FINANCIAL DEEPENING, EXTENT OF BUSINESS FLUCTUATIONS, AND POLICY EFFECTIVENESS[1]

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Abstract

We develop a model of financial deepening, economic growth, and business fluctuations. We explore the effects of the changes in financial depth on resource allocation (credit allocation), an aggregate TFP level, and an economic growth rate. Moreover, we investigate the relationship between the degree of financial deepening and policy effectiveness such as the effectiveness of monetary policy as well as the extent of business fluctuations. We find that there are three stages of financial deepening. Which stages the financial system reaches is crucial in determining the economic growth rate, the extent of business fluctuations, and the effectiveness of monetary policy.

1. Introduction

Economists have long held the view that the development of the financial system (financial deepening), economic growth, and business fluctuations (volatility of economic growth) are closely intertwined[2]. However, there are relatively few formal models in the literature[3]. In this paper, we develop a model of financial deepening, economic growth, and business fluctuations. We explore the effects of the changes in financial depth on resource allocation (credit allocation), an aggregate TFP level, and an economic growth rate. Moreover, we investigate the relationship between the degree of financial deepening and policy effectiveness such as the effectiveness of monetary policy as well as business fluctuations.

For this purpose, we develop a dynamic general equilibrium model with mainly two characteristics, the borrowing constraint and the presence of heterogeneous agents. The borrowing constraint arises because there may be a limit on the extent to which the debtors can credibly promise to repay the creditors. To be more specific, we assume that a debtor can credibly commit to repay at most a fraction θ of his or her future output. The parameter θ partly reflects the legal structure and contractual redress available to a creditor in the event of defaults. In this sense, θ provides a simple measure of financial depth, capturing the degree of the development of the financial system.

The presence of heterogeneous agents implies that at each date, there are two types of agents—high-productivity agents and low-productivity agents. Both have the technology to invest goods in the present period. They obtain returns in the subsequent period. High-productivity agents have a higher rate of return. Over time, some high-productivity agents (low-productivity agents) stochastically become low-productivity agents (high-productivity agents) in the subsequent periods.

We examine the following three questions: (1) How do the changes in financial depth affect resource allocation (credit allocation), an aggregate TFP level, and an economic growth rate? (2) How does the extent of business fluctuations depend on the degree of the development of the financial system? (3) What is the relationship between the degree of the development of the financial system and policy effectiveness such as the effectiveness of monetary policy?

We find that the degree of financial deepening plays a crucial role in determining the economic growth rate, the extent of business fluctuations, and the effectiveness of monetary policy. In fact, in this model, there are three stages of financial deepening, corresponding to three different regions of the θ value. In Region 1, where θ is low and the financial system is not well developed, the borrowing constraint of high-productivity agents binds. The real interest rate is low, so that even low-productivity firms have incentives to invest, which means that some of aggregate savings are allocated to low-productivity agents. Production and resource allocation are inefficient. The aggregate TFP level is low, so that the economic growth rate is also low. In this region, shocks to wealth distribution results in business fluctuations through persistent changes in aggregate TFP. That is, economic growth is volatile to the shocks. Moreover, the more developed the financial system, the greater is the effectiveness of monetary policy. If we interpret the shocks on θ as the shocks on the function of the financial system, this implies that the effectiveness of monetary policy could be reduced by negative shocks on the function of the financial system.

In Region 2, where θ becomes higher and the financial system is developed to some extent, the real interest rate starts to rise, so that low-productivity agents lose incentives to invest. All the savings are allocated only to high-productivity agents. In this sense, production and resource allocation becomes efficient even though the borrowing constraint of high-productivity agents binds. In Region 3, where θ becomes extremely high and the financial system is well developed, the borrowing constraint of high-productivity agents does not bind anymore. Production and resource allocation are efficient. In Region 2 and 3, the aggregate TFP level becomes high, so that the economic growth rate is high. In fact, both are the same as the case in the first best. In these regions, business fluctuations would not occur even if there are shocks to wealth distribution. That is, economic growth would not be volatile to the shocks. Thus, a well-developed financial system is important for reducing the volatility in the macroeconomic performance. Further, the effectiveness of monetary policy is independent of the degree of financial deepening. This implies that the effectiveness of monetary policy would not be affected by negative shocks on the function of the financial system.

Literature related to this paper is as follows. With regard to business fluctuations, there are, for example, Aghion et al. [1999], Bernanke and Gertler [1989, BG], and Kiyotaki and Moore [1997, KM]. They present a powerful propagation mechanism to show how the effects of a temporarily small productivity shock amplify and persist through the changes in the collateral value in the economy with imperfect credit markets. However, they do not necessarily discuss the relationship between the propagation mechanism and financial deepening.

With regard to financial deepening, examples include Jeong and Townsend [2004] and Aghion et al. [2005]. Jeong and Townsend examine the relationship between financial deepening and aggregate TFP growth. However, they do not necessarily investigate the relationship between financial deepening and the extent of business fluctuations as well as policy effectiveness, such as the effectiveness of monetary policy. Aghion et al. investigate the relationship between financial deepening and volatility in economic growth. However, neither do they explore the effects of the changes in financial deepening on policy effectiveness.

In addition, this paper would be related to the studies on the sources of aggregate TFP.

Modern business cycle models give a central role to aggregate TFP shocks as a major factor of business cycles. However, aggregate TFP shocks itself are mostly considered as exogeous events. That is, the underlying sources of aggregate TFP are not necessarily well-known. Lagos [2006] develops a model of aggregate TFP based on a search theory. He endogenously derives the level of aggregate TFP by focusing on labor markets and shows how labor market policies affect it. This paper endogenously derives it by focusing on credit markets instead and examines by what factors it is affected in both the short run and long run[4].

This paper is organized as follows. In section 2, we will explain the basic structure of our model. Three stages of financial development are identified. We show that resource allocation, the aggregate TFP level, and the economic growth rate change according to these three stages. In section 3, we analyze the short-run effect of an unanticipated monetary policy shock on the economic growth rate and the aggregate TFP level in each of three stages. We show that the extent of business fluctuations through the changes in aggregate TFP will differ in each stage. In section 4, we examine how both the aggregate TFP level and the economic growth rate are determined endogenously in a credit-constrained economy in the long run. We present a mechanism for regime switching in an economy, i.e., how an economy with a low economic growth rate, a low aggregate TFP level, and an inefficient production switches to one with a high economic growth rate, a high aggregate TFP level, and an efficient production. In section 5, we analyze the effectiveness of monetary policy. We present a theoretical relationship between the degree of financial deepening and the effectiveness of monetary policy. In section 6, we provide conclusions.

2. The Model

Our model is an extension of Kiyotaki's model [1998]. Consider a discrete-time economy with a single homogenous good and a continuum of agents. In each period, young agents are born. Each young agent lives for two periods. Each has the same preference, i.e.,

(1) $$V = a_1 \log c_t + a_2 \log c_{t+1} + a_3 \log e_{t+1}$$

where c_t and c_{t+1} represent consumption at dates *t* and *t* + 1, respectively, e_{t+1} is the bequest to the next young agent, and a(< 1) is the weight for each utility and satisfies $a_1 + a_2 + a_3 = 1$.

At each date *t*, there is a competitive one-period credit market in which one unit of consumption goods is exchanged for a claim to r_t units of consumption goods at date *t* + 1. At each date, some agents are high-productivity agents while the others are low-productivity agents. The high-productivity agents possess the following production technology:

(2) $$y_{t+1} = \alpha^H z_t^H$$

Where z_t^H is the investment at date *t* and y_{t+1} is the output from the investment at date *t* + 1. It takes one period to produce goods from the investment project. Further, although the low- and high-productivity agents have similar production technology, the marginal productivity of the former is lower.

(3) $$y_{t+1} = \alpha^L z_t^L \qquad 1 < \alpha^L < \alpha^H$$

The marginal productivity of each firm follows the Markov process. This implies that there is a switching probability between the high- and low-productivity states; this probability is 1–p. On the other hand, the probability of switching from the low-productivity state to the high-productivity state is X (1 – p). This switching probability is exogenous, and it is independent across agents over time. We assume that the initial measures of the high- and low-productivity agents are X and 1, respectively. From the assumption of the Markov process, it follows that the population ratio of the high-productivity agents to low-productivity agents is X over time.

There is an enforcement problem between the creditors and the debtors. The creditors can not enforce the debtors to repay debts unless the debts are secured by collateral. We assume that the creditors are assured a fraction θ^H, θ^L of the maximum returns. This means that the debtors can credibly commit to repay at most this fraction. This fraction θ can be collateral in borrowing. In such a situation, in order for the debts contract to be credible, the creditors limit the amount of credit at date *t*, so that the debt repayment does not exceed the value of the collateral.

(4) $$r_t b_t^i \leq \theta^s \alpha^s z_t^s \qquad r_t \equiv (1 + i_t)\, P_t \big/ P_{t+1}^e \qquad \text{s = H or L,}$$

where *i* is index of each firm, b_t^i is the amount of borrowing, r_t and i_t are the gross real interest rate and the net nominal interest rate between dates *t* and *t* + 1, respectively, $r_t b_t^i$ is the debt repayment, P_t and P_{t+1} are the price levels at date *t* and *t* + 1, respectively, and $\theta^s \alpha^s$ is the collateral value of the unit investment project. Here, for simplicity, we assume that θ^H is equal to θ^L and defined as θ[5]. The parameter θ in part reflects the legal structure and contractual redress available to a creditor in the event of defaults. For example, in the economy where property rights, bankruptcy laws or bankruptcy procedures are well established, the creditors are legally protected and they could seize enough assets of the debtors in the event of bankrupt, so that θ would be high. In this paper, we call an increase in θ the development of the financial system, financial deepening[6]. We also assume that marginal productivity of each firm is known to the public. So people have perfect foresight about both debt repayment and future output aside from an unanticipated shock.

Each agent determines the consumption, bequest, investment, borrowing, and money holding in order to maximize the utility (equation 1), subject to the production technology constraints (equations 2 and 3), the borrowing constraint (equation 4), the following first-period and second-period budget constraints, and the cash-in-advance constraint[7].

The first-period budget constraint is as follows: $\frac{m_t^i}{P_t} + c_t^i + z_t^{i,s} \leq (1-\tau_t)e_t^i + b_t^i$.

The second-period budget constraint is as follows: $c_{t+1}^i + e_{t+1}^i \leq y_{t+1}^{i,s} - r_t b_t^i + \frac{m_t^i}{P_{t+1}}$.

The cash-in-advance constraint is as follows: $m_t^i \geq P_{t+1} c_{t+1}^i$.

Here, τ_t is the tax rate at date t and m_t^i is the nominal money demand of each agent. For now, we assume that τ_t is constant over time, i.e., $\tau_t = \tau$.

2.1 The case without the borrowing constraint

First, we will describe the equilibrium for an economy without the borrowing constraint. In this case, the high-productivity agents would prefer to borrow an unlimited amount of credit as long as the marginal rate of return on investment exceeds the real interest rate, $\alpha^H > r_t$. Both the high- and low-productivity agents would not borrow if the marginal rate of return on investment was less than the real interest rate, $\alpha^H < r_t$. Thus, the equilibrium interest rate in the credit market would be equal to the marginal rate of return on the investment of the high-productivity agents, $\alpha^H = r_t$.

Then, in the equilibrium, the low-productivity agents would not undertake any investment and would prefer lending because the marginal rate of return on lending is higher than the marginal rate of return on their investment. Consequently, only the high-productivity agents would undertake investment. The aggregate investment of the high-productivity agents would be equal to the aggregate savings for investment of these agents, which is equal to a fraction $(1-\tau)a_3$ of the aggregate output of the economy.

(5) $$Z_t^H = a_3(1-\tau)Y_t$$

The aggregate output of the economy is simply the output from the investment undertaken by the high-productivity agents.

(6) $$Y_{t+1} = \alpha^H a_3 (1-\tau) Y_t$$

Therefore, the aggregate TFP level in the economy without the borrowing constraint is as follows.

(7) $$AggregateTFP_t = \alpha^H$$

Moreover, in this case, the economic growth rate defined as G_t^{NBC} is as follows.

(8) $$G_{t+1}^{NBC} \equiv \frac{Y_{t+1}}{Y_t} = \alpha^H a_3 (1-\tau)$$

Here, it is important to note that the aggregate investment, the aggregate output, the aggregate TFP level, and the economic growth rate are independent of the distribution of net worth between the high- and low-productivity agents.

2.2 The case with the borrowing constraint

We now discuss the case of an economy with the borrowing constraint. First, let us discuss the case where θ is strictly less than $\hat{\theta}$, which is defined as $1-p \Big/ \left(\frac{\alpha^H}{\alpha^L} - p + X(1-p) \right)$. In this case, the real interest rate is equal to the marginal rate of return on investment of the low-productivity agents, α^L, the borrowing constraint of the high-productivity agents is binding, and both high- and low-productivity agents invest in the neighborhood of the steady state. This can be verified in Proposition 1 after we describe the equilibrium in a credit-constrained economy.

The high-productivity agents invest by borrowing up to the credit limit because the real interest rate is lower than the marginal rate of return on their investment. Thus, the high-productivity agents' investment is as follows.

(9)
$$z_t^{i,H} = \frac{a_3(1-\tau)e_t^i}{1-\dfrac{\theta\alpha^H}{r_t}}$$

Since $\dfrac{\theta\alpha^H}{r_t}$ is the present value of the collateral of the unit investment, the numerator is the required down payment for the unit investment. Equation 9 implies that the high-productivity agents use a fraction a_3 of the net worth after tax $(1-\tau)e_t$ to finance the required down payment. With regard to equation 9, it is important to note that the investment of the high-productivity agents is an increasing function of their net worth, e_t and their productivity, α^H and a decreasing function of the real interest rate, r_t and the parameter of the degree of the development of the financial system, θ . Since the utility function is log utility, the consumption and money demand of each agent at date t are a fraction a_1 and a_2 of the net worth after tax, respectively. These are presented as $c_t^i = a_1(1-\tau)e_t^i$ and $m_t^i/P_t = a_2(1-\tau)e_t^i$

If the real interest rate is equal to the marginal rate of return on their investment, the low-productivity agents are indifferent between investing and lending. Thus, investment and lending are not determined at the individual level. However, the aggregate investment and aggregate lending are determined from the goods market clearing condition. Since consumption, debt, money demand, and investment are linear functions of the net worth, by aggregating across agents, we can obtain the equations of motion of the aggregate output, Y_t and the aggregate net worth of the high-productivity agents, E_t^H .

(10)
$$Y_{t+1} = \alpha^H Z_t^H + \alpha^L Z_t^L = \alpha^L a_3(1-\tau)Y_t + (\alpha^H - \alpha^L)\frac{a_3(1-\tau)E_t^H}{1-\dfrac{\theta\alpha^H}{\alpha^L}}$$

(11)
$$E_{t+1}^H = p(Y_{t+1}^H - r_t B_t) + X(1-p)(Y_{t+1}^L + r_t B_t)$$

$$= p(1-\theta)\alpha^H \frac{a_3(1-\tau)E_t^H}{1-\frac{\theta\alpha^H}{\alpha^L}} + X(1-p)a_3(1-\tau)\alpha^L(Y_t - E_t)$$

From equation 10, we understand that the aggregate output comprises two parts: the returns on the investments of the high- and low-productivity agents. From equation 11, we understand that the aggregate net worth of the high-productivity agents also comprises two parts: the net worth of the agents that continue to function as high-productivity agents from the previous period and the net worth of the agents that after the previous period switch from being low-productivity agents to being high-productivity agents. The important point in the case with the borrowing constraint is that the aggregate output is an increasing function of the net worth share of the high-productivity agents. Intuitively, the high-productivity agents have a greater net worth, and they can obtain greater credit and finance increased investment because the default probability reduces. Thus, a greater output is produced.

The aggregate TFP level in the case with the borrowing constraint is as follows.

(12)
$$AggregateTFP_t \equiv \frac{Y_{t+1}}{Z_t^H + Z_t^L} = \alpha^L + (\alpha^H - \alpha^L)\frac{1}{1-\frac{\theta\alpha^H}{\alpha^L}}\frac{E_t^H}{Y_t}$$

From equation 12, we understand that the aggregate TFP level is also an increasing function of the net worth share of the high-productivity agents. The greater the net worth of the high-productivity agents relative to that of the low-productivity agents, the higher is the aggregate TFP level. Intuitively, if the net worth share of the high-productivity agents is high, a greater amount of resources are allocated to them because the borrowing constraint weakens.

From equation 10, the economic growth rate with the borrowing constraint defined as G_t^{BC} is as follows.

$$G_{t+1}^{BC} \equiv \frac{Y_{t+1}}{Y_t} = \alpha^L a_3(1-\tau) + (\alpha^H - \alpha^L)\frac{a_3(1-\tau)}{1-\frac{\theta\alpha^H}{\alpha^L}}\frac{E_t^H}{Y_t}$$

(13) $$= a_3(1-\tau) \times AggregateTFP_{t+1}$$

From equation 13, it follows that the economic growth rate is determined once the aggregate TFP is determined. Thus, the shocks which affect the aggregate TFP result in a change in the economic growth rate. In a standard real business cycle model, the technology shock is the main factor which affects the aggregate TFP. But, in this model, it is also affected by different factors such as monetary policy or the degree of financial deepening. From equations 11 and 13, the net worth share of the high-productivity agents evolves according to the following equation.

(14) $$\frac{E_{t+1}^H}{Y_{t+1}} = \frac{p\alpha^H(1-\theta)\frac{1}{1-\frac{\theta\alpha^H}{\alpha^L}}\frac{E_t^H}{Y_t} + X(1-p)\alpha^L(1-\frac{E_t^H}{Y_t})}{\alpha^L + (\alpha^H - \alpha^L)\frac{1}{1-\frac{\theta\alpha^H}{\alpha^L}}\frac{E_t^H}{Y_t}} \equiv \phi\left((\frac{E_t^H}{Y_t}), \theta, \alpha^H, X\right)$$

Government Budget Constraint (GBC)

The government finances its expenditure by printing money and imposing taxes.

(15) $$P_t G_t = M_t - M_{t-1} + P_t T_t$$

where G_t is the government expenditure at date *t,* M_t and M_{t-1} are the nominal money supply at dates *t* and *t* – 1, respectively, and T_t is the tax revenue at date *t* imposed on the agents. We assume that government expenditure does not affect the utility of the agents.

Government Policy

In this paper, with regard to government policy, we analyze two cases. First let us consider case 1 in which the government adopts the following policy.

(16) $$M_t = (1+\mu)M_{t-1} \quad \text{and} \quad \tau_t = \tau$$

The government considers the growth rate of money supply, μ and the tax rate as constant over time. We assume that μ is positive and large enough.

Money Market Clearing Condition

(17) $$M_t = P_t a_2 (1 - \tau_t) Y_t$$

Here, we confine our attention to equilibria in which the cash-in-advance constraint is binding. This can hold for a large enough μ. The right-hand side is the aggregate nominal money demand of the young agents at date *t*, and the left-hand side is the money supply at date *t*. M_{t-1} is supplied by the old agents and $M_t - M_{t-1}$ is supplied by the government. Since Y_t is a predetermined variable and M_t is determined by the government policy, from equation 17, the price level P_t is determined.

Goods Market Clearing Condition

(18) $$C_t + Z_t^H + Z_t^L + G_t = Y_t^H + Y_t^L$$

The right-hand side is the aggregate output of the high- and low-productivity agents. The left-hand side is the aggregate consumption of the old agents and the young agents, the aggregate investment of the high- and low-productivity agents, and the government expenditure.

From equations 15, 16, and 17, the government budget constraint becomes

(19) $$G_t = (\frac{\mu}{1+\mu}) a_2 (1 - \tau) Y_t + \tau Y_t$$

The first and second terms on the right-hand side are real seigniorage and tax revenues, respectively. From equation 19, we understand that the more money the government prints, the greater will be the seigniorage revenues.

The Definition of Competitive Economy (Case 1)

Given the initial Y_1, E_1^H, M_0, the exogenous variables $\theta, \alpha^H, \alpha^L, p, a$, and the monetary and fiscal policies $\mu_t = \mu$ and $\tau_t = \tau$, *a competitive equilibrium* is a sequence of prices and quantities, $\left\{C_t, Z_t^H, Z_t^L, B_t, G_t, i_t, P_t, P_{t+1}^e, Y_{t+1}, E_{t+1}^H, TFP_t\right\}_{t=1}^{\infty}$ that satisfies the following conditions:

(1) each high- and low-productivity agent maximizes his/her utility

(2) money and goods market clearing conditions

(3) perfect foresight with regard to the future price level

(4) the GBC for all t

In this model, equilibrium in the credit market changes depending on the three conditions of the net worth share of the high-productivity agents, i.e., (1) $\frac{E_t^H}{Y_t} < 1 - \frac{\theta\alpha^H}{\alpha^L}$, (2) $1 - \frac{\theta\alpha^H}{\alpha^L} \le \frac{E_t^H}{Y_t} < 1 - \theta$, and (3) $1 - \theta \le \frac{E_t^H}{Y_t} < 1$.

The dynamic movement of the net worth share of the high-productivity agents also changes according to the above three conditions. Dynamic equations in each condition are as follows;

In (1), $\frac{E_{t+1}^H}{Y_{t+1}} = \phi\left((\frac{E_t^H}{Y_t}), \theta, \alpha^H, X \right)$, In (2), $\frac{E_{t+1}^H}{Y_{t+1}} = p(1-\theta) + X(1-p)\theta$

and In (3), $\frac{E_{t+1}^H}{Y_{t+1}} = p(\frac{E_t^H}{Y_t}) + X(1-p)(1-\frac{E_t^H}{Y_t})$

We describe the dynamic movement of the net worth share of high-productivity agents in Figure 1. In this figure, we depict a case in which p is greater than $p > X/(1+X)$, which implies that there is a positive correlation between the current state and the following state.

In this economy, there is a unique stationary equilibrium in which the net worth share of the high-productivity agents, the aggregate TFP level, and the economic growth rate are constant over time. We can obtain the stationary equilibrium, G_t^{BC*}, $(\frac{E_t^H}{Y_t})^*$, and the aggregate TFP* level by using equations 12, 13, 14, and $\frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t}$. The stationary equilibrium is characterized by the following three equations.

(20) $$(\frac{E_{t+1}^{H}}{Y_{t+1}})^{*} = \phi\left((\frac{E_{t}^{H}}{Y_{t}})^{*}, \theta, \alpha^{H}, X \right)$$

(21) $$AggregateTFP^{*} = \alpha^{L} + (\alpha^{H} - \alpha^{L}) \frac{1}{1 - \frac{\theta\alpha^{H}}{\alpha^{L}}} (\frac{E_{t}^{H}}{Y_{t}})^{*}$$

(22) $$\left(\frac{Y_{t+1}}{Y_{t}} \right)^{*} = a_{3}(1 - \tau) \times AggregateTFP^{*}$$

The net worth share of the high-productivity agents in the steady state is determined from equation 20. Once the net worth share of the high-productivity agents is determined, the aggregate TFP level in the steady state is determined from equation 21, and once the aggregate TFP level is determined, the economic growth rate in the steady state is determined from equation 22.

The difference between this case and the one without the borrowing constraint is that in a credit-constrained economy, the aggregate TFP level and the economic growth rate in the steady state are dependent on the distribution of the net worth between the high- and low-productivity agents; moreover, these are increasing functions of the net worth share of the high-productivity agents. The stationary equilibrium depends on the degree of financial deepening. Therefore, depending on θ , there exist three different regions.

Proposition 1: There are three stages of financial deepening, corresponding to three different regions of θ value. The characteristics of each region are as follows.

In Region 1, where $\theta < \hat{\theta}$, (1) the real interest rate is α^{L}; (2) the borrowing constraint of the high-productivity agents binds; (3) production and resource allocation are inefficient. Both the high- and low-productivity agents undertake investment; and (4) both the aggregate TFP level and the economic growth rate are lower than those in the case without the borrowing constraint.

In Region 2, where $\hat{\theta} \le \theta < 1/(1+X)$, (1) the real interest rate is $r^{*} \in [\alpha^{L}, \alpha^{H})$; (2) the borrowing constraint of the high-productivity agents still binds; (3) production and resource allocation are efficient. Only the high-productivity agents undertake investment (4) both the aggregate TFP level and the economic growth rate are the same as those in the case without the borrowing constraint.

In Region 3, where $1/(1+X)\leq\theta\leq 1$, (1) the real interest rate is α^H; (2) the borrowing constraint of the high-productivity agents no longer binds; (3) production and resource allocation are efficient. Only the high-productivity agents undertake investment; (4) both the aggregate TFP level and the economic growth rate are the same as those in the case without the borrowing constraint.

Proof: see appendix

Intuitively, in a case in which the financial system is not well developed, $\theta < \hat{\theta}$, the amount of credit that the high-productivity agents can borrow is relatively low because the borrowing constraint is strong. The real interest rate becomes low in the credit market, so that even low-productivity agents have incentives to invest. This implies that savings for investment are allocated not only to high-productivity firms, but also to low-productivity agents. Production and resource allocation are inefficient. Therefore, the aggregate TFP level and the economic growth rate are lower than those in the case without the borrowing constraint. We refer to this region, where $\theta < \hat{\theta}$, as Region 1.

Next, let us discuss the case in which θ is greater than $\hat{\theta}$. Here, there are two regions, i.e., $\hat{\theta} \leq \theta < 1/(1+X)$ and $1/(1+X) \leq \theta \leq 1$. We refer to these two regions as Region 2 and Region 3, respectively. In Region 2, since the financial system is developed to some extent, the collateral value of the investment increases such that the high-productivity agents can borrow more credit. In the credit market, the real interest rate begins to rise. In this situation, low-productivity agents lose incentives to invest because the rate of return on lending is higher than that of investing. Production is efficient. In other words, through the credit market, all the savings for investment are allocated only to the high-productivity agents. Resource allocation becomes efficient (however, income distribution between the high-and-low agents is not the first best because the borrowing constraint of the high-productivity agents is binding and they earn higher rate of return than low-productivity agents). Thus, the aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint.

In Region 3, since the financial system is well developed, the collateral value of the investment increases and the amount of credit that the high-productivity agents is so high. Consequently, the equilibrium real interest rate in the credit market rises to α^H, which is equal to the marginal rate of return on the investment of the high-productivity agents. Thus, the borrowing constraint of the high-productivity agents is no longer binding. In this situation, through the credit market, all the savings for investment are allocated only to the high-productivity agents. Production and resource allocation are efficient (income distribution becomes the first best because high-and-low agents earn the same rate of return). The aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint.

3. The short-run story

In this section, we investigate the short run effect of an unanticipated monetary policy shock on credit allocation, the aggregate TFP level, and the economic growth rate in each of three regions. In particular, we focus on an impulse response to an unanticipated monetary tightening policy shock. First, we discuss an impulse response in Region 1.

Suppose that at date $s-1$, the economy is in the steady state. Then, there occurs an unanticipated monetary tightening policy shock, i.e., the growth rate of money supply between dates s and $s-1$ becomes lower than what was expected. Consequently, following from money market clearing condition (equation 17), the price level declines. In such a situation, if the debt contract between the borrowers and lenders is nominal and not indexed, the borrowers' real burden of debt repayment increases. This also implies that to the lenders, the real rate of return from lending increases, and consequently, there occurs redistribution of wealth from the debtors to the creditors[8].

This has two effects—positive and negative—on the aggregate net worth of the high-productivity agents at date s. Assuming that p is strictly greater than X/(1 + X), the negative effect is greater than the positive effect[9]. Thus, the aggregate net worth of the high-productivity agents at date s decreases. Consequently, the high-productivity agents have to reduce their investment because the borrowing constraint becomes stronger. Moreover, a greater amount of resources are allocated to the low-productivity agents through the credit market, and resource allocation gets worse.

From equation 12, it follows that as the aggregate net worth of the high-productivity agents at date s decreases, the aggregate TFP level at date $s + 1$ begins to decline. Further, from equation 13, it follows that as the aggregate TFP level begins declining, the economic growth rate at date $s + 1$ also begins to decline. This unexpected monetary tightening policy shock results in business fluctuations through persistent changes in aggregate TFP because it takes a certain amount of time for the aggregate net worth of the high-productivity agents to recover. That is, economic growth is volatile to the shocks. Moreover, the interesting point is that the impulse response of aggregate investment of high-and-low productivity agents is asymmetric. Aggregate investment of low-productivity agents increases while that of high-productivity firms decreases. The impulse response in Region 1 is depicted in Figure 2.1.

Here, it is important to highlight the following two points. First, as evident in equation 13, in this model, aggregate TFP is the main factor that generates business cycles; this is similar to the scenario presented in a standard real business cycle (RBC) model[10]. However, the difference between the two models is that in this model, in the short run, the aggregate TFP level could be affected by an unexpected monetary policy shock. On the other hand, in a standard RBC model, it is affected mainly by the technology shock, which is aggregate TFP shock itself in its theory.

Second, this effect of an unanticipated monetary policy on aggregate TFP is different from BG's model. In BG's model, aggregate TFP is not the main factor generating business fluctuations. Indeed, it is independent of credit allocation changes. In their model, business fluctuations occur because capital input changes over time due to temporal productivity shocks which affect the net worth of borrowers. However, in this model, the main factor generating business fluctuations is aggregate TFP and aggregate TFP itself could be affected by credit allocation changes triggered by an unanticipated monetary policy.

Next, let us discuss an impulse response in Region 2. In Region 2, even if there is an unanticipated monetary tightening shock that causes the net worth share of the high-productivity agents to decrease, as long as the shock is relatively small, the real interest rate declines such that all the savings for investment are still allocated only to the high-productivity agents. Therefore, production does not change. Thus, even if the debt contract is nominal, both business fluctuations would not occur by an unanticipated monetary policy shock[11]. A similar explanation is applicable to an impulsive response in Region 3; however, the only difference is that in Region 3, the real interest rate does not change as long as the shock is relatively small. Impulse responses in Region 2 and Region 3 are depicted in Figure 2.2. We summarize the above discussion as Proposition 2.

Proposition 2: (1) If an economy exists in Region 1, an unanticipated monetary policy results in business fluctuations through persistent changes in aggregate TFP in the short run. Economic growth is volatile to the shocks. The movement of aggregate investment of H-and-L agents is asymmetric. (2) On the other hand, if the economy exists in either Region

2 or 3, even if the debt contract is nominal, as long as a monetary shock is relatively small, business fluctuations through persistent changes in aggregate TFP would not occur. Economic growth would not be volatile to the shocks.

Proof: see appendix

Based on this analysis, we could state that if the financial system is not well developed, the macroeconomic performance is vulnerable to the shocks to the net worth distribution of the agents. On the other hand, if the economy has a well-developed financial system, the volatility in economic performance would be reduced. Thus, a well-developed financial system is important for economic stability. In the case of an unexpected shock to the productivity of both agents, the same results are obtained.

Next, we present a brief discussion with regard to the following two cases: (1) a case in which the monetary shock is large and (2) the case without the borrowing constraint. Suppose that the economy exists in Region 2 and there occurs an unanticipated large monetary tightening shock. Since the net worth share of the high-productivity agents decreases to a large extent, their borrowing constraint becomes stronger. In the credit market, the real interest rate declines to α^L, so that—unlike the case in which the shocks are small—the low-productivity agents begin to undertake investment because it becomes worth investing for them. Thus, both the aggregate TFP level and the economic growth rate will be lower than those in the steady state until the high-productivity agents accumulate enough net worth such that the low-productivity agents no longer invest. Eventually, the economy will return to the original steady state in which only the high-productivity agents undertake investment and production is efficient.

In the case without the borrowing constraint, the investment of the agents does not depend on their net worth. Thus, even if an unanticipated monetary tightening shock reduces the net worth of the high-productivity agents, both the aggregate TFP level and the economic growth rate remain unchanged. This is similar to the result in the case of a standard RBC model with no heterogeneity among agents and no friction in the credit market.

4. The long-run story

In this section, we examine the mechanism with regard to the endogeous determination of resource allocation, the aggregate TFP level and the economic growth rate in a credit-constrained economy in the long run. Here, we present the mechanism for regime switching in an economy.

Proposition 3.1: Suppose that the economy exists in Region 1. Due to a permanent positive (negative) shock on θ, α^H, or X, a greater amount of resources are allocated to the high-productivity agents (low-productivity agents). Consequently, both the aggregate TFP level and the economic growth rate increase (decline) permanently.

Proof: see appendix

From the above proposition, it follows that the permanent positive shock on either θ or α^H results in an increase in the collateral value of investment[12]. Since the borrowing constraint of the high-productivity agents becomes weak, the share of aggregate investment of these agents increases while that of the low-productivity agents decreases. In other words, resource allocation improves so that both the aggregate TFP level and the economic growth rate increase permanently.

With regard to the shock on X, suppose that X declines, for example. This implies that the ratio of the low-productivity agents increases beyond its previous value. Even if θ or α^H does not change, this shock results in a decline in the net worth share of the high-productivity agents, and a greater amount of resources are allocated to the low-productivity agents in the credit market. Therefore, both the aggregate TFP level and the economic growth rate decline permanently[13].

In both these cases, as evident in equation 22, it is important to note that aggregate TFP is the main factor affecting the economic growth rate; this is similar to a standard RBC model. However, the difference between the two models is that in this model, in the long run, the aggregate TFP level could be affected not only by a technology shock but also by the changes in the degree of financial deepening and the shocks to the ratio of the high-productivity agents to low-productivity agents. On the other hand, in a standard RBC model, it is affected mainly by the technology shock.

Proposition 3.2 (Regime Switching Story): Suppose that the economy exists in Region 1. Given p and X, due to relatively large permanent positive shock on either θ or α^H, the economy switches from a regime with a low economic growth rate, a low aggregate TFP level, and inefficient in production to one with a high economic growth rate, a high aggregate TFP level, and efficient in production permanently. In the case of relatively large negative shock on either θ or α^H, the economy switches from Region 2 or 3 to Region 1.

Proof: see appendix

Intuition of the above proposition is as follows. Due to relatively large permanent positive shock on either θ or α^H, the collateral value of investment increases and the borrowing constraint of the high-productivity agents becomes weak. Since the positive shock is relatively large, these agents could increase their investment to a large extent. In the credit market, the real interest rate starts to increase. Because of the increase in the real interest rate, it is no longer worth investing for low-productivity agents. All the savings for investment would be allocated only to high-productivity agents. Therefore, the economy switches from a regime with a low growth rate, a low aggregate TFP level, and inefficient in production to one with a high growth rate, a higher aggregate TFP level, and efficient in production permanently[14].

This regime switching story is different from what has been proposed in KM's model. In their paper, in a credit-constrained economy, there is only one region—Region 1 in this paper—in which production is inefficient. However, in this model, there also exist Regions 2 and 3 in which production is efficient even in the case of a credit-constrained economy. Thus, it could provide scope for a discussion on the economy's switching from an inefficient production equilibrium to an efficient production equilibrium.

With regard to propositions 3.1 and 3.2, it is important to note the following two things. One is that from equation 21, it is evident that the aggregate TFP level in the steady state is independent of the monetary policy parameter (μ). The other is that if θ is greater than $1/(1+X)$, even if there occurs a permanent negative shock on α^H, as long as α^H is greater than α^L, the economy will not enter Region 1 from Region 3. In this sense, a well-developed financial system is important as it prevents an economy from entering an equilibrium with inefficient resource allocation.

5. The relationship between the degree of the development of the financial system and the effectiveness of monetary policy

In this section, we analyze the relationship between the degree of financial deepening and the effectiveness of monetary policy[15]. In order to conduct this analysis, we will modify the model presented in Section 3, and present case 2 about the government policy.

In case 2, the government adopts the following policy.

(23) $$M_t = (1+\mu)M_{t-1} \text{ and } G_t = \beta Y_t$$

Government expenditure G_t is a fraction β of the aggregate output. This is the only difference between cases 1 and 2.

From equations 15, 17, and 23, we obtain the government budget constraint as follows:

(24) $$\beta Y_t = \left(\frac{\mu}{1+\mu}\right) a_2 (1-\tau_t) Y_t + \tau_t Y_t$$

Since Y_t is a predetermined variable, τ_t is endogenously determined in order to satisfy

24. From 24, we can obtain the following:

(25)
$$\tau_t = \frac{\beta - (\frac{\mu}{1+\mu})a_2}{1-(\frac{\mu}{1+\mu})a_2} \equiv \varphi(\mu)$$

From equation 25, it follows that the tax rate at date *t* is a decreasing function of the money growth rate. Intuitively, the more money the government prints, the greater will be the seigniorage revenues. In order to satisfy the government budget constraint, the government can reduce taxes. This implies that the government transfers some of the seigniorage revenues to the agents through a tax cut policy[16].

The Definition of a Competitive Economy (Case 2)

Given the initial Y_1, E_1^H, M_0, the exogenous variables $\theta, \alpha^H, \alpha^L, p, a$, and the monetary and fiscal policies $\mu_t = \mu$ and $G_t = \beta Y_t$, respectively, *a competitive equilibrium* is a sequence of prices and quantities $\left\{C_t, Z_t^H, Z_t^L, B_t, G_t, i_t, P_t, P_{t+1}^e, Y_{t+1}, E_{t+1}^H, TFP_t\right\}_{t=1}^{\infty}$ that satisfies the following conditions:

(1) each high- and low-productivity agent maximizes his/her utility

(2) money and goods market clearing conditions

(3) perfect foresight with regard to the future price level

(4) the GBC for all *t*.

In this case, there exists a unique stationary equilibrium. If the economy exists in Region 1, we can obtain the stationary equilibrium G_t^{BC*}, $(\frac{E_t^H}{Y_t})^*$ and the aggregate TFP* level by using equations 12, 13, 14, 25, and $\frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t}$. Both the aggregate TFP level and the economic growth rate in the steady state are as follows.

(26) $$AggregateTFP^* = \alpha^L + (\alpha^H - \alpha^L)\frac{1}{1-\frac{\theta\alpha^H}{\alpha^L}}(\frac{E_t^H}{Y_t})^*$$

(27) $$(\frac{Y_{t+1}}{Y_t})^* = a_3\left(1-\varphi(\mu)\right)\times AggregateTFP^*$$

As evident in equation 26, the aggregate TFP level in the steady state is an increasing function of the net worth share of the high-productivity agents; however, it is independent of the money growth rate. This is because an increase in the net worth of the agents caused by the monetary easing policy would result in a proportionately equal increase in the aggregate output and the aggregate input. As evident in equation 27, the economic growth rate in the steady state is an increasing function of the money growth rate. Intuitively by printing more money the government can earn an increased seigniorage and thus implement the tax cut policy. Due to this tax cut policy, the agents' net worth increases, and the borrowing constraint of the high-productivity agents weakens. Therefore, the high-productivity agents can finance more investment. Hence, the economic growth rate will increase. By differentiating equation 27, we can obtain the following.

(28) $$\frac{d(\frac{Y_{t+1}}{Y_t})^*}{d\mu} = -a_3\times\varphi'(\mu)\times AggregateTFP^* > 0$$

(29) $$\frac{\partial}{\partial AggregateTFP^*}\left(\frac{\partial(\frac{Y_{t+1}}{Y_t})^*}{\partial\mu}\right) = -a_3\times\varphi'(\mu) > 0$$

From 28 and 29, it follows that the effectiveness of monetary policy on the economic growth rate is an increasing function of aggregate TFP level as long as the economy exists in Region 1. In order to examine the mechanism with regard to this, we first analyze the manner in which the aggregate TFP level in the steady state is determined. From equation 26 and Proposition 1, we understand that the aggregate TFP level in the steady state is an increasing function of θ in Region 1. Therefore, if the economy has a better developed financial system, it can allocate more resources to the high-productivity agents, and consequently, the aggregate TFP level will increase. Thus, 29 implies the following.

(30)
$$\frac{\partial}{\partial\theta}\left(\frac{\partial(\frac{Y_{t+1}}{Y_t})^*}{\partial\mu}\right) = -a_3 \times \varphi'(\mu) \times \frac{\partial AggregateTFP^*}{\partial\theta} > 0$$

From 30, in Region 1, it is evident that the more developed the financial system, the higher is the effectiveness of monetary policy. Intuitively, in order to determine the effectiveness of monetary policy, the sensitivity of investment to the changes in the net worth is crucial. From equation 9, we understand that investment in economies with better developed financial systems displays greater sensitivity.

For example, when a monetary easing policy is adopted, the net worth of the agents increases. In the case of economies with better developed financial systems, the sensitivity of investment to this increase in the net worth is high. In another word, in this model, because of monetary easing policy, redistribution from old agents to young agents occurs. That is, more resources will flow to young agents. In better developed financial system, greater fraction of them will be allocated to high-productivity agents. Consequently, the effectiveness of monetary policy is also high. If we interpret the shocks on θ as the shocks on the function of the financial system, this implies that the effectiveness of monetary policy could be reduced by negative shocks on the function of the financial system[17].

However, in case the economy exists in either Region 2 or 3, the aggregate TFP level in the steady state is independent of θ (the same as equation (7)). Thus, the effectiveness of monetary policy does not depend on the degree of the development of the financial system. This implies that even if there is a negative shock on θ, production does not become inefficient. Only high-productivity agents invest. This is because the economy has already had better developed financial system before the shock. Therefore, the effectiveness of monetary policy would not be affected. We summarize the above discussion as Proposition 4.

Proposition 4: (1) In Region 1, the more developed the financial system is, the higher is the effectiveness of monetary policy. (2) In Regions 2 and 3, the effectiveness of monetary policy does not depend on the degree of the development of the financial system.

Proof: see appendix

This result is applicable to the case of X. Think about the case where X declines. This might be interpreted as a decline of the ratio of high-productivity industry to low-productivity industry. For example, if a delay of the changes in the industrial structure from low to high industry occurs, X would decline. If the economy exists in region 1, the effectiveness of monetary policy will be reduced with it. This is because even if monetary easing policy is conducted, more resources will be allocated to low-productivity industry, so that the effectiveness of monetary policy will be reduced. However, in region 2 or 3, the effectiveness is independent of X.

6. Conclusion

In this paper, we developed theoretical analyses in order to answer the following questions. (1) How do the changes in financial depth affect resource allocation (credit allocation), an aggregate TFP level, and an economic growth rate? (2) How does the extent of business fluctuations depend on the degree of the development of the financial system? (3) What is the relationship between the degree of the development of the financial system and policy effectiveness such as the effectiveness of monetary policy?

With regard to question 1, there are three stages of financial deepening, corresponding to three different regions of the θ value. In Region 1, the borrowing constraint of high-productivity agents binds, production and resource allocation are inefficient. The aggregate TFP level and the economic growth rate are low. In Region 2, the borrowing constraint of high-productivity agents becomes weak and production and resource allocation become efficient because all the savings are allocated only to high-productivity agents. In Region 3, the borrowing constraint of high-productivity agents no longer binds. Production and resource allocation are efficient. In Region 2 and Region 3, the aggregate TFP level and the economic growth rate are high.

With regard to question 2 and question 3, volatility of economic growth and the effectiveness of monetary policy depend on the degree of the development of the financial system. If the financial system is not well developed and is in Region 1, an unanticipated monetary policy shock results in business fluctuations through persistent changes in aggregate TFP. That is, economic growth is volatile to the shocks. And the more developed the financial system, the higher is the effectiveness of monetary policy. This implies that the effectiveness of monetary policy could be reduced by negative shocks on the function of the financial system.

However, if the financial system is developed to some extent and is in Region 2 or 3, business fluctuations would not occur as long as the shock is small. That is, economic growth would not be volatile to the shocks. In this sense, well-developed financial system is important to reduce volatility of macroeconomic performance. And the effectiveness of monetary policy is independent of the degree of the development of the financial system. This implies that the effectiveness of monetary policy would not be affected by negative shocks on the function of the financial system. In future studies, we aim to conduct theoretical analyses on how both investment (capital) and labor adjust according to the degree of financial deepening when shocks occur.

Appendix

1. Proof of Proposition 1

Here, we will verify that in the case where $\theta < \hat{\theta}$, the real interest rate is equal to the marginal rate of return on the investment of the low-productivity agents, the borrowing constraint of the high-productivity agents binds, and both the high- and low-productivity agents undertake investment in the neighborhood of the steady state. In order to verify this, we only need to assess whether the low-productivity agents invest a positive amount of goods.

If the net worth share of the high-productivity agents in the steady state $(\frac{E_t^H}{Y_t})^*$ takes the value $1-\frac{\theta\alpha^H}{\alpha^L}$, the low-productivity agents' aggregate investment becomes zero in the steady state. Introducing the condition $(\frac{E_t^H}{Y_t})^* = 1-\frac{\theta\alpha^H}{\alpha^L}$ in equation 20, we obtain the value of θ $(=\hat{\theta})$, where the low-productivity agents' aggregate investment becomes zero in the steady state. If θ is strictly less than $\hat{\theta}$, the low-productivity agents' aggregate investment takes a positive value in the neighborhood of the steady state. If these agents invest a positive amount, the real interest rate is equal to the marginal rate of return on their investment such that the borrowing constraint of the high-productivity agents is binding. Since both high and low productive agents invest, both the aggregate TFP level and the economic growth rate are lower than those of the case without the borrowing constraint.

If $\hat{\theta} \leq \theta < 1/(1+X)$, from the following two equations, we obtain a unique stationary equilibrium in which the net worth share of the high-productivity agents remains constant over time. $\frac{E_{t+1}^H}{Y_{t+1}} = p(1-\theta) + X(1-p)\theta$ and $\frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t}$.

$$\therefore (\frac{E_t^H}{Y_t})^* = p(1-\theta) + X(1-p)\theta$$

If $\hat{\theta} \leq \theta < 1/(1+X)$ and p is not equal to 1, the following inequality is satisfied.

(31) $$1-\frac{\theta\alpha^{H}}{\alpha^{L}} \le p(1-\theta)+X(1-p)\theta < 1-\theta$$

From equation 31, the equilibrium condition in the credit market is as follows:

$$\left(\frac{E_t^H}{Y_t}\right)^* \Big/ \left(1-\frac{\theta\alpha^H}{r^*}\right)=1 \quad \text{and} \quad r^* \in [\alpha^L, \alpha^H)$$

This implies that all the funds supplied in the credit market by the low-productivity agents are provided for the borrowing needs of the high-productivity agents. Thus, both the aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint. Since the real interest rate is lower than α^H, the borrowing constraint of the high-productivity agents binds.

If $1/(1+X) \le \theta \le 1$, from the following two equations, we obtain a unique stationary equilibrium in which the net worth share of the high-productivity agents remains constant over time. $\frac{E_{t+1}^H}{Y_{t+1}} = p\left(\frac{E_t^H}{Y_t}\right) + X(1-p)\left(1-\frac{E_t^H}{Y_t}\right)$ and $\frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t}$.

$$\therefore \left(\frac{E_t^H}{Y_t}\right)^* = \frac{X}{1+X}$$

If $\theta \ge 1/(1+X)$, the following inequality is satisfied.

(32) $$X/(1+X) \ge 1-\theta$$

From inequality 32, the equilibrium condition in the credit market is as follows:

$$\left(\frac{E_t^H}{Y_t}\right)^* \Big/ \left(1-\frac{\theta\alpha^H}{r^*}\right) \ge 1 \quad \text{and} \quad r^* = \alpha^H$$

This implies that all the funds supplied in the credit market by the low-productivity agents are provided for the borrowing needs of the high-productivity agents. Thus, both the aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint. Since the real interest rate is equal to α^H, the borrowing constraint of the high-productivity agents does not bind.

2. Proof of Proposition (1) of 2

We define $\frac{E_t^H}{Y_t}$ as s_t. Accordingly, equation 14 becomes as follows.

(33) $$s_{t+1} = \phi(s_t)$$

At date *s*, suppose that there is an unanticipated monetary tightening shock. Consequently, the net worth share of the high-productivity agents at date *s* $\frac{E_{t+s}^H}{Y_{t+s}}$ declines. Taking linear approximation with regard to equation 33 in the neighborhood of the steady state, we obtain the following:

(34) $$s_{t+s+1} = \phi(s^*) + \phi'(s^*)(s_t - s^*)$$

From equation 34, we obtain the following equation.

(35) $$\hat{s}_{t+s+n} = \phi'(s^*)^n \hat{s}_t \qquad \text{where } \hat{s}_{t+s} \equiv \frac{s_{t+s} - s^*}{s^*},\ \phi'(s^*) < 1$$

If we take that $n \to \infty$, we obtain $\phi'(s^*)^n \to 0$ and $\hat{s}_{t+s+n} \to 0$.

Therefore, it takes a certain amount of time for the net worth share of the high-productivity agents to recover. From equations 12 and 13, if the net worth share of the high-productivity agents is changing over time, both the aggregate TFP level and the economic growth rate will also change until the net worth share of the high-productivity agents returns to the original value in the steady state.

Proof of Proposition (2) of 2

In Region 2, where $\theta < \hat{\theta}$, as long as the shock is small, the equilibrium condition in the credit market does not change. The equilibrium condition in the credit market is as follows:

$$\frac{E_t^H}{Y_t} \Big/ \left(1 - \frac{\theta\alpha^H}{r_t}\right) = 1 \quad \text{and} \quad r_t \in [\alpha^L, \alpha^H)$$

If *p* is greater than X(1 – *p*), an unanticipated monetary tightening policy shock results in a decline in the net worth share of the high-productivity agents $\frac{E_t^H}{Y_t}$. In order to satisfy the equilibrium in the credit market, the real interest rate has to decline. As a result of this adjustment, all the funds supplied in the credit market by the low-productivity agents are still provided for the borrowing needs of the high-productivity agents. Thus, both the economic growth rate and the aggregate TFP level remain unchanged.

In Region 3, where $1/1+X < \theta \leq 1$, as long as the shock is small, the equilibrium condition in the credit market does not change, and it remains as follows:

$$\left(\frac{E_t^H}{Y_t}\right)^* \Big/ \left(1 - \frac{\theta\alpha^H}{r^*}\right) \geq 1 \quad \text{and} \quad r^* = \alpha^H$$

In such a case, all the funds supplied in the credit market by the low-productivity agents are still provided for the borrowing needs of the high-productivity agents. Thus, both the economic growth rate and the aggregate TFP level are unchanged. However, in a case where $\theta = 1/1+X$, a negative (positive) shock on the aggregate net worth of high productive agents results in decreasing (maintaining) the real interest rate, and the economic growth rate and the aggregate TFP level remain unchanged.

3. Proof of Proposition 3.1

From equations 14 and $\frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t}$, we obtain the following:

$$\frac{d\left(\frac{E_t^H}{Y_t}\right)^*}{d\theta} > 0, \quad \frac{d\left(\frac{E_t^H}{Y_t}\right)^*}{d\alpha^H} > 0, \quad \frac{d\left(\frac{E_t^H}{Y_t}\right)^*}{dX} > 0$$

Considering this relationship, from equations 12 and 13, we obtain the following:

$$\therefore \frac{d\left(\frac{Y_{t+1}}{Y_t}\right)^*}{d\theta} > 0, \quad \therefore \frac{dAggregateTFP^*}{d\theta} > 0$$

$$\text{and} \quad \therefore \frac{d\left(\frac{Y_{t+1}}{Y_t}\right)^*}{d\alpha^H} > 0, \quad \therefore \frac{dAggregateTFP^*}{d\alpha^H} > 0$$

$$\text{and} \quad \therefore \frac{d\left(\frac{Y_{t+1}}{Y_t}\right)^*}{dX} > 0, \quad \therefore \frac{dAggregateTFP^*}{dX} > 0$$

Thus, a permanent negative shock on α^H, X, or θ results in a permanent decline in both the aggregate TFP level and the economic growth rate.

4. Proof of Proposition 3.2

First, we prove the case with regard to θ. We consider a positive shock on θ. Suppose that the economy exists in Region 1. In Region 1, the inequality, $\theta < \hat{\theta}$ is satisfied. If a permanent positive shock results in a rise in θ such that it is greater than $\hat{\theta}$ and $\hat{\theta} \leq \theta < 1/(1+X)$. From Proposition 1, it follows that the economy shifts from Region 1 to Region 2 permanently. If θ becomes greater than 1/(1+X), the economy shifts from Region 1 to Region 3 permanently.

Next, we prove the case with regard to α^H. We consider a negative shock on α^H. Suppose that the economy exists in Region 2. In Region 2, the following inequality is satisfied.

$$p(1-\theta) + X(1-p)\theta \geq 1 - \frac{\theta\alpha^H}{\alpha^L}$$

We define α^H satisfying $p(1-\theta) + X(1-p)\theta = 1 - \frac{\theta\alpha^{H'}}{\alpha^L}$ as $\alpha^{H'}$.

As long as θ is strictly less than 1/(1 + X), $\alpha^{H'}$ is strictly greater than α^L. If α^H is strictly less than $\alpha^{H'}$, the economy exists in Region 1 in the neighborhood of the steady state. This is because the net worth share of the high-productivity agents in the steady state $(\frac{E_t^H}{Y_t})^*$ satisfies the following inequality.

$$(\frac{E_t^H}{Y_t})^* < p(1-\theta) + X(1-p)\theta < 1 - \frac{\theta\alpha^H}{\alpha^L}$$

If a permanent negative shock results in a decline in α^H such that it is strictly lower than $\alpha^{H'}$, the economy shifts from Region 2 to Region 1.

5. Proof of Proposition (1) of 4

By differentiating equation 27, we obtain the following:

$$\therefore \frac{d\,(\frac{Y_{t+1}}{Y_t})^*}{d\,\mu} = -a_3 \times \varphi'(\mu) \times AggregateTFP^* > 0$$

From Proposition 3.1, it follows that $\frac{dAggregateTFP^*}{d\theta} > 0$. Considering this relationship, we obtain the following:

$$\therefore \frac{\partial}{\partial\theta}\left(\frac{\partial(\frac{Y_{t+1}}{Y_t})^*}{\partial\mu}\right) = -a_3 \times \varphi'(\mu) \times \frac{\partial AggregateTFP^*}{\partial\theta} > 0$$

Thus, if the economy exists in Region 1, the more efficient the financial system, the stronger is the effect of monetary policy on the economic growth rate.

Proof of Proposition (2) of 4

In regions 2 and 3, both the aggregate TFP level and the economic growth rate in the steady state are as follows.

$$AggregateTFP^* = \alpha^H$$

$$(\frac{Y_{t+1}}{Y_t})^* = \alpha^H a_3\left(1-\varphi(\mu)\right)$$

Therefore, it is obvious that the effect of monetary policy on the economic growth rate is independent of the degree of efficiency of the financial system, θ.

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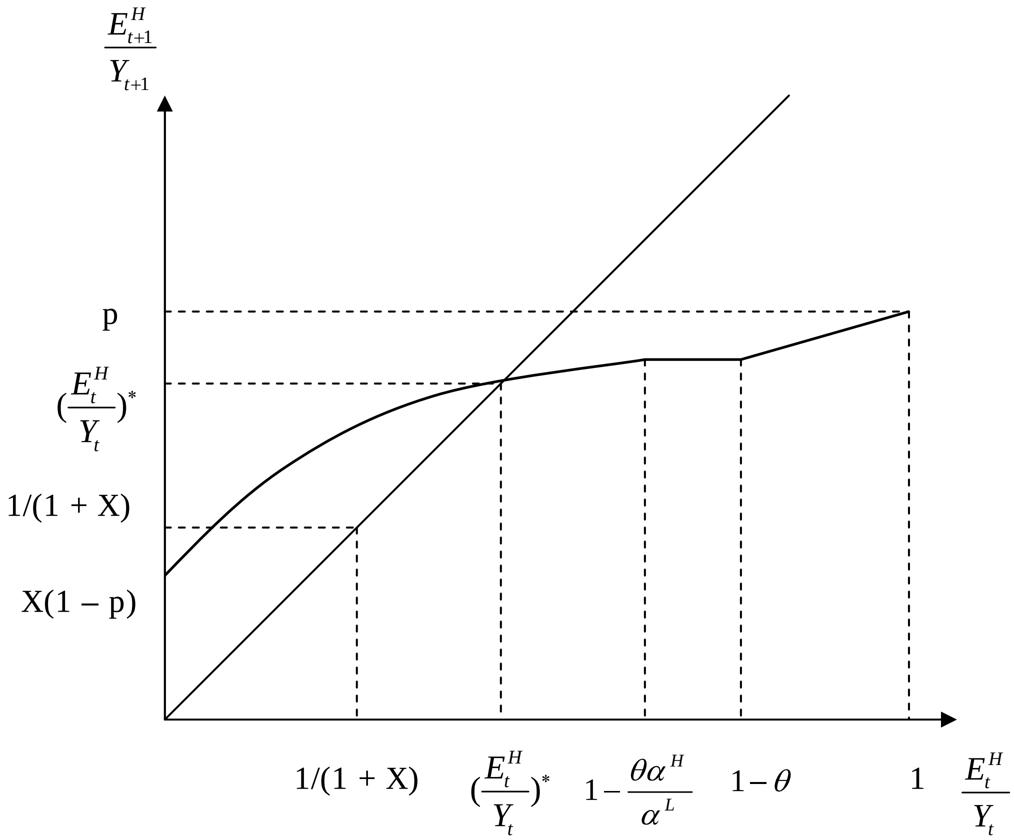
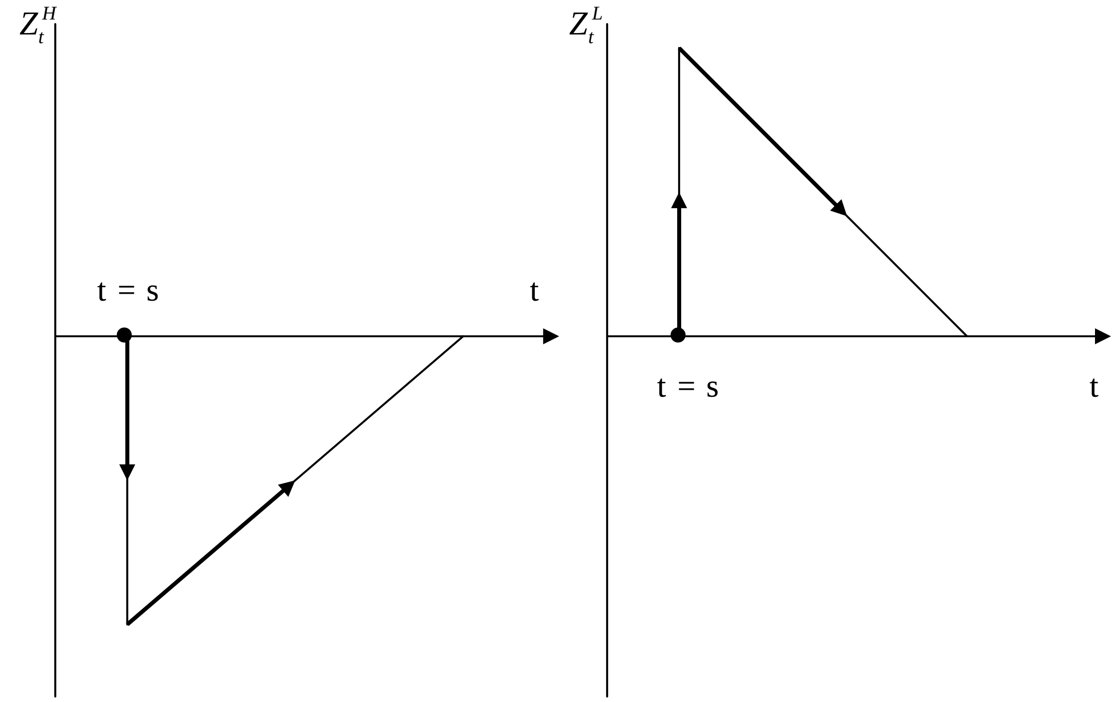
Figure 1

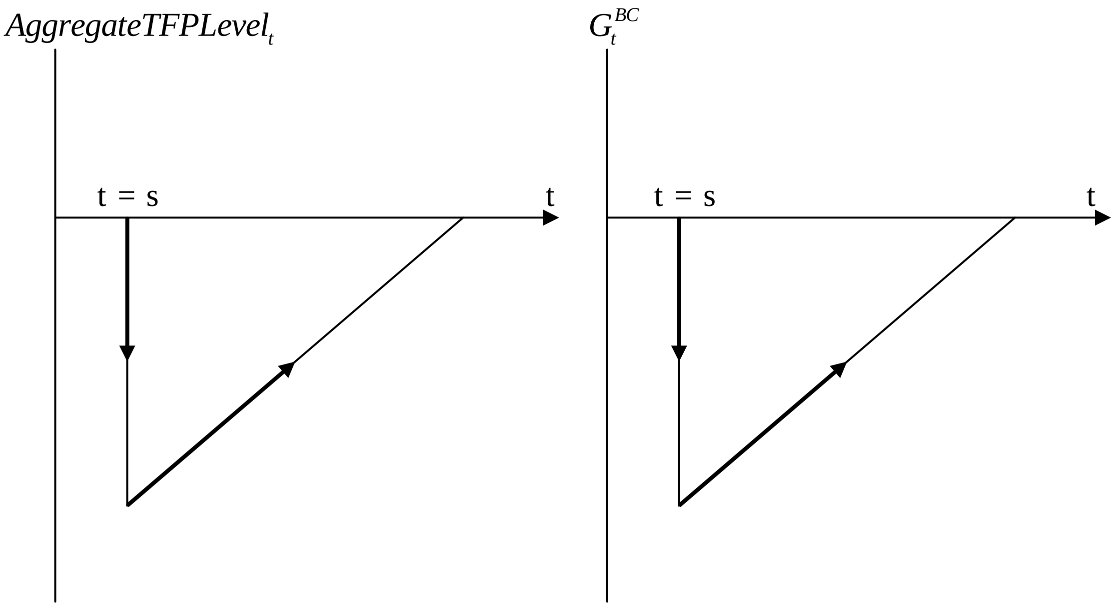


The Movement of the Net Worth Share of High-Productivity Agents

Figure 2.1

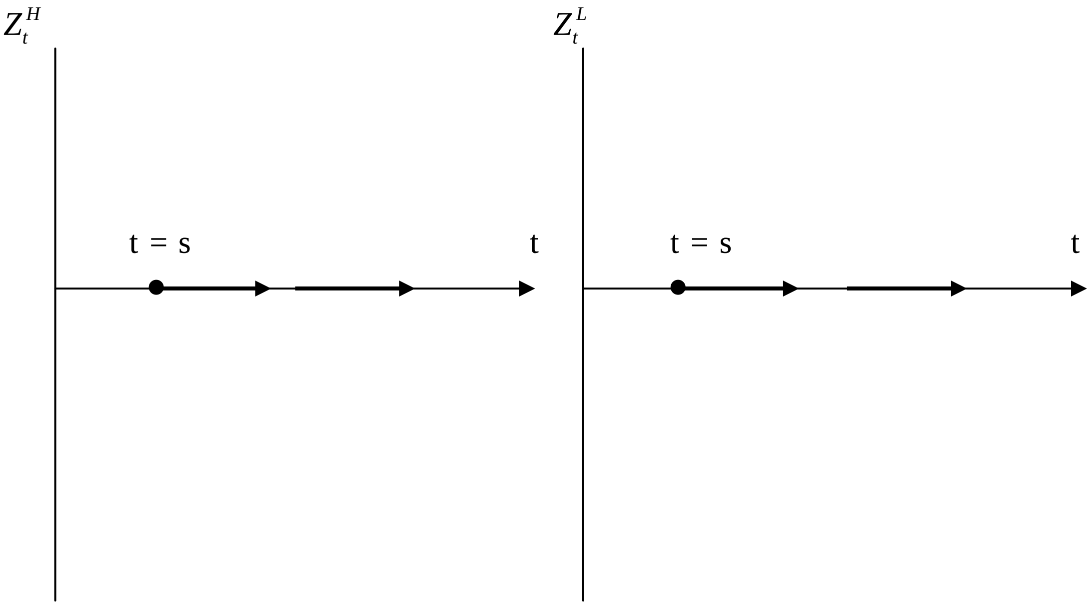


An Impulse Response in Region 1

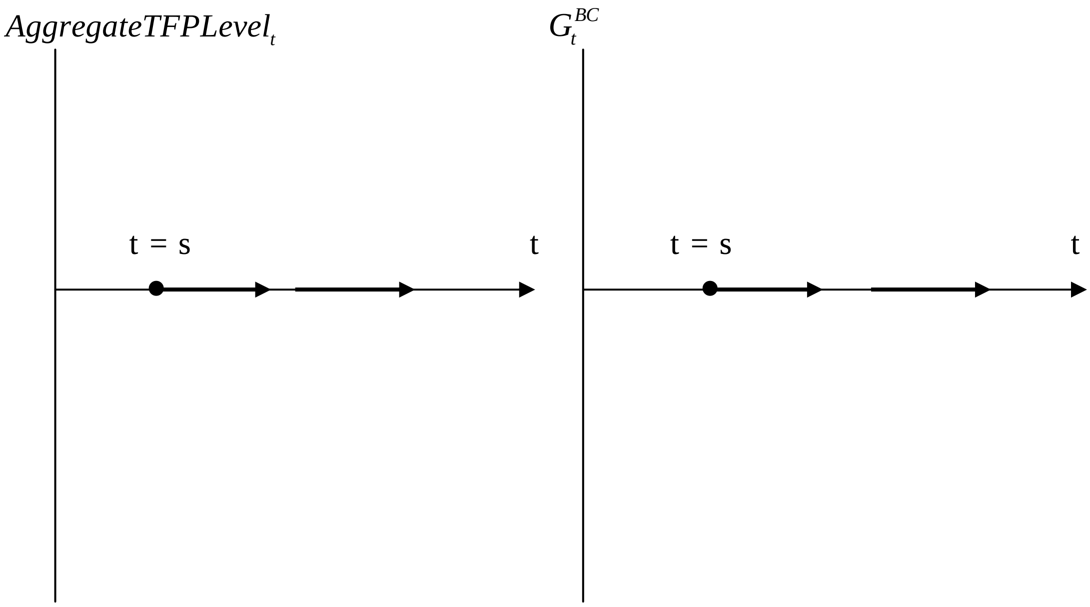


An Impulse Response in Region 1

Figure 2.2



An Impulse Response in Region 2 or 3



An Impulse Response in Region 2 or 3

Footnotes

1. I am grateful for feedback from participants of seminars at Keio University, Sophia University, the University of Tokyo, and the Annual Meeting of Japanese Economic Association, Osaka City University, 20-21 October 2006. In particular, I would like to thank Toni Braun, Fumio Hayashi, Kazuhito Ikeo, Kikuo Iwata, Satoshi Kawanishi, Nobuhiro Kiyotaki, Ryutaro Komiya, Fukuju Yamazaki and Noriyuki Yanagawa for their helpful discussions and comments. Please address any correspondence to Tomohiro Hirano, COE Researcher, Department of Economics, The University of Tokyo, Japan, email: tomohih@hotmail.com.
2. Goldsmith [1969], McKinnon [1973], and Shaw [1973] are among the pioneer contributions. For more recent contributions, see e.g., see Levine [1997] and Demirguc-Kunt and Levine [2001].
3. See Townsend [1983] and Banerjee and Newman [1993] as pioneer works.
4. In this regard, this paper might be also related to a recent series of depression studies such as Hayashi and Prescott [2002], and Kehoe and Prescott [2002]. They found that during recent Japan's stagnation, the U.S. Great depression, and recent depressions in some countries, aggregate TFP declined. They concluded that it was this decline in aggregate TFP that was crucial in accounting for these depressions. However, the mechanism of its decline is not necessarily clear. Since this paper presents theoretical analyses on the underlying sources of aggregate TFP, the present analyses might have some implications about it from a theoretical perspective.
5. We can obtain the same result with regard to a case in which these two values are different. As a matter of fact, in this model, the important factor that affects the aggregate variables is θ^H, and not θ^L

6. This is based on Hart and Moore [1994]. See La Porta, Lopez-de-Silanes, Shleifer, and Vishny [1998, 2002] for legal protection. See Aghion, Hart, and Moore for the analyses of bankruptcy procedures.

7. The formulation of cash-in-advance constraint in this paper is the same as the one in Hahn and Solow's [1995, chapter 2] model. As they pointed out, this is somewhat ad hoc manner. Since we are not concerned with the first principles of monetary theory, we shall adopt this device. Crettez B, Philippe M, and Bertrand W [1999] discuss monetary policy effect using overlapping generations model with several formulations on cash-in-advance constraints including the type in this paper.

8. Fisher [1933] and Tobin [1980] emphasize this redistribution of wealth as the monetary transmission mechanism.

9. If p is equal to X/(1 + X), the size of the positive and negative effects are same. In such a case, no dynamics will occur. If p is less than X/(1 + X), the positive effect is greater than negative effect. In such a case, an oscillation will occur.

10. See Kydland and Prescott [1988] for a standard RBC model.

11. We exclude the case in which $\theta = \hat{\theta}$ in Region 2. In such a case the negative shock on the aggregate net worth of the high-productivity agents, persistent occurs; however, by the positive shock, persistent does not occur. This is because by the positive shock, the real interest rate increases in the credit market such that all the savings for investment are allocated only to the high-productivity agents.

12. The reforms in bankruptcy laws and bankruptcy procedures could affect θ.

13. Cabellero al et. [2005] proposed a zombie hypothesis to explain recent Japan's stagnation. According to this hypothesis, because of the ever-greening policy by banks, zombie agents could survive while potential and more productive agents could not enter the market. If we consider their views in terms of our model, the

ratio of the low-productivity agents increases; this implies that X declines.

14. Raphael al et. [2002] showed that although Chile and Mexico experienced severe economic crisis in the early 1980s, Chile recovered much faster than Mexico because of the earlier reforms in banking and bankruptcy procedures. This case might be thought of as a positive shock on θ. Azariadis and Smith present a model of Regime-Switching in Business Cycles in the presence of adverse seletion.
15. Hirano [2006] theoretically showed ineffectiveness of monetary policy focusing on the balance sheet effect. He showed that the balance sheet effect is nonlinear depending on a firm's balance sheet condition. Once the economy is in a state of deep recessions, where a firm's balance sheet condition is relatively bad, the effectiveness of monetary policy becomes extremely weak because the balance sheet channel does not function effectively. However, this paper does not relate the effectiveness of monetary policy to the degree of the development of the financial system.
16. The use of seigniorage is critical for determining monetary policy effect. If the government transfers seigniorage only to the retiring agents, who do not possess investment project, monetary policy does not affect the real variables in this model.
17. Because θ implies the liquidation value of the agents (the investment project), it could be affected by the smoothness of coordination between the creditors in the event of borrowers' default. Considering this point, one possible example of negative financial system shocks is that for example, in Japan, since the shock of bubble burst in the beginning of the 1990s was so large, many banks were damaged to a large extent. Because of this, it was said that coordination between the creditors in the liquidation of the bankrupt agents did not proceed smoothly because banks wanted to avoid additional damages. This might have reduced θ temporarily. At the same time, the effectiveness of monetary policy in Japan after the bubble burst does not

appear to be strong enough to push the economy even though Bank of Japan has been taking a monetary easing policy for a long period of time. From the present analysis, this ineffectiveness might be related to negative financial system shocks mentioned above.