Relative Performance Evaluation between Multitask Agents[1]

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First Version: March 10, 2003

This Version: April 20, 2006

[1] Matsushima (2003) was an earlier version of this paper; it was based on Matsushima (1994), which was written in Japanese. This research was supported by a Grant-In-Aid for Scientific Research (KAKENHI 15330036) from the Japan Society for the Promotion of Science (JSPS) and the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of the Japanese government.

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Abstract

We investigate the moral hazard problem in which the principal delegates multiple tasks to two agents. She imperfectly monitors the action choices by observing the public signals that are correlated through the macro shock and that satisfy conditional independence. When the number of tasks is sufficiently high, relative performance evaluation functions effectively for unique implementation, where the desirable action choices are supported by an approximate Nash equilibrium, and any approximate Nash equilibrium virtually induces the first-best allocation. Thus, this is an extremely effective method through which the principal divides the workers into two groups and makes them compete with each other.

Keywords: Multitask Agency, Moral Hazard, Relative Performance Evaluation, Unique Implementation, Group Incentives.

JEL Classification Numbers: D20, D80, J33, L23

1. Introduction

This paper investigates the agency problem in which a principal delegates *multiple* tasks to *two* agents. In this case, the principal is faced with a *moral hazard* problem as she is unable to directly observe the action choices adopted by the agents for their respective tasks, but can only imperfectly monitor them by observing the *public signal* for each task that is drawn randomly and is dependent on the action choice for the task. In order to incentivize the agents to adopt the *desirable* action choices for all the tasks, the principal designs a *punishment rule* that is dependent on the observed public signals. Based on this, the principal decides whether or not she should fine each agent a *monetary* amount.

In this paper, we will show that as compared with a case in which only few tasks are delegated to each agent, it is easier for the principal to incentivize the agents when a large number of tasks are delegated. This statement implies the following. Consider a case in which the principal hires multiple workers and assigns each worker a single task. Instead of contracting with each worker individually, the principal divides them into two working groups and regards these groups as the agents with whom she makes a contract. Therefore, the members of each group agree to jointly adopt the action choices for their tasks and maximize the sum of their expected payoffs. In this paper, we show that the establishment of such working groups along with relative performance evaluation might be an extremely effective method enabling the principal to resolve the moral hazard problem.

We assume that the public signals are correlated through a randomly drawn *macro shock*; the realization of this shock is *unobservable* by the agents and the principal. The public signals for all the tasks depend not only on this common macro shock but also on their respective *private* factors; these factors are also unobservable by the agents and the principal. In this case, we assume *conditional independence*, i.e., given the occurrence of a macro shock, the public signals are drawn randomly and independently.

We specify a punishment rule on the basis of the concept of *relative performance evaluation* as follows. Each agent’s performance is measured by the proportion of tasks performed by him/her for which *good* public signals occur. If an agent’s performance is unsatisfactory as compared with that of the other agent, the principal will fine this agent according to the relative performance evaluation method. However, if an agent’s performance is almost identical to that of the other agent, but sufficiently unsatisfactory in the absolute sense, the principal will fine this agent according to the *absolute* performance evaluation method.

In this paper, we show that the concept of relative performance evaluation functions very effectively, particularly when the number of tasks is sufficiently large. Note that according to the Law of Large Numbers, the private factors for all the tasks delegated to each agent can cancel out each other. This implies that by using the relative performance evaluation method, the principal can almost perfectly detect whether or not an agent deviated from the desirable action choices, as long as the other agent adopts the desirable action choices for all the tasks assigned to him/her. Hence, it follows that the agents have an incentive to adopt the desirable action choices for all the tasks as an *approximate* Nash equilibrium, where each agent's gains from the deviation are either negligible or less than zero.

Moreover, note that each agent is incentivized to perform slightly better than the other agent whenever the latter deviates from the desirable action choices for a non-negligible number of the tasks. By doing so, the agent can almost certainly escape the punishment based on the absolute performance evaluation method and consequently help the principal in detecting the deviation of the other agent. This implies that *unique* implementation is virtually possible, i.e., *any* approximate Nash equilibrium can induce the agent to adopt the desirable action choices for almost all the tasks.

Many previous works have studied relative performance evaluation in the context of the moral hazard problem, for instance, Holmstrom (1982), Lazear and Rosen (1981), Green and Stokey (1983), and Nalebuff and Stiglitz (1983). However, these studies generally investigated the case in which each agent is delegated a single task.[3] They showed that in comparison with independent evaluation, relative performance evaluation provides for better risk sharing. However, the presence of the private factors generally prevents relative performance evaluation from achieving the first-best allocation even in the approximate sense, i.e., from achieving the desirable action choices without any substantial welfare loss that is caused by the risk-averse agents' risk sharing. In contrast, this paper shows that relative performance evaluation can virtually achieve the first-best allocation when the number of tasks is sufficiently large and conditional independence is assumed; this is because the private factors can be cancelled out.

This permissive result is robust with respect to the agents' *limited liability* constraints. In fact, in cases where the principal can perfectly monitor the agents' action choices and the number of tasks is sufficiently large, whenever the upper bound of the monetary fine is large enough to incentivize the agents, the principal can generally incentivize them even when faced with the moral hazard problem with imperfect

[3] An exception is Franckx, D'Amato, and Brose (2004), which extended Lazear and Rosen (1981) to a multitask setting. See also Battaglini (2005).

monitoring.[4]

Several works, such as Mookherjee (1984), Demski and Sappington (1984), Ma (1988), and Battaglini (2005), have investigated unique implementation in the context of the moral hazard problem. When each agent is delegated only a few tasks and there exist private factors with regard to these tasks, the concept of relative performance evaluation does not function effectively for unique implementation. Hence, instead of employing this concept, these papers demonstrated alternative concepts of mechanism design in order to eliminate the unwanted equilibria; some of these were related to the concepts of mechanism design that were explored in the adverse selection literature regarding the implementation of social choice functions.[5] In contrast, this paper shows that if a sufficiently large number of tasks are delegated, relative performance evaluation can compel each agent to "blow the whistle" with regard to the other agent's deviation in exchange for an exemption from the punishment based on the absolute performance evaluation method; thus, this will be the driving force behind the relative performance evaluation method and would enable the principal to eliminate any unwanted equilibria.

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[4] Legros and Matsushima (1991) investigated mechanism design in the context of the moral hazard problem for partnerships with limited liability.

[5] See, for instance, Moore (1992), Palfrey (1992), Osborne and Rubinstein (2004, Chapter 10), and Maskin and Sjöström (2002).

In the agency literature, several studies such as Varian (1990), Holmstrom and Milgrom (1990), and Itoh (1993) have analyzed cases in which there exist multiple workers and demonstrated the superiority of group decisions over individual decisions. In these studies, it was assumed that the members of each group mutually observe their action choices and design a side contract contingent on these choices; this contract is enforceable in non-judicial ways such as word of honor. For example, Tirole (1992) explains the manner in which the hidden side-contracting technology can be specified. Generally, these works only studied the behavior of a group that includes all the workers; nevertheless, in case of the occurrence of a *macro shock*, even this group has the incentive to deviate. In contrast, this paper examines a case in which there exist two separate groups that are identical in terms of the number of members and that compete with each other based on relative performance evaluation—which functions effectively, particularly when a macro shock occurs.

Moreover, some previous works regarding multitask incentives used the Law of Large Numbers to cancel out the private factors. For instance, see bundling goods by a monopolist (Armstrong (1999)), multimarket contact (Matsushima (2001)), and linking mechanisms (Jackson and Sonnenschein (2005) and Matsushima, Miyazaki, and Yagi (2006)).

This paper is organized as follows. Section 2 presents the model. Section 3 specifies a punishment rule based on relative performance evaluation. The main theorem and its logical core are presented in section 4. Section 5 provides the complete proof of this theorem.

2. The Model

A principal hires agents 1 and 2 and delegates *multiple* tasks to each of them as follows. n number of tasks are delegated to each agent $i \in \{1,2\}$, i.e., the tasks $(i,1)$, $(i,2)$, …, and (i,n), where $n > 0$ is a positive integer. Thereafter, each of them selects a *strategy* $a_i = (a_{i,h})_{h=1}^n$, where $a_{i,h}$ implies the *action* choice for task (i,h). Let $A_{i,h} \equiv \{0,1\}$ denote the set of actions for task (i,h). In this case, action 1 for task (i,h), i.e., $a_{i,h} = 1$, implies the *desirable action choice* for this task, whereas action 0 for task (i,h), i.e., $a_{i,h} = 0$, implies the undesirable action choice for this task. Let $A_i = \underset{h\in\{1,\dots,n\}}{\times} A_{i,h}$ denote the set of strategies for agent i. Let $A = A_1 \times A_2$ denote the set of strategy profiles and $a^n = a = (a_1, a_2) \in A$ denote a strategy profile. Therefore, the *desirable strategy profile* is denoted by $a^{n*} = a^* = (a_1^*, a_2^*) \in A$, which is defined as

$$a_{i,h}^* = 1 \text{ for all } i \in \{1,2\} \text{ and all } h \in \{1,\dots,n\}.$$

The principal is faced with a *moral hazard* problem in which she cannot observe the agents' action choices but can only imperfectly monitor them by observing the *public signal* $\omega_{i,h}$ for each task (i,h). Let $\Omega_{i,h} \equiv \{1,2\}$ denote the set of public signals for task (i,h). The public signal $\omega_{i,h} \in \Omega_{i,h}$ for each task (i,h) is *randomly* drawn according to the probability function that depends on the action choice $a_{i,h}$ for this task. With regard to task (i,h), the public signal $\omega_{i,h} = 1$ implies the *good* signal, whereas the public signal $\omega_{i,h} = 0$ implies the bad signal for this task. Let $\Omega_i \equiv \underset{h \in \{1,...,n\}}{\times} \Omega_{i,h}$ denote the set of public signal profiles for agent $i's$ tasks, and let $\Omega \equiv \Omega_1 \times \Omega_2$ denote the set of public signal profiles.

The public signals are imperfectly *correlated* across all the tasks as follows. There exists a *macro shock* θ that is *unobservable* by not only the principal but also the agents, and it is randomly drawn according to the probability density distribution $f(\theta)$ on the interval $[0,1]$, where $f(\theta) > 0$ for all $\theta \in [0,1]$, and $\int_{\theta=0}^{1} f(\theta) d\theta = 1$. Let us fix an arbitrary non-negative real number $\alpha \geq 0$. Therefore, there exists an increasing and continuous function $p : [0,1+\alpha] \to [0,1]$ such that for each $(i,h) \in \{1,2\} \times \{1,...,n\}$, $p(a_{i,h} + \alpha\theta)$ is the probability that the principal will observe the good signal $\omega_{i,h} = 1$ for task (i,h), provided the macro shock θ occurs and agent i selects action $a_{i,h}$ for this task. Hence, the public signals are correlated *through* the randomization of the macro shock. Since p is increasing with respect to θ, it follows that the stronger the macro shock θ, the better is the business for each task. Since $p(1+\alpha\theta)$ is greater than $p(\alpha\theta)$ for all $\theta \in [0,1]$, it follows that the probability of the occurrence of the good signal for a task when action 1 is adopted is greater than that when action 0 is adopted. Since the principal is unable to observe the occurred macro shock and the chosen strategy profile, she is unable to verify whether the occurrence of the good public signals for the tasks was due to the agent's adoption of the desirable action choices or the occurrence of a strong macro shock. We assume *conditional independence*, i.e., given the occurrence of the macro shock, the public signals are drawn randomly and *independently*. This implicitly assumes that there exist some *private factors* for each task that are drawn randomly and independently of each other and that influence the realization of the public signal.

The payoff for agent i when he/she selects a strategy $a_i \in A_i$ and receives a *monetary* transfer $t_i \in R$ is given by

$$u(t_i) - \frac{\sum_{h=1}^{n} a_{i,h}}{n},$$

where $u : R \to R$ is an increasing function and $\frac{a_{i,h}}{n}$ implies the cost of selecting the action for task (i,h). Note that the desirable action choice is more costly than the undesirable action choice. Without loss of generality, we assume that

$u_i(0) = 0$ for each $i \in \{1,2\}$.

In order to incentivize the agents to select the desirable strategy profile a^*, the principal will design a *punishment rule for each agent* $i \in \{1,2\}$ that is defined as a function $x_i : \Omega \to [-H, 0]$, where $H > 0$ implies the *upper bound* of the monetary fine. Given a punishment rule x_i for each agent $i \in \{1,2\}$, when a strategy profile $a \in A$ is selected, the expected payoff for agent i is defined by

$$v_i(a; x_i) \equiv \sum_{\omega \in \Omega} u_i(x_i(\omega)) p(\omega \mid a) - \frac{\sum_{h=1}^{n} a_{i,h}}{n},$$

where $p(\omega \mid a)$ denotes the probability that the public signal profile ω occurs when a is chosen, i.e.,

$$p(\omega \mid a) \equiv \int_{\theta=0}^{1} \prod_{\substack{(i,h) \in \{1,2\} \times \{1,\dots,n\}: \\ \omega_{i,h}=0}} \{1 - p(a_{i,h} + \alpha\theta)\}.$$

$$\prod_{\substack{(i,h) \in \{1,2\} \times \{1,\dots,n\}: \\ \omega_{i,h}=1}} p(a_{i,h} + \alpha\theta) f(\theta) d\theta.$$

Let $x = (x_1, x_2)$ denote a *punishment rule*. This paper uses the approximate Nash equilibrium concept, which is defined as follows. For each positive real number $\varepsilon > 0$, a strategy profile $a \in A$ is said to be an $\varepsilon -$*Nash equilibrium* for a punishment rule x if for every $i \in \{1,2\}$, every $a_i' \in A$, and for $j \neq i$,

$$v_i(a; x_i) \geq v_i(a_i', a_j; x_i) - \varepsilon.$$

This implies that for each agent, the gain from deviating from an $\varepsilon -$Nash equilibrium is less than or equal to ε, provided that the other agent plays this $\varepsilon-$Nash equilibrium. Note that the $\varepsilon-$Nash equilibrium concept is equivalent to the standard Nash equilibrium concept when $\varepsilon = 0$.

This paper examines a case in which the principal delegates a large number of tasks to each agent. An interpretation is as follows. In order to adopt the desirable action choices for $2n$ tasks, the principal hires $2n$ workers and divides them into two separate *groups*; each group has the same number of workers. The members of each group enter into a binding agreement to jointly adopt the action choices for the n tasks that the principal delegates and maximize the sum of their expected payoffs.

This paper aims to design a punishment rule for which the desirable strategy profile a^* is an approximate Nash equilibrium; moreover, every approximate Nash equilibrium induces the desirable action choices for almost all the tasks and rarely fines the agents. The reasons why the principal dislikes fining the agents even though they adopt the desirable action choices for all the tasks are as follows. Suppose that each agent $i \in \{1,2\}$ has an *outside opportunity* that provides him/her with a payoff that is less than but close to $u_i(0) - \frac{\sum_{h=1}^{n} a_{i,h}^*}{n} = -1$. If he/she is fined with a positive probability, playing the desirable strategy a_i^* would not satisfy the *participation constraint*; therefore, the agent is incentivized not to participate in the principal's business. Hence, in order to prevent this agent from leaving, the principal must provide him/her with an extra monetary payment; consequently, the principal fails to extract the full surplus. Next, suppose that the agents are risk averse with respect to the monetary transfers, i.e., u_i is concave for each $i \in \{1,2\}$. If each agent is fined with a positive probability, the principal fails to achieve the first-best allocation because of the welfare distortion caused by this risk-averse agent's risk sharing.

3. Relative Performance Evaluation

Let us arbitrarily fix a positive integer $n \geq 1$ and a positive integer $\lambda = \lambda(n) \in \{1,...,n\}$. We specify a punishment rule $x^n = x$ as follows. For every $i \in \{1,2\}$ and for $j \neq i$,

(1) $$x_i(\omega) = -H \quad \text{if } \sum_{h=1}^{n} \omega_{i,h} \leq \max\left[\sum_{h=1}^{n} \omega_{j,h} - \lambda, \min\left[np(1), \sum_{h=1}^{n} \omega_{j,h} + \lambda - 1\right]\right],$$

and

(2) $$x_i(\omega) = 0 \quad \text{otherwise.}$$

We can regard the above specification of x^n as a hybrid of the *relative performance evaluation* method and the *absolute* performance evaluation method as follows. If the absolute value of the difference between the number of good signals for agent $i's$ tasks and that of the other agent $j's$ tasks is greater than or equal to λ, i.e.,

$$\left|\sum_{h=1}^{n}\omega_{1,h}-\sum_{h=1}^{n}\omega_{2,h}\right|\geq\lambda,$$

then the principal will evaluate each agent's performance according to the relative performance evaluation method as follows. If the number of good signals for agent $i's$ tasks is relatively small as compared with that of the other agent $j's$ tasks such that

$$\sum_{h=1}^{n}\omega_{j,h}-\sum_{h=1}^{n}\omega_{i,h}\geq\lambda,$$

then agent i is fined a monetary amount H, whereas agent j is never fined.

On the other hand, if the absolute value of the difference between the number of good signals for agent $i's$ tasks and that of the other agent $j's$ tasks is less than λ, i.e.,

$$\left|\sum_{h=1}^{n}\omega_{1,h}-\sum_{h=1}^{n}\omega_{2,h}\right|<\lambda,$$

then the principal will evaluate each agent's performance according to the absolute performance evaluation method as follows. Let us consider $np(1)$ as the *threshold* to determine whether an agent should be fined or not. Here, $p(1)$ implies the probability that a good signal will occur for a task when action 1 is chosen, and the *weakest* macro shock $\theta=0$ occurs. If the number of good signals for agent $i's$ tasks is less than or equal to this threshold, i.e.,

(3) $$\sum_{h=1}^{n} \omega_{i,h} \leq np(1) \,,$$

then agent i is fined a monetary amount H. However, if the number of good signals for agent $i's$ tasks is greater than this threshold, he/she is not fined. Note that although the inequality (3) holds, agent i is not fined if this number is relatively larger than that of the other agent $j's$ tasks, i.e.,

$$\sum_{h=1}^{n} \omega_{i,h} \geq \sum_{h=1}^{n} \omega_{j,h} + \lambda \,.$$

4. The Theorem

In this paper, we will assume that

(4) $$u(-H) \leq -1 \,,$$

which should be considered as a *necessary* condition for the existence of a well-behaved punishment rule. In fact, without this assumption (4), it is impossible for the principal to resolve the incentive problem even for the perfect monitoring case. Let us consider a situation in which the principal can perfectly monitor the agents' action choices. With assumption (4), by fining any agent who selects action 0 for m tasks irrespective of $m \in \{1,...,n\}$ with a monetary amount $\frac{mH}{n}$, the principal can incentivize the agents to select a^* as a Nash equilibrium. Without this assumption, however, the principal is unable to compel the agents to select a^* as a Nash equilibrium although she can perfectly monitor their action choices. The following theorem shows that generally, this assumption, i.e., (4), is almost *sufficient* for unique implementation in the *imperfect* monitoring case.

The Theorem: *There exists an infinite sequence of positive integers* $(\lambda(n))_{n=1}^{\infty}$ *that satisfies the following three properties.*

(i) $\lambda(n) \in \{1,...,n\}$ *for all* $n \geq 1$.

(ii) *For every* $\varepsilon > 0$, *there exists* $\bar{n}$ *such that for every* $n \geq \bar{n}$ *and every* $i \in \{1,2\}$, *when* a^{n*} *is selected, the probability that agent* i *is fined, i.e.,* $x_i^n(\omega) = -H$ *, is less than* ε.

(iii) *For every* $\varepsilon > 0$, *there exists* $\bar{n}$ *such that for every* $n \geq \bar{n}$, a^{n*} *is a* $\varepsilon -$ *Nash equilibrium for* x^n.

(iv) *If the strict inequality of (4) holds, i.e.,*

$$u(-H) < -1,$$

then for every $\eta > 0$, *there exist* $\varepsilon > 0$ *and* $\bar{n}$ *such that for every* $n \geq \bar{n}$, *there exists no* $\varepsilon -$ *Nash equilibrium* $a^n = a$ *for* x^n *that satisfies*

$$\frac{\sum_{h=1}^{n} a_{i,h}}{n} \le 1-\eta \quad \textit{for some} \quad i \in \{1,2\}.$$

The above theorem states that if the number of tasks that each agent is delegated is sufficiently large, then a^* is an approximate Nash equilibrium; moreover, every approximate Nash equilibrium can induce the agents to adopt the desirable action choices for almost all the tasks and rarely fines them. Hence, the principal succeeds in achieving the desirable action choices for all the tasks and extracts the full surplus without any substantial welfare loss.

Although the complete proof of this theorem will be presented in the next section, the following is a brief outline of it. Consider a sufficiently large n. The Law of Large Numbers implies that when each agent $i \in \{1,2\}$ selects a_i^*, irrespective of the macro shock θ, it is almost certain that $\frac{\sum_{h=1}^{n} \omega_{i,h}}{n}$ is around $p(1+\theta)$. This implies that when the agents select a^*, it is almost certain that $\frac{\sum_{h=1}^{n} \omega_{1,h}}{n}$ and $\frac{\sum_{h=1}^{n} \omega_{2,h}}{n}$ are almost identical. Hence, it is almost certain that in the relative performance evaluation method, the agents are never fined. Moreover, given that $\frac{\lambda(n)}{n}$ is close to zero, it is almost certain that $\frac{\sum_{h=1}^{n} \omega_{i,h}}{n}$ is greater than $p(1)$ for each $i \in \{1,2\}$; therefore, the agents are never fined under the absolute performance evaluation method as well. Hence, with a sufficiently large n, $a^* = a^{n*}$ almost surely induces $x_i(\omega) = 0$ for each $i \in \{1,2\}$, i.e., property (ii) holds.

Let us arbitrarily fix $\varepsilon \in (0,1)$. When each agent $i \in \{1,2\}$ adopts action 0 for approximately $n\varepsilon$ tasks, irrespective of θ, it is almost certain that $\frac{\sum_{h=1}^{n} \omega_{i,h}}{n}$ is around $p(1+\theta) - \varepsilon\{p(1+\theta) - p(\theta)\}$, which is less than $p(1+\theta)$ by the positive value $\varepsilon\{p(1+\theta) - p(\theta)\}$. This implies that the relative performance evaluation method can almost certainly detect agent $i's$ deviation, as long as the other agent $j \neq i$ plays a_j^*. This along with (3) implies that a^* is an approximate Nash equilibrium, i.e., property (iii) holds.

Finally, consider any strategy profile a, according to which an agent adopts action 0 for some tasks. If the macro shock that occurred is sufficiently weak, it is almost certain that $\sum_{h=1}^{n} \omega_{i,h} < np(1)$. Hence, in the absolute performance evaluation method, some agents are fined with a positive probability. On the other hand, the Law of Large Numbers implies that if an agent can alter the proportion of tasks for which he/she adopts the desirable action choices to be *slightly* greater than that for which the other agent adopts the desirable action choices, then this agent can almost certainly evade this fine. This contradicts the approximate Nash equilibrium concept. Hence, we have proved that every approximate Nash equilibrium can induce the agents to adopt the desirable action choices for almost all the tasks, and that it rarely fines them, i.e., property (iv).[6]

5. Proof of the Theorem

The Law of Large Numbers implies that irrespective of θ, with a sufficiently large n, it is almost certain that for each $i \in \{1, 2\}$, $\frac{\sum_{h=1}^{n} \omega_{i,h}}{n}$ is around

$$\frac{p(1+\alpha\theta)\sum_{h=1}^{n} a_{i,h} + p(\alpha\theta)(n - \sum_{h=1}^{n} a_{i,h})}{n},$$

provided that agent i selects a_i and the macro shock θ occurs. Hence, it is almost certain that $\frac{\sum_{h=1}^{n} \omega_{i,h}}{n} - \frac{\sum_{h=1}^{n} \omega_{j,h}}{n}$ is around

$$\frac{\{p(1+\alpha\theta) - p(\alpha\theta)\}(\sum_{h=1}^{n} a_{i,h} - \sum_{h=1}^{n} a_{j,h})}{n},$$

provided that the agents select a and the macro shock θ occurs. Hence, we can

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[6] In this paper, we do not consider the possibility that the agents *overwork*. However, it is very easy to resolve this issue by modifying the specification of the punishment rule such that each agent is fined whenever the proportion of his/her tasks for which good public signals occur is greater than $p(1+\alpha)$.

select $(\lambda(n))_{n=1}^{\infty}$ that satisfies property (i) and the following two properties.

(iv) $\lim_{n\to\infty}\frac{\lambda(n)}{n}=0$.

(v) The probability that $\left|\sum_{h=1}^{n}\omega_{1,h}-\sum_{h=1}^{n}\omega_{2,h}\right|<\lambda(n)$ when a^{n*} is selected converges to unity as n increases.

Moreover, for each $i\in\{1,2\}$,

(vi) the probability that $\sum_{h=1}^{n}\omega_{i,h}>np(1)$ when agent i selects a_i^{n*} converges to unity as n increases.

Property (v) implies that it is almost certain that under the relative performance evaluation method, the agents are never fined. Property (vi) implies that it is almost certain that the agents are never fined under the absolute performance evaluation method. Hence, with a sufficiently large n, a^{n*} almost surely induces $x_i(\omega)=0$ for each $i\in\{1,2\}$, i.e., property (ii) holds.

Let us arbitrarily fix $\varepsilon>0$. We prove property (iii), i.e., with a sufficiently large n, $a^{n*}=a^*$ is an $\varepsilon-$Nash equilibrium, as follows. Suppose that there exists $a_i\neq a_i^*$ such that

(5) $$u_i(a_i,a_j^*)>u_i(a^*)+\varepsilon.$$

Note that the number of tasks for which agent i chooses action 0 must be greater than $n\varepsilon$, i.e.,

$$n-\sum_{h=1}^{n}a_{i,h}>n\varepsilon .$$

Irrespective of θ, it is almost certain that $\dfrac{\sum_{h=1}^{n}\omega_{j,h}}{n}-\dfrac{\sum_{h=1}^{n}\omega_{i,h}}{n}$ is around

$$\frac{\{p(1+\alpha\theta)-p(\alpha\theta)\}(n-\sum_{h=1}^{n}a_{i,h})}{n},$$

which is greater than the positive value

$$\{p(1+\alpha\theta)-p(\alpha\theta)\}\varepsilon>0 .$$

This along with the fact that $\dfrac{\lambda(n)}{n}$ is close to zero implies that it is almost certain that $\dfrac{\sum_{h=1}^{n}\omega_{j,h}}{n}-\dfrac{\sum_{h=1}^{n}\omega_{i,h}}{n}\geq\dfrac{\lambda(n)}{n}$, and therefore, agent i is almost certainly fined. Hence, $u_i(a_i,a_j^*)-u_i(a^*)$ is approximated by

$$1-\frac{\sum_{h=1}^{n}a_{i,h}}{n}+u_i(-H),$$

which is non-positive, because of (4). This contradicts (5). Hence, we have proved that with a sufficiently large n, a^{n*} is an $\varepsilon-$Nash equilibrium.

We prove property (iv) as follows. Since $p(\cdot)$ is continuous and increasing, it follows that for every $\eta>0$, there exists $\theta=\theta^*(\eta)>0$ such that

$$p(1)=\eta p(\alpha\theta)+(1-\eta)p(1+\alpha\theta) .$$

Let us arbitrarily fix $\eta>0$. From the strict inequality of (4), we can choose $\varepsilon>0$ such that

(6) $$\min[-u_i(-H)\theta^*(\eta), -u_i(-H)-1] > 3\varepsilon .$$

For each $i \in \{1,2\}$, consider any strategy profile a such that

$$\frac{\sum_{h=1}^{n} a_{i,h}}{n} \le 1-\eta ,$$

and

$$\sum_{h=1}^{n} a_{i,h} \le \sum_{h=1}^{n} a_{j,h} .$$

From the definition of $\theta^*(\eta)$, it follows that when the occurred macro shock is weaker than $\theta^*(\eta)$, in the case of a sufficiently large n, it is almost certain that

$$\sum_{h=1}^{n} \omega_{i,h} \le \min\left[np(1), \sum_{h=1}^{n} \omega_{j,h} + \lambda(n) \right].$$

This implies that the probability that agent i is fined is at least around $\theta^*(\eta)$.

Suppose that

$$0 \le \sum_{h=1}^{n} a_{j,h} - \sum_{h=1}^{n} a_{i,h} < n\varepsilon .$$

When agent i chooses another strategy $\tilde{a}_i \ne a_i$ such that $\frac{\sum_{h=1}^{n} \tilde{a}_{i,h} - \sum_{h=1}^{n} a_{i,h}}{n}$ is around 2ε, for a sufficiently large n, it is almost certain that $\sum_{h=1}^{n} \omega_{i,h} - \sum_{h=1}^{n} \omega_{j,h} \ge \lambda(n)$; therefore, agent i is almost certainly never fined. Hence, $u_i(\tilde{a}_i, a_j) - u_i(a)$ is at least around

$$-u_i(-H)\theta^*(\tilde{\eta}) - 2\varepsilon ,$$

which is greater than ε, because of (6). This implies that a is not an $\varepsilon-$Nash equilibrium.

Next, suppose that

$$\sum_{h=1}^{n} a_{j,h} - \sum_{h=1}^{n} a_{i,h} \geq n\varepsilon \ .$$

Irrespective of θ, for a sufficiently large n, it is almost certain that $\frac{\sum_{h=1}^{n}\omega_{j,h}}{n} - \frac{\sum_{h=1}^{n}\omega_{i,h}}{n}$ is at least around

$$\{p(1+\alpha\theta) - p(\alpha\theta)\}\varepsilon \, ,$$

which is positive. This along with property (i) implies that it is almost certain that $\frac{\sum_{h=1}^{n}\omega_{i,h}}{n} - \frac{\sum_{h=1}^{n}\omega_{j,h}}{n} \leq -\lambda(n)$; therefore, agent i is almost certainly fined. When agent i selects a_i^* instead of a_i, for a sufficiently large n, it is almost certain that he/she is never fined. Hence, $u_i(a_i^*, a_j) - u_i(a)$ is around

$$-u_i(-H) - 1 + \frac{\sum_{h=1}^{n} a_{i,h}}{n} \geq -u_i(-H) - 1 \, ,$$

which is greater than ε, because of (6). This implies that a is not an $\varepsilon-$Nash equilibrium. Hence, we have proved that with a sufficiently large n, there exists no $\varepsilon-$Nash equilibrium a such that $\frac{\sum_{h=1}^{n} a_{i,h}}{n} \leq 1-\eta$, i.e., property (iv) holds.

Q.E.D.

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