$ because of the inclusion of the second term, which denotes the profit yielded by refinancing bad projects. Furthermore, the expected payoff to bank lending at $t = 0$ $E\pi_0^S$ becomes:

$$E\pi_0^S = \left(\int_{\hat{w}_0^S}^1 \alpha\pi_G f(w_0)dw_0 - r_0 D_0 \right) r_1 + \int_{\hat{w}_0^S}^1 \alpha\pi_G f(w_0)dw_0 \\ + \Delta W_0^S(1-\alpha)p\pi_R - r_0 r_1 \bar{E} - (1-\alpha)\Delta W_0^S p r_1.$$

The third term denotes the expected profit yielded by refinancing bad projects and the fifth term denotes the opportunity cost of refinancing. As in the former case, the first term can be rewritten as $r_1 E_1^S$. Using $E\pi^S$ to denote the sum of the bank's expected payoff if it does not monitor at $t = 1$, we have $E\pi^S = E\pi_0^S + E\pi_1^S - \pi_R L_1^R$[19].

Having described the amount of bank capital, the amount of bank lending at $t = 1$, and the expected profits of the bank, we are now in a position to consider what monetary policy should be implemented in a situation where the bank can monitor new firms at $t = 1$. The analysis in the previous section shows that monetary policy does not always solve the soft budget constraint problems. On the other hand, in a situation where the bank has an option to monitor new firms at $t = 1$, these problems can be solved if the bank is induced to monitor new firms and to harden the budget constraints of firms.

Concerning the method of solving the soft budget constraint problems in this context, we can derive the following proposition.

Proposition 3

Suppose that monitoring new firms at $t = 1$ is costly but that it improves the profitability of new lending at $t = 1$(i.e., $\Delta\beta\pi_G/\pi_R - m_1 > 0$). Moreover, $E\pi^S > E\pi^M$ holds at $r_0 = r_1 = \hat{r}$ so that firms face soft budget constraints. Under these settings, a monetary expansion and a capital injection can induce the bank to monitor at $t = 1$, which then solves the soft budget constraints

[19] $\pi_R L_1^R$ is subtracted to avoid double counting.

problems.

To investigate the rational behind Proposition 3, suppose that $E\pi^S(\hat{r}) > E\pi^M(\hat{r})$ is satisfied at the equilibrium interest rate $r_0 = r_1 = \hat{r}$. That is, the cost of monitoring (a decrease in the amount of lending at $t = 1$) outweighs its benefits (an improvement in profits yielded from new firms) so that the bank does not have an incentive to monitor at $t = 1$. In order to solve the soft budget constraint problems, a policy should be implemented so that $E\pi^S(r) < E\pi^M(r)$ holds.

First, consider the effect of a monetary expansion. The effect of monetary expansion if the bank does not monitor at $t = 1$ has been analyzed in the former section. Therefore, we now consider the case where the bank monitors at $t = 1$. Note that if the bank monitors new firms at $t = 1$, it allocates all funds to new firms at $t = 1$. In other words, as Figure 4 shows, hard budget constraints prevail in this case. Thus, in the light of Lemma 2, a decrease in r_0 increases L_0. As E_1^{HM} is given by (17), an increase in L_0 (i.e., ΔW_0^M) increases E_1^{HM}, thereby increasing L_1^M. Under hard budget constraints, an increase in L_1^M leads the bank to finance more new firms at $t = 1$. This means that $\hat{w}_1^{M'}$ in Figure 4 moves to the left. Thus, a monetary expansion increases $E\pi^M$ ($\partial E\pi^M/\partial r_t < 0$). Note that a monetary expansion increases $E\pi^M$ regardless of the level of α. In contrast, as the arguments in the former section suggest, whether a monetary expansion increases $E\pi^S$ depends on the level of α. Moreover, if β' is large enough, $|\partial E\pi^M/\partial r_t| > |\partial E\pi^S/\partial r_t|$ is satisfied. This means that even if a monetary expansion increases $E\pi^S$, it increases $E\pi^M$ more than $E\pi^S$. This is because if the bank monitors new firms, an increased amount of bank lending at $t = 1$ is allocated to new profitable projects. By contrast, if the bank does not monitor, an increased amount of bank lending at $t = 1$ is allocated to old unprofitable projects. Therefore, if monetary policy is expanded sufficiently, $E\pi^M > E\pi^S$ can be achieved, and then the soft budget constraint problems can be solved.

In solving the soft budget constraint problems through monetary expansion, increasing E_1^{HM} plays a crucial role. Considering this point, we can have another policy instrument: a capital injection. Recall that the bank does not have an incentive to monitor at $t = 1$ because the monitoring cost m_1 is so high that the amount of bank lending at $t = 1$ should be decreased under a capital requirement. This suggests that if capital is injected and if the bank can hold sufficient capital at $t = 1$, it can lend to many new firms even if it monitors them at $t = 1$[20]. That is, in Figure 4, a capital injection shifts $\hat{w}_1^{M'}$ to the left, which increases L_1^M, thereby increasing $E\pi^M$. Therefore, if a sufficient amount of capital is injected, $E\pi^M > E\pi^S$ is obtained, and thus the soft budget constraint problems can be solved.

The effect of a monetary expansion and a capital injection is depicted in Figures 5.1 and 5.2. At the equilibrium interest rate $r = \hat{r}$, $E\pi^S > E\pi^M$ is satisfied. In case of a monetary expansion, $E\pi^M > E\pi^S$ can be satisfied by lowering the interest rate from $\hat{r}$ to $r^{'}$. On the other hand, a capital injection causes $E\pi^M$ to shift upward while $\hat{r}$ remains unchanged. Thus, the government should inject capital into the bank until $E\pi^M > E\pi^S$ is satisfied.

The arguments in this section show that the soft budget constraint problems can be solved by either a monetary expansion or a capital injection. However, as is shown in Figure 5.2, a capital injection is more favorable than a monetary expansion if $\hat{r}$ is located near one. This is because, in this case, r cannot be reduced sufficiently, and thus $E\pi^M > E\pi^S$ is not satisfied. Therefore, when the equilibrium interest rate is too low, a capital injection is more desirable.

Finally, we consider how to achieve the desired aggregate income Y^* defined in subsection 4.2. The answer is obvious. Initially, the government solves the soft budget constraint problems either by a monetary expansion or a capital injection, with the result that aggregate income shifts from Y^S to Y^H. Subsequently, the government has only to lower r until $Y^M(r) = Y^*$ is satisfied.

These arguments provide important policy implications for the Japanese economy in the recent recession. After the collapse of stock and asset bubbles, Japanese banks suffered from bad loan problems. At the same time, the progress of information technology drastically altered the

[20]For simplicity, as in Berglof and Roland (1997), capital is injected into the bank as a subsidy.

industrial structure of the economy, which impelled banks to design a new monitoring scheme. Such evidence implies that the level of α is low and the level of m_1 is high in the context of this paper. In addition, in order to stimulate the economy, the equilibrium interest rate was kept at a very low level. The argument in this section suggests that, under these parameters, a capital injection is preferable to a monetary expansion because only capital injection can improve the efficiency of bank lending and solve the soft budget constraint problems. Therefore, injecting capital into the bank is justified if it provides the bank with sufficient capital and contributes to restoring the efficiency of bank lending.

6 Issuing bank equity

So far, we have assumed that bank equity at $t = 1$ reflects only retained earnings. In this section, the bank is allowed to issue its equity at $t = 1$. Then, we show that even if the bank issues its equity in the capital market, the bank does not monitor new firms at $t = 1$. As the soft budget constraint problems are not solved by issuing bank equity, it is shown that a capital injection remains effective.

For simplicity, the bank owns 100% of its capital at $t = 0$[21]. If the bank issues equity, it owns a proportion $1 - \theta (0 < \theta < 1)$ of its capital. In other words, a proportion θ is owned by outside investors. Obviously, the amount the bank raises by issuing equity depends on the market value of the bank, which, as discussed in section 5, is affected by the bank's decision to monitor new firms at $t = 1$.

First, consider how much capital the bank issues if it does not monitor new firms at $t = 1$. Z is used to denote the amount the bank raises by issuing equity, and MV denotes the market

[21] This assumption represents the bank manager's intention to maximize the expected profit of existing shareholders.

value of the bank. Considering the market clearing condition, the following equation is satisfied:

$$Z = \frac{\theta MV}{r_1}. \tag{20}$$

The bank's problem is to choose the optimal θ to maximize $(1-\theta)MV$ subject to (20).

Next, consider the same problem faced by the bank if it monitors new firms at $t = 1$. Z^M, θ^M, and MV^M denote the amount the bank raises by issuing equity, the proportion of bank capital owned by outside investors, and the market value of the bank, respectively. Then, the following equation is satisfied:

$$Z^M = \frac{\theta^M MV^M}{r_1}. \tag{21}$$

As before, the bank's problem is to choose the optimal θ^M to maximize $(1-\theta^M)MV^M$ subject to (21).

Comparing the solution of these maximization problems, we can derive the following proposition.

Proposition 4

If monitoring costs at $t = 1$ (m_1) are large, the bank issues less new equity if it monitors new firms at $t = 1$ than if it does not ($Z^M < Z$).

Proof See Appendix 6.

Proposition 4 indicates that if m_1 is sufficiently large, the bank does not have an incentive to issue enough capital at $t = 1$. Consequently, the bank does not monitor new firms at $t = 1$, and thus the soft budget constraint problems prevail in the economy. The intuition behind this can be explained as follows. In order for the bank to monitor new firms at $t = 1$, it is necessary to increase the amount of bank lending to these firms sufficiently. In other words, $\hat{w}_1^{M'}$ in Figure 4 should be shifted leftward sufficiently. In order to do so, the bank should raise a large amount of capital in the market under a capital requirement. However, if this issuance were carried out, a proportion the bank owns its capital would be so small that the expected payoff of the bank be sufficiently diluted. Anticipating this, the bank does not have an incentive to issue enough capital and thus to monitor the firms. Instead, even if it issues capital, it prefers to refinance old unprofitable projects[22]. As the soft budget constraint problems prevail in this case, a capital injection remains effective.

7 Conclusion

This paper has analyzed the effect of monetary policy in a situation where soft budget constraint problems prevail in the economy and the bank's asset–liability management is constrained by a capital requirement. We have derived several conclusions. First, under a capital requirement, the effect of monetary policy is seriously affected by the proportion of good firms in the economy. That is, if the proportion of good firms is small, an increase in bank lending induced by a monetary expansion today does not increase the amount of bank capital tomorrow, which in turn does not increase the amount of bank lending tomorrow. Next, a monetary expansion will not be effective in stimulating the economy even if it increases the amount of bank lending. This is because the increased amount of bank lending is allocated to refinancing old unprofitable projects under soft budget constraints. Third, in a situation where the bank is allowed to monitor new projects, a capital injection is more effective than a monetary expansion in resolving the misallocation of bank lending and stimulating the economy when the equilibrium interest rate is too low. Finally, a capital injection remains effective even if the bank is allowed to issue new equity. This is because solving the soft budget constraint problems forces the bank to raise a large amount of funds by issuing new equity, which significantly dilutes the expected payoff of the bank.

[22] Note that the funds raised by issuing new equity are allocated to refinancing old projects. Hosono and Sakuragawa (2003) showed that, in the 1990s, Japanese banks allocated funds to troubled firms such as real estate and construction firms by issuing subordinated debt. The mechanism in this model confirms this behavior.

These conclusions suggest that the misallocation of bank lending because of the soft budget constraint problems seriously damages not only the banking sector but also the macroeconomy, and that improving the *quality* of bank lending is more important than simply increasing the *volume* of it. These arguments will provide important viewpoints not only on how moral hazard problems in the banking sector damage the macroeconomy but also on how macroeconomic policy (i.e., monetary policy) and microeconomic policy (i.e., capital injections) should be related.

An interesting topic for future research is to extend the model to a multiperiod setting in order to explore the dynamics of bank capital. Another extension is to introduce the money market so that the interest rate is determined endogenously, which would enable solutions for deflation to be analyzed. These extensions should be investigated to provide a more systematic analysis of the operation of monetary policy.

Appendix

Appendix 1: Proof of Lemma 1

Consider the bank's lending behavior at $t = 1$. The bank has two options at $t = 1$: to finance new projects or to refinance old projects. A new project requires the amount $1 - w_1$ to be financed, and its verifiable expected return is $\beta\pi_G$. On the other hand, refinancing an old project requires the amount 1 to be financed, and its verifiable expected return is π_R. Thus, the bank prefers financing new projects if:

$$\frac{\beta\pi_G}{1-w_1} \geq \pi_R$$
$$w_1 \geq 1 - \frac{\beta\pi_G}{\pi_R} \equiv \hat{w}_1.$$

The left-hand side (LHS) of the first inequality denotes the marginal expected return of financing a new project. In addition, it is obvious that $\partial\hat{w}_1/\partial r_t = 0 (t = 0, 1)$ (QED).

Appendix 2: Proof of Lemma 2

Under hard budget constraints, all the firms financed at $t = 0$ yield π_G at $t = 1$ and $t = 2$. Thus, at $t = 0$, the bank lends to a firm, satisfying:

$$(1 + r_1)\pi_G \geq (1 - w_0 + m_0)r_0 r_1$$
$$w_0 \geq 1 + m_0 - \frac{(1 + r_1)\pi_G}{r_0 r_1} \equiv \hat{w}_0^H.$$

The LHS of the first inequality denotes the return yielded from firms financed at $t = 0$, whereas the RHS denotes the opportunity cost of bank lending at $t = 0$. From the definition of $\hat{w}_0^H$, we can derive the following:

$$\frac{\partial \hat{w}_0^H}{\partial r_0} = \frac{(1 + r_1)\pi_G}{r_0^2 r_1} > 0$$
$$\frac{\partial \hat{w}_0^H}{\partial r_1} = \frac{\pi_G}{r_0 r_1^2} > 0.$$

(QED).

Appendix 3: Proof of Lemma 3

Under soft budget constraints, firms with bad projects do not exert effort at $t = 1^-$. Thus, they do not yield a return at $t = 2$ unless they are refinanced at $t = 1$. In this case, the bank lends to a firm at $t = 0$, which satisfies:

$$(1 + r_1)\alpha\pi_G + (1 - \alpha)p\pi_R \geq (1 - w_0 + m_0)r_0 r_1 + (1 - \alpha)p r_1$$
$$w_0 \geq 1 + m_0 - \frac{(1 + r_1)\alpha\pi_G}{r_0 r_1} - \frac{(1 - \alpha)p(\pi_R - r_1)}{r_0 r_1} \equiv \hat{w}_0^S.$$

The second term of the LHS of the first inequality denotes the return yielded from firms with bad projects refinanced at $t = 1$, whereas the second term of the RHS denotes the opportunity cost of refinancing. From the definition of $\hat{w}_0^S$, we can derive:

$$\frac{\partial \hat{w}_0^S}{r_0} = \frac{(1 + r_1)\alpha\pi_G}{r_0^2 r_1} + \frac{(1 - \alpha)p(\pi_R - r_1)}{r_0^2 r_1} - \frac{(1 - \alpha)(\pi_R - r_1)}{r_0 r_1}\frac{\partial p}{\partial r_0}$$

$$\frac{\partial \hat{w}_0^S}{\partial r_1} = \frac{(1+r_1)\alpha\pi_G}{r_0 r_1^2} + \frac{(1-\alpha)p(\pi_R - r_1)}{r_0 r_1^2} + \frac{(1-\alpha)p - \alpha\pi_G}{r_0 r_1} - \frac{(1-\alpha)(\pi_R - r_1)}{r_0 r_1}\frac{\partial p}{\partial r_1}.$$

Thus, $\partial \hat{w}_0^S/\partial r_t > 0$ if $\partial p/\partial r_t < 0$, but $\partial \hat{w}_0^S/\partial r_t < 0$ if $\partial p/\partial r_t > 0$ and $|\partial p/\partial r_t|$ is large (QED).

Appendix 4: Proof of Lemma 4

Substituting $D_0 = L_0 - E_0$, (5), and (7) into (10), we obtain:

$$E_1^S = \alpha\pi_G \Delta W_0^S - r_0\Big(\frac{(\Delta W_0^S)^2}{2} - \bar{E} + m_0 \Delta W_0^S\Big).$$

Differentiating this with respect to ΔW_0^S, we obtain:

$$\frac{\partial E_1^S}{\partial \Delta W_0^S} = \alpha\pi_G - r_0(\Delta W_0^S + m_0).$$

Then, $\partial E_1^S/\partial \Delta W_0^S < 0$ for low values of α (QED).

Appendix 5: Proof of Proposition 1

Suppose that the bank credit market at $t = 0, 1$ is balanced at $r_0 = r_1 = \hat{r}$. $\hat{p}$ denotes the probability that firms with bad projects are refinanced at $t = 1$ when $r_0 = r_1 = \hat{r}$ are satisfied. Then, $\hat{p} > B_L/B$ is supposed to be satisfied.

First, consider the derivatives of D_1^R and L_1^R with respect to ΔW_0^S. From (12) and (13), we can derive:

$$\frac{\partial D_1^R}{\partial \Delta W_0^S} = 1 - \alpha > 0 \tag{22}$$

$$\frac{\partial L_1^R}{\partial \Delta W_0^S} = \frac{1}{\gamma}(\alpha\pi_G - r_0(m_0 + \Delta W_0^S)). \tag{23}$$

Next, consider how D_1^R and L_1^R change with respect to r_0. Again, from (12) and (13):

$$\begin{aligned}\frac{\partial D_1^R}{\partial r_0} &= \frac{\partial D_1^R}{\partial \Delta W_0^S}\frac{\partial \Delta W_0^S(r_0, r_1, p)}{\partial r_0} \\ &= (1-\alpha)\Big(\frac{\partial \Delta W_0^S(r_0, \hat{r}, \hat{p})}{\partial r_0} + \frac{\partial \Delta W_0^S(\hat{r}, \hat{r}, p)}{\partial p}\frac{\partial p}{\partial r_0}\Big) \qquad (24)\end{aligned}$$

$$\begin{aligned}\frac{\partial L_1^R}{\partial r_0} &= \frac{\partial L_1^R}{\partial r_0} + \frac{\partial L_1^R}{\partial \Delta W_0^S}\frac{\partial \Delta W_0^S(r_0, r_1, p)}{\partial r_0} \\ &= -\frac{D_0}{\gamma} + \frac{1}{\gamma}(\alpha\pi_G - r_0(m_0 + \Delta W_0^S))\Big(\frac{\partial \Delta W_0^S(r_0, \hat{r}, \hat{p})}{\partial r_0} + \frac{\partial \Delta W_0^S(\hat{r}, \hat{r}, p)}{\partial p}\frac{\partial p}{\partial r_0}\Big). \quad (25)\end{aligned}$$

Therefore, we have to derive $\partial \Delta W_0^S/\partial r_0$ and $\partial p/\partial r_0$. From (8), we obtain:

$$\frac{\partial \Delta W_0^S}{\partial r_0} = -\frac{(1+r_1)\alpha\pi_G}{r_0^2 r_1} - \frac{(1-\alpha)p(\pi_R - r_1)}{r_0^2 r_1} + \frac{(1-\alpha)(\pi_R - r_1)}{r_0 r_1}\frac{\partial p}{\partial r_0}. \quad (26)$$

On the other hand, as $p = L_1^R/D_1^R$,

$$p = \frac{(\alpha\pi_G - r_0 m_0)\Delta W_0^S - r_0(\Delta W_0^S)^2/2 + r_0\bar{E} - \gamma L_1^N}{(1-\alpha)\gamma\Delta W_0^S}.$$

Differentiating p with respect to r_0, we obtain:

$$\frac{\partial p}{\partial r_0} = -\frac{D_0}{(1-\alpha)\gamma\Delta W_0^S} + \frac{\alpha\pi_G - r_0(m_0 + \Delta W_0^S)}{(1-\alpha)\gamma\Delta W_0^S}\frac{\partial \Delta W_0^S}{\partial r_0} - \frac{L_1^R}{(1-\alpha)(\Delta W_0^S)^2}\frac{\partial \Delta W_0^S}{\partial r_0}. \quad (27)$$

The first term in (27) can be derived by using $D_0 = L_0 - (\bar{E} - m_0\Delta W_0)$ and (7).

Therefore, from (26) and (27), we obtain:

$$\begin{pmatrix} \partial p/\partial r_0 \\ \partial \Delta W_0^S/\partial r_0 \end{pmatrix} = \begin{pmatrix} 0 & a_{12} \\ a_{21} & 0 \end{pmatrix}\begin{pmatrix} \partial p/\partial r_0 \\ \partial \Delta W_0^S/\partial r_0 \end{pmatrix} - \begin{pmatrix} b_1 \\ b_2, \end{pmatrix} \quad (28)$$

where $a_{12} = \frac{(\alpha\pi_G - r_0(m_0+\Delta W_0^S))\Delta W_0^S - \gamma L_1^R}{(1-\alpha)\gamma(\Delta W_0^S)^2}$, $a_{21} = \frac{(1-\alpha)(\pi_R - r_1)}{r_0 r_1}$, $b_1 = \frac{D_0}{(1-\alpha)\gamma\Delta W_0^S}$, and $b_2 = \frac{(1+r_1)\alpha\pi_G}{r_0^2 r_1} + \frac{(1-\alpha)p(\pi_R - r_1)}{r_0^2 r_1}$. Note that $a_{21} > 0$, $b_1 > 0$, and $b_2 > 0$.

Rearranging (28), we obtain:

$$\begin{pmatrix} \partial p/\partial r_0 \\ \partial \Delta W_0^S/\partial r_0 \end{pmatrix} = \frac{1}{1 - a_{12}a_{21}}\begin{pmatrix} 1 & a_{12} \\ a_{21} & 1 \end{pmatrix}\begin{pmatrix} -b_1 \\ -b_2. \end{pmatrix} \quad (29)$$

It is necessary to consider the following two cases separately.

When α is small

When α is small, a_{12} becomes negative, which implies that $a_{12}a_{21} < 0$. Thus, $\partial \Delta W_0^S/\partial r_0 < 0$ can be derived. In addition, because a_{12} is negative, $a_{12}b_2 < 0$. However, if α is small enough, $-b_1 - a_{12}b_2 > 0$, which implies that $\partial p/\partial r_0 > 0$. Using these inequalities, we can derive $\partial L_0/\partial r_0 < 0$ and, in the light of Lemma 4, $\partial L_1/\partial r_0 > 0$.

When α is large

When α is large, a_{12} is positive and $|a_{21}|$ is small, implying that $a_{12}a_{21} > 0$ but that $|a_{12}a_{21}|$ is small. Thus, $1 - a_{12}a_{21} > 0$. Moreover, $a_{12}b_2 > 0$. Using these inequalities, we can derive $\partial p/\partial r_0 < 0$ and $\partial \Delta W_0^S/\partial r_0 < 0$. Therefore, $\partial L_0/\partial r_0 < 0$ and $\partial L_1/\partial r_0 < 0$.

Next, consider the effect of monetary policy at $t = 1$. Similarly to the former case:

$$\begin{aligned}\frac{\partial D_1^R}{\partial r_1} &= \frac{\partial D_1^R}{\partial \Delta W_0^S}\frac{\partial \Delta W_0^S}{\partial r_1}\\ &= (1-\alpha)\Big(\frac{\partial \Delta W_0^S(\hat{r}, r_1, \hat{p})}{\partial r_1} + \frac{\partial \Delta W_0^S(\hat{r}, \hat{r}, p)}{\partial p}\frac{\partial p}{\partial r_1}\Big) \qquad (30)\end{aligned}$$

$$\begin{aligned}\frac{\partial L_1^R}{\partial r_0} &= \frac{\partial L_1^R}{\partial \Delta W_0^S}\frac{\partial \Delta W_0^S}{\partial r_1}\\ &= \frac{1}{\gamma}(\alpha\pi_G - r_0(m_0 + \Delta W_0^S))\Big(\frac{\partial \Delta W_0^S(\hat{r}, r_1, \hat{p})}{\partial r_1} + \frac{\partial \Delta W_0^S(\hat{r}, \hat{r}, p)}{\partial p}\frac{\partial p}{\partial r_1}\Big). \qquad (31)\end{aligned}$$

Note that (30) is very similar to (24), and that (25) and (31) are also very similar. The only difference between (25) and (31) is that the first term in (25) disappears in (31). All other effects are the same as the case where monetary policy is implemented at $t = 0$ (QED).

Appendix 6: Proof of Proposition 2

Note that $Y = Y^S(\hat{r})$ at the equilibrium interest rate $r_0 = r_1 = \hat{r}$. In addition, without loss of generality, we can focus on the relation between aggregate income and a monetary policy implemented at $t = 0$.

First, consider case (a). As Proposition 1(a) denotes, a decrease in r_0 increases L_0 but decreases L_1. This immediately leads to an increase in Y_0^S but a decrease in Y_1^S. Thus, whether a monetary expansion increases Y^S or not is ambiguous. On the other hand, in terms of the change in p, Proposition 1(a) indicates that a decrease in r_0 decreases p ($\partial p/\partial r_0 > 0$). Therefore, if r_0 decreases sufficiently, p becomes so small that (6) does not hold. This means that firms come to face hard budget constraints. Let $\tilde{r}_a$ denote the threshold interest rate. Then, when $1 \leq r_0 \leq \tilde{r}_a$, firms face hard budget constraints and aggregate income becomes Y^H. When $\tilde{r}_a < r_0$, firms face soft budget constraints and the aggregate income becomes Y^S.

Second, consider case (b). As Proposition 1(b) indicates, a decrease in r_0 increases ΔW_0^S, thus increasing Y_0^S. Moreover, an increase in ΔW_0^S increases E_1, which increases L_1 and therefore Y_1^S. Thus, in this case, a monetary expansion increases Y^S ($\partial Y^S/\partial r_0 < 0$). On the other hand, in terms of the change in p, Proposition 1(b) indicates that an increase in r_0 decreases p ($\partial p/\partial r_0 < 0$). Therefore, if r_0 increases sufficiently, p becomes so small that (6) does not hold. Let $\tilde{r}_b$ denote the threshold interest rate. Then, when $1 \leq r_0 \leq \tilde{r}_b$, firms face soft budget constraints and aggregate income becomes Y^S. When $\tilde{r}_b < r_0$, firms face hard budget constraints and aggregate income becomes Y^H (QED).

Appendix 7: Proof of Proposition 4

First, consider the case where the bank issues new equity but does not monitor new projects at $t = 1$. Using $E_1^{'}$ to denote an equity base at $t = 1$, $E_1^{'}$ is represented by:

$$E_1^{'} = \alpha \pi_G \Delta W_0^S - r_0 D_0 + Z,$$

where Z denotes the amount that the bank raises by issuing new equity. Now, the amount of bank lending at $t = 1$ ($L_1^{'}$) is denoted by: $L_1^{'} = E_1^{'}/\gamma$.

As the soft budget constraint problems prevail in this case, the market value of the bank at $t = 1$ is given by:

$$MV = \int_{\hat{w}_0}^{1} \alpha \pi_G f(w_0) dw_0 + \int_{\hat{w}_1}^{1} \beta \pi_G f(w_1) dw_1 + \pi_R L_1^{R'} - r_1 D_1.$$

The first, second, and third terms represent the expected return yielded from good projects, the expected return yielded from new projects financed at $t = 1$, and the return yielded from bad projects refinanced at $t = 1$, respectively. The last term represents the repayment to depositors.

The bank chooses a proportion θ to maximize the following objective function.

$$\max_{\theta} \quad (1-\theta)MV \quad \textit{subject to } (20)$$

The first-order condition is:

$$-MV + (1-\theta)\frac{\partial MV}{\partial \theta} = 0. \tag{32}$$

As $0 < L_1^{R'} < (1-\alpha)\Delta W_0$, the amount raised by issuing new equity Z is allocated to refinancing bad projects. Therefore,

$$\frac{\partial MV}{\partial \theta} = \pi_R \frac{1}{\gamma}\frac{\partial E_1}{\partial \theta}.$$

From (20),

$$\frac{\partial E_1}{\partial \theta} = \frac{MV}{r_1}.$$

Using these equations, (32) can be rewritten as:

$$-MV + \frac{(1-\theta)\pi_R}{\gamma}\frac{MV}{r_1} = 0.$$

The first term represents the dilution cost of issuing new equity, whereas the second term represents the marginal market value yielded from a bad project refinanced at $t = 1$. Thus, the optimal θ^* becomes:

$$\theta^* = 1 - \frac{\gamma MV}{\frac{MV}{r_1}\pi_R}. \tag{33}$$

Next, consider the case where the bank issues new equity and monitors new projects. Using $E_1^{M'}$ to denote the equity base of the bank, $E_1^{M'}$ is represented by:

$$E_1^{M'} = \pi_G \Delta W_0^M - r_0 D_0 - m_1 \Delta W_1^M + Z^M.$$

As monitoring hardens the budget constraint, the market value of the bank $MV^{M'}$ is given by:

$$MV^{M'} = \int_{\hat{w}_0^H}^{1} \pi_G f(w_0) dw_0 + \int_{\hat{w}_1}^{1} \beta^{'} \pi_G f(w_1) dw_1 - r_1 D_1.$$

The first and second terms denote the return yielded from all firms financed at $t = 0$, and the return yielded from new firms financed at $t = 1$, respectively.

Then, the maximization problem becomes:

$$\max_{\theta^M} \quad (1 - \theta^M) MV^{M'} \quad \textit{subject to } (23).$$

The first-order condition is:

$$-MV^{M'} + (1-\theta)\frac{\partial MV^{M'}}{\partial \theta^M} = 0. \tag{34}$$

As the amount Z^M is allocated to financing new projects under hard budget constraints,

$$\frac{\partial MV^{M'}}{\partial \theta^M} = \frac{\beta^{'} \pi_G}{\gamma} \frac{\partial MV^{M'}}{\partial \theta^M}.$$

From (21),

$$\frac{\partial MV^{M'}}{\partial \theta^M} = \frac{MV^M}{r_1} - m_1 \frac{\partial \Delta W_1^M}{\partial \theta^M}.$$

Using these equations, (34) can be rewritten as:

$$-MV^{M'} + \frac{(1-\theta^M)\beta\pi_G}{\gamma}\left(\frac{MV^{M'}}{r_1} - m_1 \frac{\partial \Delta W_1^M}{\partial \theta^M}\right) = 0.$$

Thus, the optimal θ^{M^*} becomes:

$$\theta^{M^*} = 1 - \frac{\gamma MV^{M'}}{\left(\frac{MV^{M'}}{r_1} - m_1 \frac{\partial \Delta W_1^M}{\partial \theta^M}\right)\beta' \pi_G}. \tag{35}$$

Comparing (33) and (35), $\theta^{M^*} < \theta^*$ for large values of m_1. In addition, as $MV > MV^M$ under $r_0 = r_1 = \hat{r}$, $Z^M < Z$ can be derived[23] (QED).

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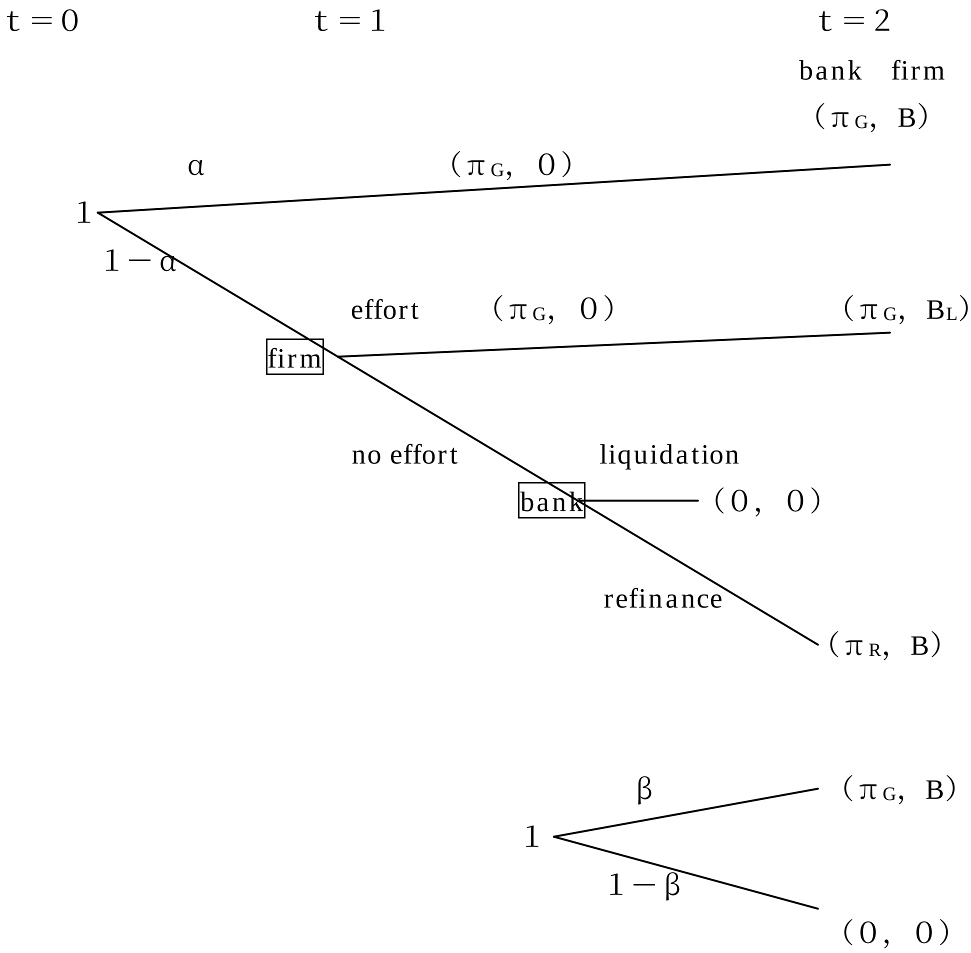
t = 0
t = 1
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bank firm
(πG, B)
α
(πG, 0)
1
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effort
(πG, 0)
(πG, BL)
firm
no effort
liquidation
bank
(0, 0)
refinance
(πR, B)
β
(πG, B)
1
1 − β
(0, 0)

Figure 1

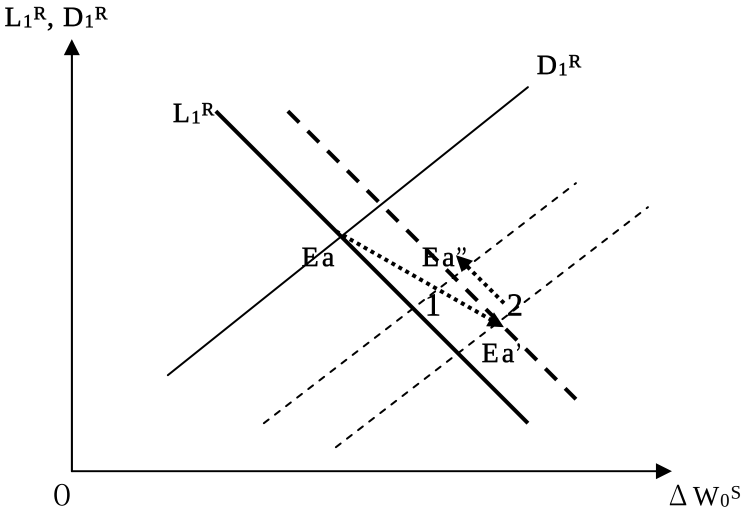

Figure 2(a)

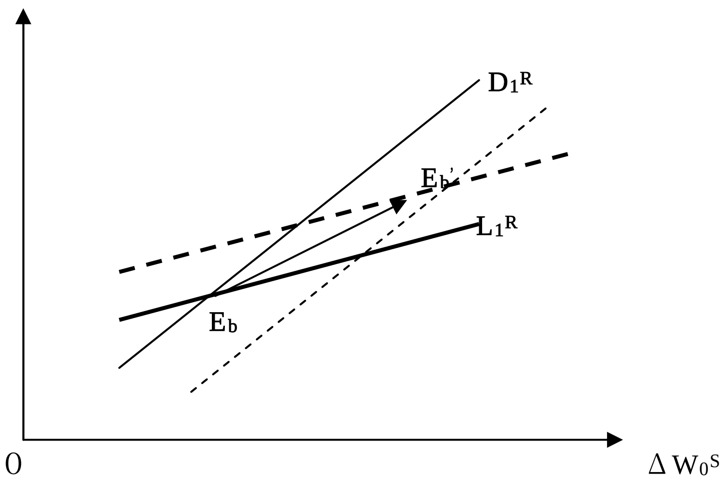

Figure 2(b)

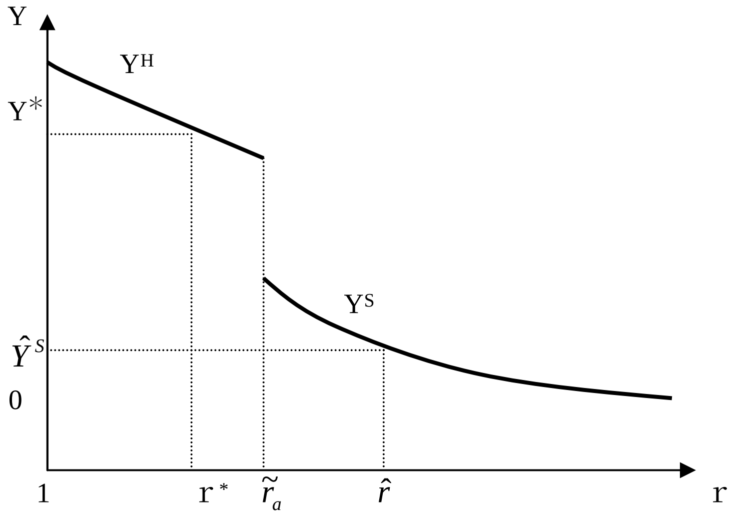

Figure 3 (a)

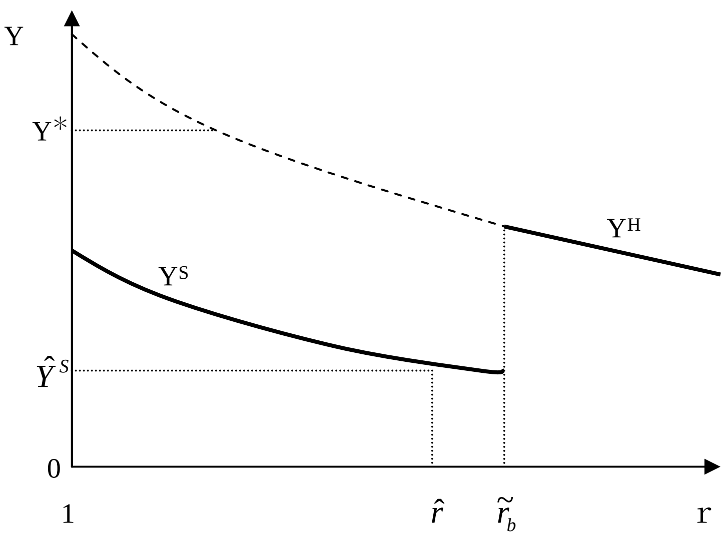

Figure 3 (b)

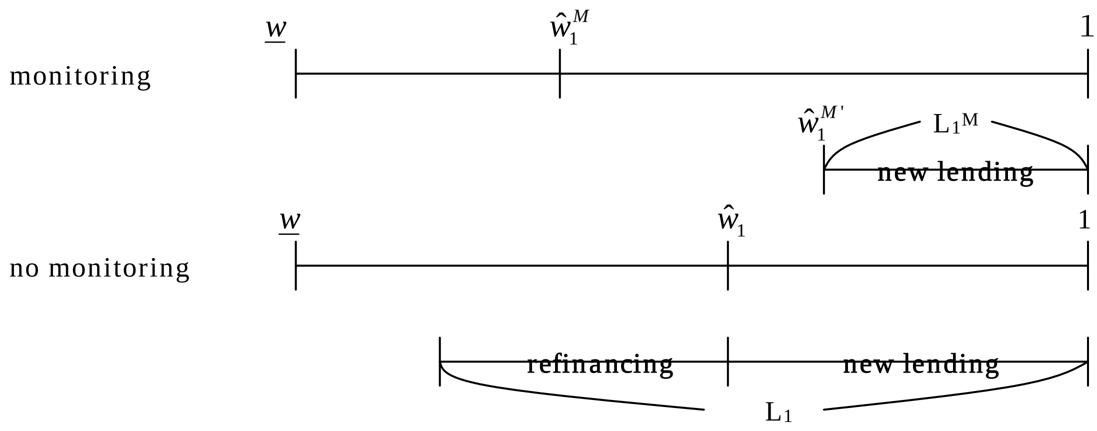

Figure 4

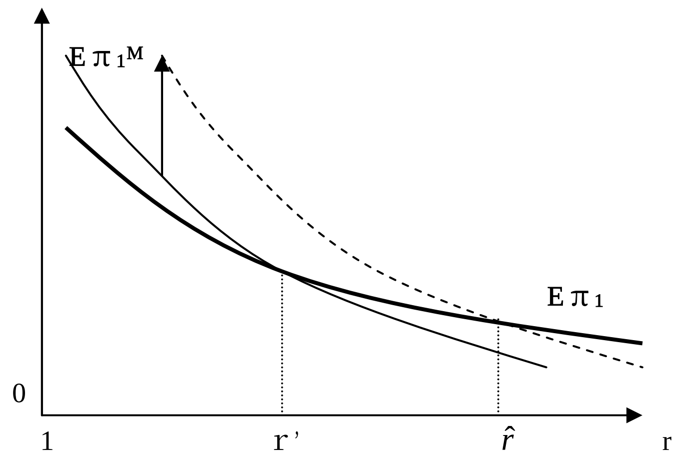

Figure 5.1

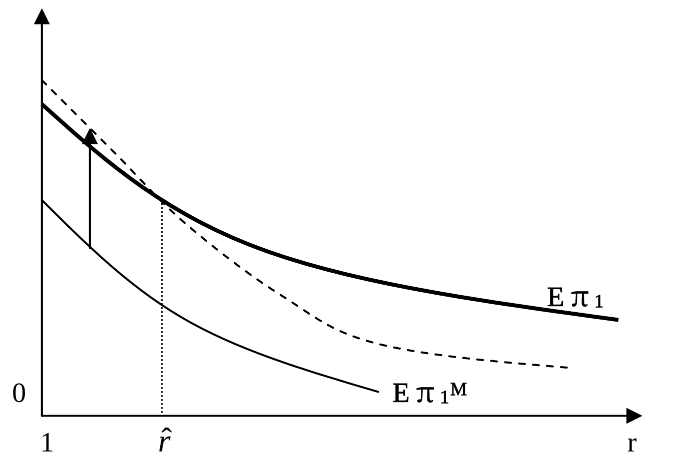

Figure 5.2