Credit Allocation, Aggregate TFP, and the Effectiveness of Monetary Policy

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Abstract

Japan has been experiencing prolonged stagnation since the 1990s. Several hypotheses have emerged to explain Japan’s stagnation so far, such as a misallocation of credit, a decrease in the function of the financial system, and a failure of monetary policy. This paper theoretically investigates whether these hypotheses make it possible to account for both a decline in aggregate TFP and a low economic growth rate. Moreover, this paper theoretically examines relationships between an effectiveness of monetary policy and an aggregate TFP level as well as the degree of the function of the financial system.

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1. Introduction

The Japanese economy has been stagnating for over a decade. Why has the Japanese economy stagnated? Is this stagnation a short-run or long-run event? According to Hayashi and Prescott [2002, HP], the main reason for Japan's stagnation is the decline of aggregate TFP growth rate. They concluded that this decline in aggregate TFP was crucial in accounting for Japan's stagnation. However, in their paper, aggregate TFP's decline itself was considered as an exogenous event. That is, the mechanism of aggregate TFP's decline is not necessarily clear yet.

In addition to HP's study, several hypotheses have emerged to account for Japan's stagnation so far: a misallocation of credit, a decrease in the function of the financial system, and a failure of monetary policy. Although previous studies presenting these hypotheses point out the linkage between these hypotheses and the low economic growth rate, they do not necessarily mention the relationship between these hypotheses and aggregate TFP’s decline. Even if we try to account for Japan's stagnation with these hypotheses, we need to explain a simple fact, aggregate TFP's decline. This paper theoretically investigates whether these hypotheses make it possible to account for both a decline of aggregate TFP and a low economic growth rate. By doing so, this paper attempts to give some interpretations about the causes of Japan's lost decade from a theoretical perspective.

Moreover, we analyze relationships between an effectiveness of monetary policy and an aggregate TFP level as well as a degree of the function of the financial system. The motivation of conducting this analysis is that for example, in Japan, after the economy stagnated, Bank of Japan has adopted a monetary easing policy for a long period of time. However, the effect does not appear to be strong enough to spur the economy. Why is that? During stagnation, as HP's study showed, the aggregate TFP growth rate has been low. At
the same time, it was said that since the shock of bubble burst was so large, many banks were damaged to a large extent, so that the function of the financial system decreased. This gives rise to the following question: Does there exist any relationship between an effectiveness of monetary policy and a decline in aggregate TFP as well as a decrease in the function of the financial system? This paper attempts to clarify these relationships as well.

For this purpose, we develop a dynamic general equilibrium model with mainly two characteristics, the borrowing constraint and the presence of heterogeneous firms. The borrowing constraint arises because there may be a limit on the extent to which the debtors can credibly promise to repay the creditors. To be more specific, we assume that a debtor can credibly commit to repay at most a fraction $\theta$ of his or her future output. The parameter $\theta$ partly reflects the legal structure and contractual redress available to a creditor in the event of defaults. In this sense, $\theta$ provides a simple measure of the degree of the function of the financial system.

The presence of heterogeneous firms implies that at each date, there are two types of firms—high-productivity firms and low-productivity firms. Both have the technology to invest goods in the present period. They obtain returns in the subsequent period. High-productivity firms have a higher rate of return. Over time, some high-productivity firms (low-productivity firms) stochastically become low-productivity firms (high-productivity firms) in the subsequent periods.

The contributions of this paper show the following. First, a decline in aggregate TFP could be caused not only by a negative technology shock but also by a misallocation of credit triggered by a decrease in the function of the financial system or an unanticipated monetary tightening policy. Second, this decline in aggregate TFP could eventually result in a slump of economic growth rate (see Figure 1). Third, the effectiveness of monetary policy could be reduced because of certain negative shocks on the financial system.
In this regard, the present analysis might have some implications on the causes of the U.S. Great Depression or the recent depressions in some countries. Recent studies such as Kehoe and Prescott [2002] have found that during these depressions, aggregate TFP declined. They concluded that this decline in aggregate TFP was the main cause for these depressions. However, why aggregate TFP declined is not necessarily clear in these studies. During these depressions, a number of similar problems arose, such as credit misallocation problems, banking problems, and monetary policy problems. Thus, it appears that there could be a certain relationship between these problems and a decline in aggregate TFP and a slump of economic growth\(^5\).

In terms of theoretical perspective, there are some contributions in this paper. With regard to business fluctuations, literature related to this paper are, for example, Aghion et al. [1999], Bernanke and Gertler [1989, BG], and Kiyotaki and Moore [1997, KM]. They present a powerful propagation mechanism to show how the effects of a temporarily small productivity shock amplify and persist through the changes in the collateral value in the economy with imperfect credit markets. However, they do not necessarily discuss the relationship between the propagation mechanism and the degree of the function of the financial system.

With regard to the function of the financial system, examples include Jeong and Townsend [2004] and Aghion et al. [2005]. Jeong and Townsend examine the relationship between the degree of the function of the financial system and aggregate TFP growth. However, they do not necessarily investigate the relationship between the degree of the function of the financial system and the extent of business fluctuations as well as the effectiveness of monetary policy. Aghion et al. investigate the relationship between the degree of the function of the financial system and volatility in economic growth. However, neither do they explore the effects of the changes in the degree of the function of the
financial system on the effectiveness of monetary policy.

In addition, this paper would be related to the studies on the sources of aggregate TFP. Modern business cycle models give a central role to aggregate TFP shocks as a major factor of business cycles. However, aggregate TFP shocks itself are mostly considered as exogenous events. That is, the underlying sources of aggregate TFP are not necessarily well-known. Lagos [2006] develops a model of aggregate TFP based on a search theory. He endogenously derives the level of aggregate TFP by focusing on labor markets and shows how labor market policies affect it. This paper endogenously derives it by focusing on credit markets instead and examines by what factors it is affected in both the short run and long run.

This paper is organized as follows. In section 2, we will explain the basic structure of our model. In section 3, we analyze how an unexpected monetary policy shock could affect an aggregate TFP level and an economic growth rate in a credit-constrained economy in the short run. In section 4, we examine how both an aggregate TFP level and an economic growth rate could be determined endogenously in a credit-constrained economy in the long run. Thereafter, we discuss the factors that might possibly explain Japan's economic stagnation from a theoretical perspective. In addition, we present the factors that could affect a capital-output ratio in both the short and the long run. In section 5, we analyze the effectiveness of monetary policy. In section 6, we provide conclusions.

2 The Model

Our model is an extension of Kiyotaki's [1998] model. Consider a discrete-time economy with a single homogenous good and a continuum of firms. Each firm is operated by one manager. A manager retires after two periods. Therefore, a new manager takes over the retiring manager. Each manager has the same preference, i.e.,

\[ V = a_1 \log c_t + a_2 \log c_{t+1} + a_3 \log e_{t+1} \]
where \( c_t \) and \( c_{t+1} \) represent consumption at dates \( t \) and \( t+1 \), respectively, \( e_{t+1} \) is the bequest to the next new manager, and \( a(<1) \) is the weight for each utility and satisfies \( a_1 + a_2 + a_3 = 1 \).

At each date \( t \), there is a competitive one-period credit market in which one unit of consumption goods is exchanged for a claim to \( r_t \) units of consumption goods at date \( t+1 \). At each date, some firms are high-productivity firms while the others are low-productivity firms. The high-productivity firms possess the following production technology:

\[
y_{t+1}^{H} = \alpha^H z_{t}^{H}
\]

Where \( z_{t}^{H} \) is the investment at date \( t \) and \( y_{t+1}^{H} \) is the output from the investment at date \( t + 1 \). It takes one period to produce goods from the investment project. Further, although the low- and high-productivity firms have similar production technology, the marginal productivity of the former is lower.

\[
y_{t+1}^{L} = \alpha^L z_{t}^{L}
\]

1 < \( \alpha^L < \alpha^H \)

The marginal productivity of each firm follows the Markov process. This implies that there is a switching probability between the high- and low-productivity states; this probability is 1 - p. On the other hand, the probability of switching from the low-productivity state to the high-productivity state is \( X (1 - p) \). This switching probability is exogenous, and it is independent across firms over time. We assume that the initial measures of the high- and low-productivity firms are \( X \) and 1, respectively. From the assumption of the Markov process, it follows that the population ratio of the high-productivity firms to low-productivity firms is \( X \) over time.

There is an enforcement problem between the creditors and the debtors. The creditors can not enforce the debtors to repay debts unless the debts are secured by collateral. We
assume that the creditors are assured a fraction $\theta^H, \theta^L$ of the maximum returns. This means that the debtors can credibly commit to repay at most this fraction. This fraction $\theta$ can be collateral in borrowing. In such a situation, in order for the debts contract to be credible, the creditors limit the amount of credit at date $t$, so that the debt repayment does not exceed the value of the collateral.

\[ r_t b^i_t \leq \theta^z \alpha^z z^s_i \quad r_t \equiv (1 + i_t) P_t / P_{t+1} \quad s = H \text{ or } L, \]

where $i$ is index of each firm, $b^i_t$ is the amount of borrowing, $r_t$ and $i_t$ are the gross real interest rate and the net nominal interest rate between dates $t$ and $t+1$, respectively, $r_t b^i_t$ is the debt repayment, $P_t$ and $P_{t+1}$ are the price levels at date $t$ and $t+1$, respectively, and $\theta^z \alpha^z$ is the collateral value of the unit investment project. Here, in order to ensure simplicity, we assume that $\theta^H$ is equal to $\theta^L$ and defined as $\theta^9$.

The value of this $\theta$ would depend on the degree of the function of the financial system. In the economy where property rights, bankruptcy laws or bankruptcy procedures are well established, the creditors can seize enough assets of the debtors when bankrupt, so that the value of $\theta$ would be high. For example, if bankruptcy procedures are not well established and bankruptcy process such as the negotiation between the creditors does not proceed smoothly in the event of bankrupt, the assets including human capital bankrupt firms have would be lost, so that $\theta$ would be low. We also assume that marginal productivity of each firm is known to the public. So people have perfect foresight about both debt repayment and future output aside from an unanticipated shock.

Each manager determines the consumption, bequest, investment, borrowing, and money holding in order to maximize the utility (equation 1), subject to the production technology constraints (equations 2 and 3), the borrowing constraint (equation 4), the following first-period and second-period budget constraints, and the cash-in-advance constraint.
The first-period budget constraint is as follows:

\[
\frac{m_i^t}{P_t} + c_i^t + z_{i,t}^i \leq (1 - \tau_t) e_i^t + b_i^t.
\]

The second-period budget constraint is as follows:

\[
e_i^{t+1} + e_i^t \leq y_{i,t+1} - r_i b_i^t + \frac{m_i^t}{P_{t+1}}.
\]

The cash-in-advance constraint is as follows:

\[
m_i^t \geq P_{t+1} c_i^{t+1}.
\]

Here, \( \tau_t \) is the tax rate at date \( t \) and \( m_i^t \) is the nominal money demand of each manager.

For now, we assume that \( \tau_t \) is constant over time, i.e., \( \tau_t = \tau \).

2.1 The case without the borrowing constraint

First, we will describe the equilibrium for an economy without the borrowing constraint.

In this case, the high-productivity firms would prefer to borrow an unlimited amount of credit as long as the marginal rate of return on investment exceeds the real interest rate, \( \alpha^H > r_i \). Both the high- and low-productivity firms would not borrow if the marginal rate of return on investment was less than the real interest rate, \( \alpha^H < r_i \). Thus, the equilibrium interest rate in the credit market would be equal to the marginal rate of return on the investment of the high-productivity firms, \( \alpha^H = r_i \).

Then, in the equilibrium, the low-productivity firms would not undertake any investment and would prefer lending because the marginal rate of return on lending is higher than the marginal rate of return on their investment. Consequently, only the high-productivity firms would undertake investment. The aggregate investment of the high-productivity firms would be equal to the aggregate savings for investment of these firms, which is equal to a fraction \( (1 - \tau) a_3 \) of the aggregate output of the economy.

\[
Z_i^H = a_3 (1 - \tau) Y_i
\]

The aggregate output of the economy is simply the output from the investment undertaken by the high-productivity firms.
Therefore, the aggregate TFP level in the economy without the borrowing constraint is as follows.

\[(7) \quad \text{Aggregate TFP}_t = \alpha^H\]

Moreover, in this case, the economic growth rate defined as \(G^{NBC}_{t+1}\) is as follows.

\[(8) \quad G^{NBC}_{t+1} \equiv \frac{Y_{t+1}}{Y_t} = \alpha^H a_t (1 - \tau)\]

Here, it is important to note that the aggregate investment, the aggregate output, the aggregate TFP level, and the economic growth rate are independent of the distribution of net worth between the high- and low-productivity firms.

### 2.2 The case with the borrowing constraint

We now discuss the case of an economy with the borrowing constraint. First, let us discuss the case where \(\theta\) is strictly less than \(\hat{\theta}\), which is defined as

\[
1 - p \left( \frac{\alpha^H}{\alpha^L} - p + X(1 - p) \right)\]

In this case, the real interest rate is equal to the marginal rate of return on investment of the low-productivity firms, \(\alpha^L\), the borrowing constraint of the high-productivity firms is binding, and both high- and low-productivity firms invest in the neighborhood of the steady state. This can be verified in Proposition 1 after we describe the equilibrium in a credit-constrained economy.

The high-productivity firms invest by borrowing up to the credit limit because the real interest rate is lower than the marginal rate of return on their investment. Thus, when the borrowing constraint binds, the high-productivity firms' investment is as follows.
Since \( \frac{\theta \alpha^H}{r^i} \) is the present value of the collateral of the unit investment, the numerator is the required down payment for the unit investment. Equation 9 implies that the high-productivity firms use a fraction \( a_3 \) of the net worth after tax \((1 - \tau)e^i\) to finance the required down payment. With regard to equation 9, it is important to note that the investment of the high-productivity firms is an increasing function of their net worth, \( e^i \) and their productivity, \( \alpha^H \) and a decreasing function of the real interest rate, \( r^i \) and the parameter of the efficiency of the financial system, \( \theta \). Since the utility function is log utility, the consumption and money demand of the managers at date \( t \) are a fraction \( a_1 \) and \( a_2 \) of the net worth after tax, respectively. These are presented as \( c^i = a_1(1 - \tau)e^i \) and \( m^i/P^i = a_2(1 - \tau)e^i \).

If the real interest rate is equal to the marginal rate of return on their investment, the low-productivity firms are indifferent between investing and lending. Thus, investment and lending are not determined at the individual level. However, the aggregate investment and aggregate lending are determined from the goods market clearing condition. Since consumption, debt, money demand, and investment are linear functions of the net worth, by aggregating across firms, we can obtain the equations of motion of the aggregate output, \( Y_t \) and the aggregate net worth of the high-productivity firms, \( E_t^H \).

\[
Y_{t+1}^H = \alpha^H Z_t^H + \alpha^L Z_t^L = \alpha^L a_3(1 - \tau)Y_t + (\alpha^H - \alpha^L) a_3(1 - \tau) \frac{E_{t+1}^H}{1 - \theta \alpha^H} \\
E_{t+1}^H = p(Y_{t+1}^H - r_t B_t) + X (1 - p)(Y_{t+1}^L + r_t B_t)
\]
\[ y = p(1 - \theta)\alpha_{\mu} \frac{a_3(1 - \tau)E_i^{\mu}}{1 - \theta\alpha_{\mu}} + X(1 - p)a_3(1 - \tau)\alpha_{\ell}(Y_i - E_i) \]

From equation 10, we understand that the aggregate output comprises two parts: the returns on the investments of the high- and low-productivity firms. From equation 11, we understand that the aggregate net worth of the high-productivity firms also comprises two parts: the net worth of the firms that continue to function as high-productivity firms from the previous period and the net worth of the firms that after the previous period switch from being low-productivity firms to being high-productivity firms. The important point in the case with the borrowing constraint is that the aggregate output is an increasing function of the net worth share of the high-productivity firms. Intuitively, the high-productivity firms have a greater net worth, and they can obtain greater credit and finance increased investment because the default probability reduces. Thus, a greater output is produced.

The aggregate TFP level in the case with the borrowing constraint is as follows.

\[
AggregateTFP_t = \frac{Y_{t+1}}{Z_i^{\mu} + Z_i^{\ell}} = \alpha_{\ell} + (\alpha^{\mu} - \alpha_{\ell}) \frac{1}{1 - \theta\alpha_{\mu}} \frac{E_i^{\mu}}{Y_i}
\]

From equation 12, we understand that the aggregate TFP level is also an increasing function of the net worth share of the high-productivity firms. The greater the net worth of the high-productivity firms relative to that of the low-productivity firms, the higher is the aggregate TFP level. Intuitively, if the net worth share of the high-productivity firms is high, a greater amount of resources are allocated to them because the borrowing constraint weakens.

From equation 10, the growth rate of an economy with the borrowing constraint defined as \( G_i^B \) is as follows.
From equation 13, it follows that the economic growth rate is determined once the aggregate TFP is determined. Thus, the shocks which affect the aggregate TFP result in a change in the economic growth rate. In a standard real business cycle model, the technology shock is the main factor which affects the aggregate TFP. But, in this model, as we will discuss later, it is also affected by different factors such as monetary policy or the degree of the function of the financial system. From equations 11 and 13, the net worth share of the high-productivity firms evolves according to the following equation.

\[
E_{t+1}^{H} \frac{1}{Y_{t+1}} = \frac{p \alpha^{H} (1 - \theta) \frac{1}{\theta \alpha^{H}} \frac{E_{t}^{H}}{Y_{t}} + X (1 - p) \alpha^{L} (1 - \frac{E_{t}^{H}}{Y_{t}})}{\alpha^{L} + (\alpha^{H} - \alpha^{L}) \frac{1}{1 - \theta \alpha^{H}} \frac{E_{t}^{H}}{Y_{t}}} \equiv \phi(\frac{E_{t}^{H}}{Y_{t}})
\]

Government Budget Constraint (GBC)

The government finances its expenditure by printing money and imposing taxes.

\[
P_{i}G_{i} = M_{i} - M_{i-1} + P_{i}T_{i},
\]

where \(G_{i}\) is the government expenditure at date \(t\), \(M_{i}\) and \(M_{i-1}\) are the nominal money supply at dates \(t\) and \(t-1\), respectively, and \(T_{i}\) is the tax revenue at date \(t\) imposed on the firms. We assume that government expenditure does not affect the utility of the managers.

Government Policy

In this paper, with regard to government policy, we analyze two cases. First let us consider case 1 in which the government adopts the following policy.

\[
M_{i} = (1 + \mu)M_{i-1} \quad \text{and} \quad \tau_{i} = \tau
\]

The government considers the growth rate of money supply, \(\mu\) and the tax rate as constant.
over time. We assume that \( \mu \) is positive and large enough.

Money Market Clearing Condition

\[
M_t = P_t a_2 (1 - \tau_t) Y_t
\]

Here, we confine our attention to equilibria in which the cash-in-advance constraint is binding. This can hold for a large enough \( \mu \). The right-hand side is the aggregate nominal money demand of the new managers at date \( t \), and the left-hand side is the money supply at date \( t \). \( M_{t-1} \) is supplied by the retiring managers and \( M_t - M_{t-1} \) is supplied by the government. Since \( Y_t \) is a predetermined variable and \( M_t \) is determined by the government policy, from equation 17, the price level \( P_t \) is determined.

Goods Market Clearing Condition

\[
C_t + Z^H_t + Z^L_t + G_t = Y^H_t + Y^L_t
\]

The right-hand side is the aggregate output of the high- and low-productivity firms. The left-hand side is the aggregate consumption of the retiring managers and the new managers, the aggregate investment of the high- and low-productivity firms, and the government expenditure.

From equations 15, 16, and 17, the government budget constraint becomes

\[
G_t = \left( \frac{\mu}{1 + \mu} \right) a_2 (1 - \tau) Y_t + \tau Y_t
\]

The first and second terms on the right-hand side are real seigniorage and tax revenues, respectively. From equation 19, we understand that the more money the government prints, the greater will be the seigniorage revenues.

The Definition of Competitive Economy (Case 1)

Given the initial \( Y_t, E^H_t, M_0 \), the exogenous variables \( \theta, \alpha^H, \alpha^L, p, a \), and the monetary
and fiscal policies $\mu_t = \mu$ and $\tau_t = \tau$, a competitive equilibrium is a sequence of prices and quantities, $\{C_t, Z_t^H, Z_t^L, B_t, G_t, i_t, P_t, P_t^e, Y_t, \tau_t, E_t^H, TFP_t\}_{t=1}^\infty$ that satisfies the following conditions:

(1) each manager of the high- and low-productivity firms maximizes his/her utility

(2) money and goods market clearing conditions

(3) perfect foresight with regard to the future price level

(4) the GBC for all $t$

In this model, equilibrium in the credit market changes depending on the three conditions of the net worth share of the high-productivity firms, i.e., (1) $\frac{E_t^H}{Y_t} < 1 - \frac{1}{\alpha^H}$, (2) $1 - \frac{1}{\alpha^H} \leq \frac{E_t^H}{Y_t} < 1 - \theta$, and (3) $1 - \theta \leq \frac{E_t^H}{Y_t} < 1$.

The dynamic movement of the net worth share of the high-productivity firms also changes according to the above three conditions. Dynamic equations in each condition are as follows:

$$\ln(1), \frac{E_{t+1}^H}{Y_{t+1}} = \phi\left(\frac{E_t^H}{Y_t}\right), \quad \ln(2), \frac{E_{t+1}^H}{Y_{t+1}} = p(1 - \theta) + X(1 - p)\theta$$

and $\ln(3), \frac{E_{t+1}^H}{Y_{t+1}} = p\left(\frac{E_t^H}{Y_t}\right) + X(1 - p)(1 - \frac{E_t^H}{Y_t})$.

We describe the dynamic movement of the net worth share of high-productivity firms in Figure 2. In this figure, we depict a case in which $p$ is greater than $p > X/(1 + X)$, which implies that there is a positive correlation between the current state and the following state.

In this economy, there is a unique stationary equilibrium in which the net worth share of the high-productivity firms, the aggregate TFP level, and the economic growth rate are constant over time. We can obtain the stationary equilibrium, $G_t^{BC*}$, $(\frac{E_t^H}{Y_t})^*$, and the aggregate TFP* level by using equations 12, 13, 14, and $\frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t}$. The stationary equilibrium is characterized by the following three equations.
The net worth share of the high-productivity firms in the steady state is determined from equation 20. Once the net worth share of the high-productivity firms is determined, the aggregate TFP level in the steady state is determined from equation 21, and once the aggregate TFP level is determined, the economic growth rate in the steady state is determined from equation 22.

The difference between this case and the one without the borrowing constraint is that in a credit-constrained economy, the aggregate TFP level and the economic growth rate in the steady state are dependent on the distribution of the net worth between the high- and low-productivity firms; moreover, these are increasing functions of the net worth share of the high-productivity firms. The stationary equilibrium depends on the degree of the function of the financial system. Therefore, depending on $\theta$, there exist three different regions.

**Proposition 1:** There are three stages of financial deepening, corresponding to three different regions of $\theta$ value. The characteristics of each region are as follows.

In Region 1, where $\theta < \theta$, (1) the real interest rate is $r^* \in [\alpha^l, \alpha^H]$; (2) the borrowing constraint of the high-productivity firms binds; (3) production and resource allocation are inefficient. Both the high- and low-productivity firms undertake investment; and (4) both the aggregate TFP level and the economic growth rate are lower than those in the case without the borrowing constraint.

In Region 2, where $\theta \leq \theta < \hat{\theta}/(1+\lambda)$, (1) the real interest rate is $r^* \in [\alpha^l, \alpha^H]$; (2) the
borrowing constraint of the high-productivity firms still binds; (3) production and resource allocation are efficient. Only the high-productivity firms undertake investment (4) both the aggregate TFP level and the economic growth rate are the same as those in the case without the borrowing constraint.

In Region 3, where \( 1/(1+X) \leq \theta \leq 1 \), (1) the real interest rate is \( \alpha^u \); (2) the borrowing constraint of the high-productivity firms no longer binds; (3) production and resource allocation are efficient. Only the high-productivity firms undertake investment; (4) both the aggregate TFP level and the economic growth rate are the same as those in the case without the borrowing constraint.

Proof: see appendix 1

Intuitively, in a case in which the financial system is not well developed, \( \theta < \hat{\theta} \), the amount of credit that the high-productivity firms can borrow is relatively low because the borrowing constraint is strong. The real interest rate becomes low in the credit market, so that even low-productivity firms have incentives to invest. This implies that savings for investment are allocated not only to high-productivity firms, but also to low-productivity firms. Production and resource allocation are inefficient. Therefore, the aggregate TFP level and the economic growth rate are lower than those in the case without the borrowing constraint. We refer to this region, where \( \theta < \hat{\theta} \), as Region 1.

Next, let us discuss the case in which \( \theta \) is greater than \( \hat{\theta} \). Here, there are two regions, i.e., \( \hat{\theta} \leq \theta \leq 1/(1+X) \) and \( 1/(1+X) \leq \theta \leq 1 \). We refer to these two regions as Region 2 and Region 3, respectively. In Region 2, since the financial system is developed to some extent, the collateral value of the investment increases such that the high-productivity firms can borrow more credit. In the credit market, the real interest rate begins to rise. In this situation, low-productivity firms lose incentives to invest because the rate of return on lending is higher than that of investing. Production is efficient. In other words, through the
credit market, all the savings for investment are allocated only to the high-productivity firms. Resource allocation becomes efficient (however, income distribution between the high-and-low firms is not the first best because the borrowing constraint of the high-productivity firms is binding and they earn higher rate of return than low-productivity firms). Thus, the aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint.

In Region 3, since the financial system is well developed, the collateral value of the investment increases and the amount of credit that the high-productivity firms is so high. Consequently, the equilibrium real interest rate in the credit market rises to $\alpha^{ii}$, which is equal to the marginal rate of return on the investment of the high-productivity firms. Thus, the borrowing constraint of the high-productivity firms is no longer binding. In this situation, through the credit market, all the savings for investment are allocated only to the high-productivity firms. Production and resource allocation are efficient (income distribution becomes the first best because high-and-low firms earn the same rate of return. This point is different from Region 2). The aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint.

3. The short-run story

In this section, we investigate the short run effect of an unanticipated monetary policy shock on credit allocation, the aggregate TFP level, and the economic growth rate in each of three regions. In particular, we focus on an impulse response to an unanticipated monetary tightening policy shock. First, we discuss an impulse response in Region 1.

Suppose that at date $s - 1$, the economy is in the steady state. Then, there occurs an unanticipated monetary tightening policy shock, i.e., the growth rate of money supply between dates $s$ and $s - 1$ becomes lower than what was expected. Consequently, following from money market clearing condition (equation 17), the price level declines. In such a
situation, if the debt contract between the borrowers and lenders is nominal and not indexed, the borrowers’ real burden of debt repayment increases. This also implies that to the lenders, the real rate of return from lending increases, and consequently, there occurs redistribution of wealth from the debtors to the creditors.

This has two effects—positive and negative—on the aggregate net worth of the high-productivity firms at dates. Assuming that \( \rho \) is strictly greater than \( X/(1 + X) \), the negative effect is greater than the positive effect. Thus, the aggregate net worth of the high-productivity firms at date \( s \) decreases. Consequently, the high-productivity firms have to reduce their investment because the borrowing constraint becomes stronger. Moreover, a greater amount of resources are allocated to the low-productivity firms through the credit market, and resource allocation becomes worse.

From equation 12, it follows that as the aggregate net worth of the high-productivity firms at date \( s \) decreases, the aggregate TFP level at date \( s + 1 \) begins to decline. Further, from equation 13, it follows that as the aggregate TFP level begins declining, the economic growth rate at date \( s + 1 \) also begins to decline. This unexpected monetary tightening policy shock results in business fluctuations through persistent changes in aggregate TFP because it takes a certain amount of time for the aggregate net worth of the high-productivity firms to recover. That is, economic growth is volatile to the shocks. Moreover, the interesting point is that the impulse response of aggregate investment of high-and-low productivity firms is asymmetric. Aggregate investment of low-productivity firms increases while that of high-productivity firms decreases. The impulse response in Region 1 is depicted in Figure 3-1.

Here, it is important to highlight the following two points. First, as evident in equation 13, in this model, aggregate TFP is the main factor that generates business cycles; this is similar to the scenario presented in a standard real business cycle (RBC) model. However,
the difference between the two models is that in this model, in the short run, the aggregate TFP level could be affected by an unexpected monetary policy shock. On the other hand, in a standard RBC model, it is affected mainly by the technology shock, which is aggregate TFP shock itself in its theory.

Second, this effect of an unanticipated monetary policy on aggregate TFP is different from BG’s model. In BG’s model, aggregate TFP is not the main factor generating business fluctuations. Indeed, it is independent of credit allocation changes and constant over time except for the case of shocks to aggregate TFP itself. In their model, business fluctuations occur because capital input changes over time due to temporal productivity shocks which affect the net worth of borrowers. However, in this model, the main factor generating business fluctuations is aggregate TFP and aggregate TFP itself could be affected by credit allocation changes triggered by an unanticipated monetary policy.

Next, let us discuss an impulse response in Region 2. In Region 2, even if there is an unanticipated monetary tightening shock that causes the net worth share of the high-productivity firms to decrease, as long as the shock is relatively small, the real interest rate declines such that all the savings for investment are still allocated only to the high-productivity firms. Therefore, production does not change. Thus, even if the debt contract is nominal, business fluctuations would not occur by an unanticipated monetary policy shock\textsuperscript{14}. A similar explanation is applicable to an impulsive response in Region 3; however, the only difference is that in Region 3, the real interest rate does not change as long as the shock is relatively small. Impulse responses in Region 2 and Region 3 are depicted in Figure 3-2. We summarize the above discussion as Proposition 2.

**Proposition 2**: (1) If an economy exists in Region 1, an unanticipated monetary policy results in business fluctuations through persistent changes in aggregate TFP in the short run. Economic growth is volatile to the shocks. The movement of aggregate investment of
high-and-low firms is asymmetric. (2) On the other hand, if the economy exists in either Region 2 or 3, even if the debt contract is nominal, as long as a monetary shock is relatively small, business fluctuations through persistent changes in aggregate TFP would not occur. Economic growth would not be volatile to the shocks.

Proof: see appendix 2

In the case of an unexpected temporary shock to the productivity of both firms or on the functioning of the financial system, the same results are obtained. Based on this analysis, we could state that if the financial system is not well developed, the macroeconomic performance is vulnerable to the shocks to the net worth distribution of the firms. On the other hand, if the economy has a well-developed financial system, the volatility in economic performance would be reduced. Thus, a well-developed financial system is important for economic stability.

Moreover, according to this analysis, we might interpret that BG's and KM's papers analyze only region 1 with low function of the financial system. In their paper, they show that in the economy with the presence of asymmetric information, or enforcement problem, business fluctuations occur by the shocks to the borrowers' net worth. However, this paper shows that even if there is asymmetric information or enforcement problem in the financial market, business fluctuations would not occur in Region 2 with middle function of the financial system or Region 3 with well function of it.

Next, we present a brief discussion with regard to the following two cases: (1) a case in which the monetary shock is large and (2) the case without the borrowing constraint. Suppose that the economy exists in region 2 and there occurs an unanticipated large monetary tightening shock. Since the net worth share of the high-productivity firms decreases to a large extent, their borrowing constraint becomes stronger. In the credit market, the real interest rate declines to $\alpha^*$ such that—unlike the case in which the shocks
are small—the low-productivity firms begin to undertake investment. Thus, both the aggregate TFP level and the economic growth rate will be lower than those in the steady state until the high-productivity firms accumulate enough net worth such that the low-productivity firms no longer invest. Eventually, the economy will return to the original steady state in which only the high-productivity firms undertake investment and resource allocation is efficient.

In the case without the borrowing constraint, the investment of the firms does not depend on their net worth. Thus, even if an unanticipated monetary tightening shock reduces the net worth of the high-productivity firms, both the aggregate TFP level and the economic growth rate remain unchanged. This is similar to the result in the case of a standard real business cycle model with no heterogeneity among firms and no friction in the credit market.

Thus, based on the above discussion, we might give the following explanation for the stagnation of the Japanese economy after 1990 from monetary policy perspective. Suppose that the Japanese economy existed in Region 1. From May 1989 to June 1991, Bank of Japan undertook monetary tightening policies on five occasions. Consequently, the net worth of the high-productivity firms decreased, and they might be required to reduce their investment because the borrowing constraint became stronger.

Therefore, a greater amount of resources were allocated to the low-productivity firms in the credit market. Thus, low-productivity firms’ investment (capital) increased while high-productivity firms’ one decreased, so that both the aggregate TFP level and the growth rate began declining.

In order to examine whether there was any inefficient allocation of resources in Japan, we examined the share of tangible fixed assets by using the data from the Financial Statements Statistics of Corporations by Industry released by the Ministry of Finance in
Japan. Here, following Sakuragawa [2002 chapter 5, 2005 chapter 2], we compared the share of tangible fixed assets in manufacturing over all industries with that in the following three industries: construction, real estate, and wholesale and retail.

Sakuragawa calculated years of debt redemption. An increase in years of debt redemption implies a delay of debt repayment. He found that years of debt redemption increased in construction, real estate, and wholesale and retail in the 1990s compared to the one in manufacturing. At the same time, despite of the increase of years of debt redemption in three industries, he points out from data that lendings outstanding increased in construction and real estate from 1992 to about 1999. He also mentions that the ratio of non-performing loans in three industries was relatively high, i.e., 57.1 percent in 2001, while that in the manufacturing industry was 7.6 percent. Accordingly, as Sakuragawa pointed out, these facts might imply that productivity, or the rate of returns in three industries would be relatively low while that in the manufacturing industry would be relatively high.

The data provided in Figure 4 reveals that between 1991 and 1992, the share of tangible fixed assets in the manufacturing industry over all industries began declining while that in these three industries began increasing. Thus, it might be possible to state that inefficient allocation of resources from high- to low-productivity firms occurred in Japan. This might provide empirical evidence to support our theoretical analysis.

4. The long-run story

In this section, we examine the mechanism with regard to the endogenous determination of both the aggregate TFP level and the economic growth rate in a credit-constrained economy in the long run. Here, we will show that the degree of the function of the financial system plays an important role in determining both the aggregate TFP level and the economic growth rate in the long run. Thereafter, we discuss the factors that might explain Japan’s economic stagnation. In addition, we mention the factors that could affect the
capital-output ratio in both the short and long runs.

**Proposition 3.1:** Suppose that the economy exists in region 1. Due to a permanent negative shock on \( \alpha'' \), \( X \), or \( \theta \), a greater amount of resources are allocated to the low-productivity firms. Consequently, both the aggregate TFP level and the economic growth rate decline permanently.

Proof: see appendix 3.1

From the above proposition, it follows that the permanent negative shock on either \( \alpha'' \) or \( \theta \) results in a decline in the collateral value of investment. Since the borrowing constraint of the high-productivity firms becomes stronger, the aggregate investment of these firms decreases while that of the low-productivity firms increases. Moreover, resource allocation becomes more inefficient. Therefore, both the aggregate TFP level and the economic growth rate decline permanently.

With regard to the shock on \( X \), suppose that \( X \) declines. This implies that the ratio of the low-productivity firms increases beyond its previous value. For example, a delay of the changes in the industrial structure from high-productivity industry to low-productivity industry occurs, \( X \) would decline. In this case, even if \( \alpha'' \) or \( \theta \) does not change, this shock results in a decline in the net worth share of the high-productivity firms, and a greater amount of resources are allocated to the low-productivity firms in the credit market. Therefore, both the aggregate TFP level and the economic growth rate decline permanently.

Concerning Japan's stagnation, Ikeo [2003, chapter 5] presents a delay of the changes in the industrial structure. He points out that the main cause of Japan's economic stagnation is that the structural change from the low-productivity to high-productivity industries did not occur smoothly. If we interpret Ikeo's views in terms of our model, \( X \) declines. Aoki and Yoshikawa [2000] attribute the main cause of Japan's economic stagnation to the absence of leading industries that can generate increased demand.

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This story might be interpreted as a decline in $X$.

In these cases, as evident in equation 22, it is important to note that the aggregate TFP is the main factor causing a decline of economic growth rate; this is similar to HP’s model. However, the difference between the two models is that in this model, in the long run, the aggregate TFP level could be affected not only by a technology shock but also by the shocks on the function of the financial system and shocks to the ratio of the high-productivity firms to low-productivity firms. On the other hand, in HP’s model, the aggregate TFP is only affected by a technology shock.

**Proposition 3.2 (Regime Switching Story):** Suppose that the economy exists in either region 2 or 3. Given $\alpha$, due to the large permanent negative shock on either $\alpha^H$, $X$, and $\theta$ the economy switches from a regime with higher growth rate, higher aggregate TFP level, and efficient resource allocation to one with lower growth rate, lower aggregate TFP level, and inefficient resource allocation permanently.

Proof: see appendix 3.2

From the above proposition, it follows that due to the large permanent negative shock on either $\alpha^H$, $X$, or $\theta$, the collateral value of investment declines and the borrowing constraint of the high-productivity firms becomes stronger. Since the negative shock is relatively large, these firms have to reduce their investment to a large extent. In the credit market, the real interest rate declines to $\alpha^L$ such that the low-productivity firms begin to undertake investment. Therefore, the economy switches from a regime with higher growth rate, higher aggregate TFP level, and efficient resource allocation to one with lower growth rate, lower aggregate TFP level, and inefficient resource allocation permanently.

This regime switching story is different from what has been proposed in KM’s model. In their paper, in a credit-constrained economy, there is only one region—region 1 in this paper—in which resource allocation is inefficient. However, in this model, there also exist
regions 2 and 3 in which resource allocation is efficient even in the case of a credit-constrained economy. Thus, it could provide scope for a discussion on the economy's switching from the efficient resource allocation equilibrium to the inefficient resource allocation equilibrium.

With regard to propositions 3.1 and 3.2, it is important to note the following two things. One is that from equation 21, it is evident that the aggregate TFP level in the steady state is independent of monetary policy. Thus, if we interpret Japan's economic stagnation as a long-run event, which means that the steady state itself changes, we might be able to interpret that the permanent negative shock on technology of high-productivity firms, the ratio of the high-productivity firms to low-productivity firms, or the function of the financial system, not the monetary tightening policy shock, were responsible for it.

The other is that if $\theta$ is greater than $1/(1+X)$, even if there occurs a permanent negative shock on $\alpha^H$, as long as $\alpha^H$ is greater than $\alpha^L$, the economy will not enter region 1. Therefore, a well-developed financial system is important as it prevents an economy from entering an equilibrium with inefficient resource allocation.

Next, we discuss the issues pertaining to the capital-output ratio. As HP pointed out, one of the characteristics of Japan's lost decade is the increase in the capital-output ratio. According to HP, this occurred due to the decline in aggregate TFP. However, in this model, we could present certain different mechanisms and factors that result in the increase in the capital-output ratio.

In our model, there are several factors that affect the capital-output ratio, i.e., the marginal productivity of firms, the ratio of the high-productivity firms to low-productivity firms, the degree of the function of the financial system, and an unanticipated monetary policy. Considering the input for production as capital, the capital-output ratio in this model could be presented as follows.
From equation 23, it follows that the capital-output ratio is a decreasing function of aggregate TFP level. Proposition 2 shows that the aggregate TFP level could be affected by an unanticipated monetary policy shock in the short run. This shock results in a change in credit allocation from the high-productivity firms to the low-productivity firms, and therefore, the aggregate TFP level declines. Thus, the capital-output ratio increases temporarily.

Proposition 3 shows that the aggregate TFP level in the steady state could be affected by the marginal productivity of firms, the ratio of the high-productivity firms to low-productivity firms, and the degree of the function of the financial system. For example, due to a permanent negative shock on the function of the financial system (a decrease in the function of the financial system), the aggregate TFP level in the steady state declines because a greater amount of resources are allocated to the low-productivity firms. Thus, the capital-output ratio increases permanently.

5. The relationship between the effectiveness of monetary policy and an aggregate TFP level as well as the degree of the function of the financial system

In this section, we analyze relationships between the effectiveness of monetary policy and an aggregate TFP level as well as the degree of the function of the financial system. In order to conduct this analysis, we will modify the model presented in Section III, and present case 2 about the government policy.

In case 2, the government adopts the following policy.

\[
M_t = (1 + \mu)M_{t-1} \quad \text{and} \quad G_t = \beta Y_t
\]

Government expenditure \(G_t\) is a fraction \(\beta\) of the aggregate output. This is the only difference between cases 1 and 2.
From equations 15, 17, and 24, we obtain the government budget constraint as follows:

\[(25) \quad \beta Y_t = \left( \frac{\mu}{1 + \mu} \right) a_2 (1 - \tau_t) Y_t + \tau_t Y_t \]

Since \( Y_t \) is a predetermined variable, \( \tau_t \) is endogenously determined in order to satisfy \( 25 \). From 25, we can obtain the following:

\[(26) \quad \tau_t = \beta - \left( \frac{\mu}{1 + \mu} \right) a_2 \equiv \phi(\mu) \]

From equation 26, it follows that the tax rate at date \( t \) is a decreasing function of the money growth rate. Intuitively, the more money the government prints, the greater will be the seigniorage. In order to satisfy the government budget constraint, the government can reduce taxes. This implies that the government transfers some of the seigniorage to the firms through a tax cut policy\(^{17}\).

The Definition of a Competitive Economy (Case 2)

Given the initial \( Y_1, E_{i1}^H, M_0 \), the exogenous variables \( \theta, \alpha^H, \alpha^L, p, a \), and the monetary and fiscal policies \( \mu_t = \mu \) and \( G_t = \beta Y_t \), respectively, a competitive equilibrium is a sequence of prices and quantities \( \{C_t, Z_t^H, Z_t^L, B_t, G_t, i, p_t, P_t, P_t^e, Y_t, E_{i1}^H, TFP_t\}_{t=1}^\infty \) that satisfies the following conditions:

1. each manager of the high- and low-productivity firms maximizes his/her utility
2. money and goods market clearing conditions
3. perfect foresight with regard to the future price level
4. the GBC for all \( t \).

In this case, there exists a unique stationary equilibrium. If the economy exists in region 1, we can obtain the stationary equilibrium \( G_t^{BC*}, \left( \frac{E_t^H}{Y_t} \right)^* \) and the aggregate TFP* level by...
using equations 12, 13, 14, 26, and $\frac{E_{t+1}^H}{\overline{Y}_t} = \frac{E_t^H}{Y_t}$. Both the aggregate TFP level and the economic growth rate in the steady state are as follows.

\begin{equation}
Agggregate TFP^* = \alpha^t + (\alpha^H - \alpha^t) \frac{1}{1 - \theta \alpha^H} (\frac{E_t^H}{Y_t})^*
\end{equation}

\begin{equation}
(\frac{Y_{t+1}}{Y_t})^* = a_3 \left(1 - \varphi(\mu)\right) \times Aggregate TFP^*
\end{equation}

As evident in equation 27, the aggregate TFP level in the steady state is an increasing function of the net worth share of the high-productivity firms; however, it is independent of the money growth rate. This is because an increase in the net worth of the firms caused by the monetary easing policy would result in a proportionately equal increase in the aggregate output and the aggregate input. As evident in equation 28, the economic growth rate in the steady state is an increasing function of the money growth rate. Intuitively by printing more money the government can earn an increased seigniorage and thus implement the tax cut policy. Due to this tax cut policy, the firms’ net worth increases, and the borrowing constraint of the high-productivity firms weakens. Therefore, the high-productivity firms can finance more investment. Hence, the economic growth rate will increase. By differentiating equation 28, we can obtain the following.

\begin{equation}
\frac{d(Y_{t+1})^*}{d \mu} = -a_3 \times \varphi'(\mu) \times Aggregate TFP^* > 0
\end{equation}

\begin{equation}
\frac{\partial}{\partial Aggregate TFP^*} \left( \frac{\partial(Y_{t+1})^*}{\partial \mu} \right) = -a_3 \times \varphi'(\mu) > 0
\end{equation}

From 29 and 30, it follows that the effect of monetary policy on the growth rate is an increasing function of aggregate TFP level as long as the economy exists in region 1. In order to examine the mechanism with regard to this, we first analyze the manner in which the
aggregate TFP level in the steady state is determined. From equation 27 and Proposition 1, we understand that the aggregate TFP level in the steady state is an increasing function of $\theta$ in region 1. Therefore, if the economy has well-functioning financial system, it could allocate more resources to the high-productivity firms, and consequently, the aggregate TFP level will increase. Thus, 30 implies the following.

$$
(31) \quad \frac{\partial}{\partial \theta} \left( \frac{\partial (Y_{t+1}^*)}{\partial \mu} \right) = -a_1 \times \varphi'(\mu) \times \frac{\partial \text{Aggregate TFP}^*}{\partial \theta} > 0
$$

From 31, in Region 1, it is evident that the more developed the financial system, the higher is the effectiveness of monetary policy. Intuitively, in order to determine the effectiveness of monetary policy, the sensitivity of investment to the changes in the net worth is crucial. From equation 9, we understand that investment in economies with better developed financial systems displays greater sensitivity.

For example, when a monetary easing policy is adopted, the net worth of the agents increases. In the case of economies with better developed financial systems, the sensitivity of investment to this increase in the net worth is high. In other words, in this model, because of monetary easing policy, redistribution from the retiring managers to the new managers occurs. That is, more resources will flow to the new managers. In better developed financial system, greater fraction of them will be allocated to high-productivity firms. Consequently, the effectiveness of monetary policy is also high. If we interpret the shocks on $\theta$ as the shocks on the function of the financial system, this implies that the effectiveness of monetary policy could be reduced by negative shocks on the function of the financial system\textsuperscript{17}.

However, in case the economy exists in either Region 2 or 3, the aggregate TFP level in the steady state is independent of $\theta$ (the same as equation (7)). Thus, the effectiveness of
monetary policy does not depend on the degree of the development of the financial system. This implies that even if there is a negative shock on $\theta$, production does not become inefficient. Only high-productivity agents invest. This is because the economy has already had better developed financial system even before the shock. Therefore, the effectiveness of monetary policy would not be affected. We summarize the above discussion as Proposition 4.

**Proposition 4**: (1) In Region 1, the more developed the financial system is, the higher is the effectiveness of monetary policy. (2) In Regions 2 and 3, the effectiveness of monetary policy does not depend on the degree of the development of the financial system.

**Proof**: see appendix 4

This result is applicable to the case of $X$. Think about the case where $X$ declines. As we mention above, for example, a delay of the changes in the industrial structure from low to high industry occurs, $X$ would decline. If the economy exists in region 1, the effectiveness of monetary policy will be reduced with it. This is because even if monetary easing policy is conducted, more resources will be allocated to low-productivity industry, so that the effectiveness of monetary policy will be reduced. However, in region 2 or 3, the effectiveness is independent of $X$.

Concerning the Japanese economy, we might give some interpretations as follows. Because $\theta$ implies the liquidation value of the firms (the investment project), it could be affected by the smoothness of coordination between the creditors in the event of firms’ default. Considering this point, one possible example of negative financial system shocks is that for example, in Japan, since the shock of bubble burst in the beginning of the 1990s was so large, many banks were damaged to a large extent. Because of this, it was said that coordination between the creditors in the liquidation of the bankrupt firms did not proceed smoothly because banks wanted to avoid additional damages. This might have reduced $\theta$ temporarily.
At the same time, the effectiveness of monetary policy in Japan after the bubble burst does not appear to be strong enough to push the economy even though Bank of Japan has been taking a monetary easing policy for a long period of time. From the present analysis, this ineffectiveness might be related to negative financial system shocks mentioned above or the delay of the changes in the industrial structure pointed by Ikeo.

6. Conclusion

In this paper, we developed theoretical analyses in order to answer the following questions. (1) How could monetary policy affect both an aggregate TFP level and an economic growth rate in a credit-constrained economy in the short run? (2) What are the factors which could determine an aggregate TFP level and an economic growth rate in a credit-constrained economy in the long run? (3) What is the relationship between the effectiveness of monetary policy and an aggregate TFP level as well as the degree of the function of the financial system?

With regard to question 1, if the degree of the function of the financial system is not high, an unanticipated monetary policy could result in business fluctuations through persistent changes in the aggregate TFP level in the short run. However, if the degree of the function of the financial system becomes high to some extent, business fluctuations would not occur as long as the monetary shock is small.

With regard to question 2, in the long run, both the aggregate TFP level and the economic growth rate could be affected by real factors such as the marginal productivity of firms, the ratio of the high-productivity firms to low-productivity firms, and the degree of the function of the financial system.

With regard to question 3, if the degree of the function of the financial system is not high, the effectiveness of monetary policy could be reduced because of negative shocks on the
function of the financial system. However, if the degree of the function of the financial system becomes high to some extent, the effectiveness of monetary policy would not be affected by negative shocks on the function of the financial system.

Examining Japan’s economic stagnation from this perspective, we might give some interpretations from a theoretical perspective. If we interpret Japan’s economic stagnation as a short-run event, an unanticipated monetary tightening shock might be considered as the reason behind it. However, if we interpret this stagnation as a long-run event, the changes in the real factors mentioned above might be considered as the reasons behind it.

In addition, we might attribute the ineffectiveness of monetary policy to a decrease in the function of the financial system. In future studies, we aim to conduct a quantitative analysis in order to identify which hypotheses are the most plausible in explaining Japan’s economic stagnation using a model that takes labor into account.
Appendix 1

Here, we will verify that in the case where \( \theta < \hat{\theta} \), the real interest rate is equal to the marginal rate of return on the investment of the low-productivity firms, the borrowing constraint of the high-productivity firms binds, and both the high- and low-productivity firms undertake investment in the neighborhood of the steady state. In order to verify this, we only need to assess whether the low-productivity firms invest a positive amount of goods.

If the net worth share of the high-productivity firms in the steady state \( \frac{E^H_t}{Y_t} \) takes the value \( 1 - \frac{\theta \alpha^H}{\alpha^L} \), the low-productivity firms’ aggregate investment becomes zero in the steady state. Introducing the condition \( \left( \frac{E^H_t}{Y_t} \right)^* = 1 - \frac{\theta \alpha^H}{\alpha^L} \) in equation 20, we obtain the value of \( \theta \) \( (= \hat{\theta}) \), where the low-productivity firms’ aggregate investment becomes zero in the steady state. If \( \theta \) is strictly less than \( \hat{\theta} \), the low-productivity firms’ aggregate investment takes a positive value in the neighborhood of the steady state. If these firms invest a positive amount, the real interest rate is equal to the marginal rate of return on their investment such that the borrowing constraint of the high-productivity firms is binding. Since both high and low productive firms invest, both the aggregate TFP level and the growth rate of the economy are lower than those of the case without the borrowing constraint.

If \( \hat{\theta} \leq \theta < \psi(1 + \chi) \), from the following two equations, we obtain a unique stationary equilibrium in which the net worth share of the high-productivity firms remains constant over time. \( \frac{E^H_{t+1}}{Y_{t+1}} = p (1 - \theta) + X (1 - p) \theta \) and \( \frac{E^H_{t+1}}{Y_{t+1}} = \frac{E^H_t}{Y_t} \).

\[ \therefore \left( \frac{E^H_t}{Y_t} \right)^* = p (1 - \theta) + X (1 - p) \theta \]

If \( \hat{\theta} \leq \theta < \psi(1 + \chi) \) and \( p \) is not equal to 1, the following inequality is satisfied.
From equation 32, the equilibrium condition in the credit market is as follows:

\[
\frac{E^H}{Y_t^*} \frac{1}{1 - \frac{\theta \alpha^H}{\alpha}} \leq p(1 - \theta) + X(1 - p)\theta < 1 - \theta
\]

This implies that all the funds supplied in the credit market by the low-productivity firms are provided for the borrowing needs of the high-productivity firms. Thus, both the aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint. Since the real interest rate is lower than \( \alpha^H \), the borrowing constraint of the high-productivity firms binds.

If \( 1/(1+X) \leq \theta \leq 1 \), from the following two equations, we obtain a unique stationary equilibrium in which the net worth share of the high-productivity firms remains constant over time.

\[
\frac{E^H_{t+1}}{Y_{t+1}^*} = p\left(\frac{E^H_t}{Y_t^*}\right) + X(1 - p)(1 - \frac{E^H_t}{Y_t^*}) \quad \text{and} \quad \frac{E^H_{t+1}}{Y_{t+1}^*} = \frac{E^H_t}{Y_t^*}.
\]

\[
\therefore \frac{E^H_t}{Y_t^*} = \frac{X}{1 + X}
\]

If \( \theta \geq 1/(1+X) \), the following inequality is satisfied.

\[
X/(1 + X) \geq 1 - \theta
\]

From inequality 33, the equilibrium condition in the credit market is as follows:

\[
\frac{E^H_t}{Y_t^*} \frac{1}{1 - \frac{\theta \alpha^H}{r^*}} \geq 1 \quad \text{and} \quad r^* = \alpha^H
\]

This implies that all the funds supplied in the credit market by the low-productivity firms are provided for the borrowing needs of the high-productivity firms. Thus, both the aggregate TFP level and the economic growth rate are exactly the same as those in the case without the borrowing constraint. Since the real interest rate is equal to \( \alpha^H \), the borrowing constraint of the high-productivity firms does not bind.

Appendix 2

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Proof of (1)

We define \( E_t^H / Y_t \) as \( x_t \). Accordingly, equation 14 becomes as follows.

\[
(34) \quad s_{t+1} = \phi(s_t)
\]

At date \( s \), suppose that there is an unanticipated monetary tightening shock. Consequently, the net worth share of the high-productivity firms at date \( s \) \( E_t^H / Y_t \) declines. Taking linear approximation with regard to equation 34 in the neighborhood of the steady state, we obtain the following:

\[
(35) \quad s_{t+1} = \phi(s^*) + \phi'(s^*)(s_t - s^*)
\]

From equation 35, we obtain the following equation.

\[
(36) \quad \hat{s}_{t+s+n} = \phi'(s^*)^n \hat{s}_t \quad \text{where} \quad \hat{s}_{t+s} \equiv \frac{S_{t+s} - s^*}{s^*}, \quad \phi'(s^*) < 1
\]

If we take that \( n \to \infty \), we obtain \( \phi'(s^*)^n \to 0 \) and \( \hat{s}_{t+s+n} \to 0 \).

Therefore, it takes a certain amount of time for the net worth share of the high-productivity firms to recover. From equations 12 and 13, if the net worth share of the high-productivity firms is changing over time, both the aggregate TFP level and the economic growth rate will also change until the net worth share of the high-productivity firms returns to the original value in the steady state.

Proof of (2)

In region 2, where \( \theta < \hat{\theta} \), as long as the shock is small, the equilibrium condition in the credit market does not change. The equilibrium condition in the credit market is as follows:

\[
\frac{E_t^H}{Y_t} \bigg/ \left(1 - \frac{\theta \alpha^H}{r_t} \right) = 1 \quad \text{and} \quad r_t \in [\alpha^L, \alpha^H)
\]

If \( \rho \) is greater than \( X(1 - \rho) \), an unanticipated monetary tightening policy shock results in a decline in the net worth share of the high-productivity firms \( E_t^H / Y_t \). In order to satisfy the
equilibrium in the credit market, the real interest rate has to decline. As a result of this adjustment, all the funds supplied in the credit market by the low-productivity firms are still provided for the borrowing needs of the high-productivity firms. Thus, both the economic growth rate and the aggregate TFP level remain unchanged.

In region 3, where \( X^\theta < 1 \), as long as the shock is small, the equilibrium condition in the credit market does not change, and it remains as follows:

\[
\frac{E_t^H}{Y_t}^* \left( 1 - \frac{\theta \alpha^H}{r^*} \right) \geq 1 \quad \text{and} \quad r^* = \alpha^H
\]

In such a case, all the funds supplied in the credit market by the low-productivity firms are still provided for the borrowing needs of the high-productivity firms. Thus, both the economic growth rate and the aggregate TFP level are unchanged. However, in a case where \( \theta = 1/X^\theta \), a negative (positive) shock on the aggregate net worth of high productive firms results in decreasing (maintaining) the real interest rate, and the economic growth rate and the aggregate TFP level remain unchanged.

Appendix 3.1

From equations 14 and \( \frac{E_{t+1}^H}{Y_{t+1}} = \frac{E_t^H}{Y_t} \), we obtain the following:

\[
\frac{d \left( \frac{E_t^H}{Y_t} \right)^*}{d \theta} > 0, \quad \frac{d \left( \frac{E_t^H}{Y_t} \right)^*}{d \alpha^H} > 0, \quad \frac{d \left( \frac{E_t^H}{Y_t} \right)^*}{d X} > 0
\]

Considering this relationship, from equations 12 and 13, we obtain the following:

\[
\therefore \frac{d \left( Y_{t+1} \right)^*}{d \theta} > 0, \quad \therefore \frac{d \text{AggregateTFP}^*}{d \theta} > 0
\]

and

\[
\therefore \frac{d \left( Y_t \right)^*}{d \alpha^H} > 0, \quad \therefore \frac{d \text{AggregateTFP}^*}{d \alpha^H} > 0
\]
and \[ \frac{\text{d} \left( \frac{Y_{t+1}}{Y_t} \right)^\ast}{\text{d}X} > 0 \text{,} \quad \frac{\text{d}\text{AggregateTFP}}{\text{d}X}^\ast > 0 \]

Thus, a permanent negative shock on \( \alpha^u \), \( X \), or \( \theta \) results in a permanent decline in both the aggregate TFP level and the economic growth rate.

Appendix 3.2

First, we prove the case with regard to \( \alpha^u \). Suppose that the economy exists in region 2.

In region 2, the following inequality is satisfied.

\[ p(1 - \theta) + X(1 - p)\theta \geq 1 - \frac{\theta\alpha^u}{\alpha^l} \]

We define \( \alpha^u \) satisfying \[ p(1 - \theta) + X(1 - p)\theta = 1 - \frac{\theta\alpha^u}{\alpha^l} \]

as \( \alpha^{u'} \).

As long as \( \theta \) is strictly less than \( 1/(1 + X) \), \( \alpha^{u'} \) is strictly greater than \( \alpha^l \). If \( \alpha^u \) is strictly less than \( \alpha^{u'} \), the economy exists in region 1 in the neighborhood of the steady state. This is because the net worth share of the high-productivity firms in the steady state \( \left( \frac{E^u_i}{Y_i} \right)^\ast \) satisfies the following inequality.

\[ \left( \frac{E^u_i}{Y_i} \right)^\ast < p(1 - \theta) + X(1 - p)\theta < 1 - \frac{\theta\alpha^u}{\alpha^l} \]

If a permanent negative shock results in a decline in \( \alpha^u \) such that it is strictly lower than \( \alpha^{u'} \), the economy shifts from region 2 to region 1.

Next, we prove the case with regard to \( \theta \). Suppose that the economy exists in region 2.

In region 2, the inequality, \( \hat{\theta} \leq \theta < 1/(1 + X) \) is satisfied. If a permanent negative shock results in a decline in \( \theta \) such that it is strictly lower than \( \hat{\theta} \), from Proposition 1, it follows that the economy shifts from region 2 to region 1 permanently.

Suppose that the economy exists in region 3. In region 3, \( 1/(1 + X) \leq \theta \leq 1 \) is satisfied. If a permanent negative shock results in a decline in \( \theta \) such that it is strictly lower than \( \hat{\theta} \),
from Proposition 1, it follows that the economy shifts from region 3 to region 1 permanently.

Appendix 4

Proof of (1)

By differentiating equation 28, we obtain the following:

\[
\frac{d (\frac{Y_{t+1}}{Y_t})^*}{d \mu} = -a_3 \times \varphi'(\mu) \times AggregateTFP^* > 0
\]

From Proposition 3.1, it follows that \( \frac{d AggregateTFP^*}{d \theta} > 0 \). Considering this relationship, we obtain the following:

\[
\therefore \frac{\partial}{\partial \theta} \left( \frac{\partial (\frac{Y_{t+1}}{Y_t})^*}{\partial \mu} \right) = -a_3 \times \varphi'(\mu) \times \frac{\partial AggregateTFP^*}{\partial \theta} > 0
\]

Thus, if the economy exists in region 1, the effectiveness of monetary policy could be reduced because of negative shocks on the financial system.

Proof of (2)

In regions 2 and 3, both the aggregate TFP level and the economic growth rate in the steady state are as follows.

\[
AggregateTFP^* = \alpha^H
\]

\[
(\frac{Y_{t+1}}{Y_t})^* = \alpha^H a_3 (1 - \varphi(\mu))
\]

Therefore, it is obvious that the effect of monetary policy on the economic growth rate is independent of the degree of the function of the financial system, \( \theta \).
References


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Footnotes

1. See Fukao [2006], Hayashi and Nomura [2005], Jorgenson and Motohashi [2005], Miyagawa et al. [2005] for related studies on aggregate TFP growth in Japan. These studies also show that aggregate TFP growth declined in Japan during the 1990s compared to the 1980s although the size of the decline is not as large as the one in Hayashi and Prescott [2002]. On the other hand, Kawamoto shows that aggregate TFP growth during the 1990s did not decline compared to the 1980s. See also Miyagawa’s [2006] excellent survey about recent studies on productivity.

2. Misallocation of credit implies underinvestment or overinvestment. The former occurs because of the borrowing constraint. The latter occurs because of the ever-greening policy by banks. Fukuda, Kasuya, and Nakajima [2005 a], Nagahata and Sekine [2002], Ogawa and Suzuki [1998], and Ogawa [2003] showed empirically that underinvestment occurred in Japan during Japan’s lost decade. Overinvestment was first pointed out by Hoshi [2000] in Japan and was empirically shown by Hosono and Sakuragawa [2005], Peek and Resengren [2005], Sakuragawa [2002], and Caballero et al. [2005]. Fukuda, Kasuya, and Nakajima [2005 b] empirically analyze whether ever-greening to unlisted companies occurred or not during the late 1990s and early 2000s. See also Shioji’s [2006] excellent survey about Japan’s lost decade.

3. A decrease in the function of the financial system hypothesis is proposed by Ikeo [2003, chapter 3 and 4], and Hoshi and Kashyap [2001, 2004]. Ikeo emphasized that although the Japanese financial system—in which the banking system plays a central role—was effective in the past, it was no longer suited to Japan’s needs, and therefore, it needed to be reformed to one with well-developed capital markets. Further, he suggested that if these reforms were not undertaken, the economy would
be unable to allocate resources to the high-productivity sectors. Ikeo proposed that one of the main reasons for Japan’s economic stagnation was the failure of the structural changes in the financial system.

4. A failure of monetary policy implies that monetary policy during the 1990s by bank of Japan was too tight to push the economy. See, e.g., Bernanke and Gertler [1999], Iwata [2000], Jinushi et al. [2000], and Noguchi and Okada [2002] for detail discussions.

5. Chari, Kehoe, and McGrattan [2002] develop a dynamic general equilibrium model to explain the U.S. Great Depression by focusing on several hypotheses, some of which are similar to the ones in this paper. However, their work does not necessarily explain the relationship between these hypotheses and aggregate TFP. Kobayashi and Inaba [2005] develop the method of business cycle accounting and Christiano and Fujiwara [2006] develop dynamic general equilibrium with calibration to account for Japan’s lost decade.

6. Basu [1996] shows that the cyclical movement of aggregate TFP is more generated by fluctuations of the utilization rate of capital and labor rather than technology shocks. He points out that fluctuations of aggregate TFP is attributed to ones of aggregate demand. This paper stands on the same side with real business cycle model in the sense that we focus on supply side of the economy to explain fluctuations of aggregate TFP. However, the innovations of this paper theoretically show that aggregate TFP could be affected not only by technology shocks, but also different factors such as monetary policy shocks or financial system shocks.

7. Hirano [2006] theoretically showed ineffectiveness of monetary policy using the balance sheet channel. He showed that the balance sheet effect is nonlinear depending on a firm’s balance sheet condition. Once the economy is in a state of deep
recessions, where a firm's balance sheet condition is relatively bad, the effectiveness of monetary policy becomes extremely weak because the balance sheet channel does not function effectively. However, this paper does not relate the effectiveness of monetary policy to aggregate TFP as well as the degree of the function of the financial system. Aoki and Yoshikawa [2003] discuss policy ineffectiveness and long stagnation of the macroeconomy focusing on uncertainty in a model with heterogeneous agents like in this paper.

8. This is based on Hart and Moore [1994].

9. We can obtain the same result with regard to a case in which these two values are different. In fact, in this model, the important factor that affects the aggregate variables is $\theta^u$, and not $\theta^l$.


11. If $\rho$ is equal to $X/(1+X)$, the size of the positive and negative effects are same. In such a case, no dynamics will occur. If $\rho$ is less than $X/(1+X)$, the positive effect is greater than negative effect. In such a case, an oscillation will occur.

12. Since Kiyotaki's [1998] framework does not have money, the paper cannot analyze the effectiveness of monetary policy. In addition, Kiyotaki implicitly assumes low enough $\theta$ (Region 1 in this paper) although the paper does not necessarily clarify this point. Furthermore, besides Region 1, there are two different regions with different properties, Region 2 and 3 which kiyotaki's paper does not mention at all. As we show in this paper, in which regions the economy exists is critical to determine credit allocation, aggregate TFP level, the volatility of business fluctuations, and the effectiveness of monetary policy.

14. We exclude the case in which $\theta = \hat{\theta}$ in region 2. In such a case the negative shock on the aggregate net worth of the high-productivity firms, persistent occurs; however, by the positive shock, persistent does not occur. This is because by the positive shock, the real interest rate increases in the credit market such that all the savings for investment are allocated only to the high-productivity firms.

15. Bernanke (1999) calculated how much redistribution of wealth from the debtors to the creditor occurred by an unanticipated lower inflation in Japan from 1991 to 1999. Although he took the extreme case as he said, the real value of the debt obligation became 22% higher by 1999.

16. Raphael et al. [2002] showed that although Chile and Mexico experienced severe economic crisis in the early 1980s, Chile recovered much faster than Mexico because of the earlier reforms in banking and bankruptcy procedures. From this model, these reforms could be interpreted as an increase in $\theta$. Yamazaki et al. [2006] point out that mortgage violation occurred in Japan in both 1989 and 1991. This could lower the value of $\theta$.

17. The use of seigniorage is critical in determining monetary policy effect. If the government transfers seigniorage only to the retiring managers, who do not possess any investment, monetary policy does not affect the real variables in this model.
Figure 1

Aggregate TFP

• Economic Growth Rate
• Business Cycles

Hayashi and Prescott’s Model

A Decrease the Function of the Financial System

Credit Allocation (Misallocation of Credit)

Monetary Policy Shock

Our Model

Aggregate TFP

• Economic Growth rate
• Business Cycles
The Movement of the Net Worth Share of High-Productivity Firms
Figure 3.1

An Impulse Response in Region 1

An Impulse Response in Region 1
Figure 3.2

An Impulse Response in Region 2 or 3

AggregateTFPLevel, $G_t^{BC}$

An Impulse Response in Region 2 or 3
Figure 4

Share of Tangible Fixed Assets