

# Volatile Youth Employment\*

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## Abstract

It is widely observed in OECD countries that age-specific employment has different features over business cycles. Youth employment has larger cyclical fluctuations than does adult employment. However, there are significant differences in the magnitude of youth employment fluctuations across countries. In this paper, I argue that these two facts can be explained by separation cost, which increases with the period of time for which a worker is employed. For a detailed analysis, I formulate a search and matching model with an overlapping generation structure, assuming higher separation costs for tenured workers.

**Keywords:** Age-heterogeneity, unemployment rate, OLG, separation cost, stochastic job matching.

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\*I am especially grateful to Toni Braun, Fumio Hayashi, Julien Esteban-Pretel, and Futoshi Narita for their detailed suggestions, encouragement, and support, and to Yuji Genda and Ryoichi Imai for helpful comments and advice. I also thank seminar participants at the University of Tokyo, Osaka University, Kobe University, and Search Theory Workshop. I am, nonetheless, solely responsible for any errors.

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# 1 Introduction

The labor market performance in OECD countries has significant generational features. The age-specific employment rates are different in level and volatility. In particular, the youth employment rate has distinct features. The youth unemployment rate, which is equal to 1 minus the youth employment rate, is several times higher than the aggregate unemployment rate. In addition, its cyclical fluctuations are larger than those observed in other rates. Although many policies that aim to solve the youth employment problems have been issued, such age-heterogeneous features still exist. This historical evidence suggests the need for a comprehensive study of the problem. In this paper, I argue that the cost of separation is a source of age-heterogeneous features in labor market performance.

In the analysis, I define "separation costs" as any kind of difficulty and cost associated with the termination of employment, except for the age limit retirement. This is a fairly inclusive and aggregate concept. For example, the employment protection legislations and labor union pressure constitute separation costs because they make separations difficult. The dismissed worker's embodied skill and the adjustment costs in layoffs, such as the relocation of workers and the handover process, are also separation costs. Essentially, separation costs increase with the worker's tenure—the years of continuous employment.

Separation costs have implications for the gaps in the youth employment performance between countries. In the existing literature, the high youth unemployment rate is explained by young people's greater preference for vocational choices. If the youth unemployment rate is actually higher for this reason only, there should not be any significant difference in its performance in different countries. However, among OECD countries, there are significant *quantitative* differences in youth employment performance, although the *qualitative* features are the same. The level of the country-specific youth employment rate differs greatly, whereas the youth employment rate is lower than the other age-group employment rates in every country. Similarly, the magnitude of fluctuations in the country-specific youth employment rate is considerably different, whereas the youth employment rate has larger cyclical fluctuations than the other age-group employment rates in every country.<sup>1</sup> These *quantitative* gaps can be explained by the different structures of the

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<sup>1</sup>The following section documents these phenomena.

country-specific separation costs.

For a detailed analysis, I develop a search and matching model that has endogenous age-heterogeneity in job-match decisions. A key assumption I make is the higher separation costs for the separation of tenured workers. Taking into account of such an exogenous separation cost structure, firms and workers decide whether or not to begin working with their partner. In addition, the model has the two-period overlapping generation (OLG) structure that allows for the analysis of different optimal decisions of agents by age-group. The model, which has these main features, has age-heterogeneity in job-match decision in equilibrium.

The separation cost creates the following mechanism. If the separation cost increases with the worker's tenure, firms have an opportunity cost of job-match with the youth, while there are no such forward-looking opportunity costs for the adult job-match. Generally, separation costs are not multiples of business-cycle indicators because they include institutional factors. Therefore, business cycles lead to a fluctuation in the firms' opportunity cost of job-match with the youth; this results in larger cyclical fluctuations in youth employment than in adult employment. In addition, the more the separation cost increases with the worker's tenure, the greater is the volatility of youth employment. This is because separation cost enlarges the effect of business cycles on the job-match decision of youth. If the structure of the separation cost differs by country, then this mechanism would account for the international differences in youth employment performance.

The separation cost, as an inclusive concept, is useful for this macroeconomic analysis. Previous labor literature has mainly focused on one particular factor and analyzed the factor-specific mechanism.<sup>2</sup> However, there are problems with such classical analyses because of their limited focus. For instance, in an analysis focusing on the so-called "firing cost," it is necessary to distinguish between firing and voluntary separation, but it is generally impossible to identify that difference in the data. Although there have been attempts to capture firing costs by assessing employment protection legislations, the results have been inconclusive. The main reason is that we are unable to reasonably measure the actual effectiveness of the legislation. Moreover, an analysis focusing on a particular factor cannot fully explain macroeconomic phenomena, which involve all existing factors and their in-

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<sup>2</sup>See, for example, Rogerson and Hopenhyan (1993) and Mortensen and Pissarides (1999).

teractions. By using separation costs and without specifying the components, we can avoid such problems in the analysis of macroeconomic phenomena.

There is another advantage to capturing age-heterogeneity in employment fluctuations by separation costs. The cross-country differences in the magnitude of the youth employment volatility can be explained by the different levels of separation cost. This is important because the data of different countries differ significantly in terms of both the level and the volatility of the age-specific employment rate.

The importance of understanding age-heterogeneity in employment has been suggested in other contexts as well. Shimer (1998) and Herbertsson et al. (2001) argued empirically that fluctuations in the age-specific unemployment rate, rather than those in cohort size, had the largest impact on fluctuations in aggregate unemployment. In addition, based on implications in the wage scar literature,<sup>3</sup> age-heterogeneity in the unemployment rate affects other business-cycle phenomena. Since the negative effect of unemployment on the labor productivity is persistent, the age-specific unemployment rates affect the macroeconomy differently in terms of persistency.

Different volatilities of employment across age groups have recently received attention in macroeconomics. Until recently, although many works focused on the life-cycle pattern of job mobility,<sup>4</sup> few studies investigated the age-heterogeneous volatilities of employment over business cycles. A recent paper by Gomme et al. (2004) argued that understanding the forces that induce different fluctuations among age groups is relevant for an understanding of aggregate fluctuations. They showed that a standard business-cycle model cannot capture the high volatilities of the youth hours worked, and thus it produces fewer fluctuations in the average hours of market work. The model developed in this paper will contribute to this line of research because it endogenously produces higher volatility in youth employment than in adult employment.

The remainder of this paper is divided into four sections. In section 2,

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<sup>3</sup>Ruhm (1991) and Jacobson et al. (1993) show that displaced workers experience long-lasting earnings loss. Gregg and Tominey (2004) suggest that youth unemployment also has negative effect of unemployment on labor productivity. They show that workers who experienced unemployment in their youth earn less than workers who did not, even 40 years later.

<sup>4</sup>Jovanovic (1979) and Neal (1999) are prominent theoretical works in this area. Mincer (1986), Topel (1986), and Topel and Wald (1992) examined empirically the implications of such theories on the age-heterogeneous job mobility.

I document age-heterogeneity in employment fluctuations for OECD countries. Section 3 provides a model to investigate the different effects that separation costs have on different generations. Next, section 4 analyzes the equilibrium properties of the model and discusses their implications. Finally, some concluding remarks are presented in section 5.

## 2 Data Analysis

In this section, I document the age-heterogeneous features of employment over business cycles; these features are commonly observed in many countries. The first feature is a well-known level fact: the youth (15-24 years) employment rate is always lower than the adult rate. The second feature is that the youth employment rate varies more in response to economic conditions than the adult rate does. In other words, youth employment changes more along with business cycles than does the adult rate. In addition, I highlight the international differences in the magnitude of these two features. The second age-heterogeneous feature and the international difference are relatively new analyses as few researchers have discussed them before.

Of the many ways of capturing employment fluctuations over business cycles, I choose the employment rate as an indicator of employment<sup>5</sup> and the output gap as that of business cycles. The employment rate can capture global phenomena because it is free of biases due to population size and non-economic factors. For example, if the employment population ratio were to be used instead, one would have to consider the bias of the increasing school enrollment rate and female labor participation rate. As for the business-cycle indicator, I use the deviation from the HP-filtered logged GDP per capita, following Kydland and Prescott (1990). In filtering, I set the adjustment coefficient to 6.25, as recommended by Ravn and Uhlig (2001) for annual data.

All data used in the analyses are taken from the OECD database and are internationally comparable. The employment rate is obtained from OECD “Labour Force Statistics.”<sup>6</sup> It is defined according to the ILO resolution, which was adopted by the 13th International Conference of Labour Statisti-

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<sup>5</sup>Genda (2004) investigated age-heterogeneous features in the Japanese labor market, using a variety of indicators.

<sup>6</sup>URL: <http://www1.oecd.org/scripts/cde/members/lfsindicatorsauthenticate.asp>.

cians in October 1982. The per capita real GDP is the one at the price levels and PPPs of 2000 (US dollars), obtained from "Part II to IV of National Accounts of OECD Countries-Vol. 1."<sup>7</sup> The data is for the period 1970 to 2003.

I decompose the data to capture age-heterogeneity between youth and adults. In accordance with the United Nations' definition, I define youth as 15-24 year-olds and adults as 25-64 year-olds. Given this decomposition, we can clearly observe age-heterogeneous features of employment. Such features can be observed even if a finer age-grouping were to be used and other characteristics, such as education and race, are considered.<sup>8</sup>

Figure 1 shows the male employment rate by age group for Japan, the United States, the 15 countries of the EU, and OECD countries. Using this simple plot of the time series, we obtain the two age-heterogeneous characteristics for every country. First, the youth employment rate is always lower than the adult rate. Second, the youth employment rate shows larger fluctuations than does the adult rate over time. As shown in Figure 2, the female employment rate shares the same two properties.

In order to capture the extent to which the employment rate by age group moves along with business cycles, or the change in the output gap, I employ relative volatility that is defined as follows:

$$\frac{\sigma_e^i}{\sigma_y} \equiv \frac{Var(E^i)}{Var(\tilde{y})}, \quad \text{for each age group } i, \quad (1)$$

where  $Var(\cdot)$ ,  $E^i$ , and  $\tilde{y}$  stand for the variance, the employment rate of the age-group  $i$ , and the deviation from the HP-filtered logged GDP per capita, respectively. Therefore,  $\sigma_e^i/\sigma_y$  captures the extent to which the employment rate in the age-group  $i$  moves when the output gap changes.

This relative volatility enables us to clearly compare the magnitude of employment volatility among age groups and countries. Figure 3 is a bar chart of the relative volatility by age group and gender. It is evident that the youth employment rate fluctuates much more than the adult one does over business cycles. The difference in the magnitude of the relative volatility by age group is significant for every country. A comparison among countries

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<sup>7</sup>URL: [http://www.oecd.org/document/28/0,2340,en\\_2825\\_495684\\_2750044\\_1\\_1\\_1\\_1,00.html](http://www.oecd.org/document/28/0,2340,en_2825_495684_2750044_1_1_1_1,00.html).

<sup>8</sup>For details, refer to Shimer (1998), which documents the time series of the unemployment rate, that is, one minus the employment rate, in the US, using age groups by 5-year period.

reveals that there are large international differences in the magnitude of the age-specific relative volatility, as is shown in Table 1. For example, the youth relative volatility of an EU male is about 15 times more than that of a US male.

In summary, the data illustrates that the youth employment rate varies more than the adult rate over business cycles, and that the magnitude of the age-specific employment volatility differs significantly across countries. In the next section, I will provide a simple model to account for these phenomena.

### 3 Model

In this section, I present a simple model that addresses the different volatilities of employment across age groups. Consider a discrete time economy that has a two-period overlapping generation structure. In each generation, there is a continuum of workers with measure one, and each worker is born unemployed. I assume that job-searching workers are ex ante identical, except for their age. Although the workers have different characteristics that affect their productivity, such characteristics can be observed by firms only after a meeting. The huge volume of firms that enter the labor market is also ex-ante identical for job-searching workers. Many characteristics of each firm cannot be observed by workers before establishing some contact.<sup>9</sup>

At the beginning of each period, the unemployed workers and a number of jobs enter the labor market and meet each other randomly. I also assume that each firm can meet at most one worker, and vice versa. After meeting, each job-worker pair finds a specific productivity level due to the resolution of many ex-ante unobserved characteristics. The specific productivity of the job-worker pair is retained until job destruction. Based on the resolved pair specific productivity, the job and the worker decide whether or not they should work together. In other words, not all job-worker pairs that randomly meet in the labor market lead to a job-match. The job-matches occur only if the specific productivity of the job-worker pair is sufficiently high.

Formally, the number of meetings is determined by a linearly homoge-

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<sup>9</sup>Pissarides (2000) terms a search and matching model with such an information structure as "stochastic job-matching model."

neous function of the amount of job-searching workers  $s_t$  and vacancies  $v_t$ :

$$m(s_t, v_t) \equiv q(\theta_t)v_t$$

where  $q(\theta_t) \equiv m\left(\frac{1}{\theta_t}, 1\right)$ ,  $\theta_t \equiv \frac{v_t}{s_t}$ .

Since  $m(s_t, v_t)$  returns the number of meetings, it satisfies

$$m(s_t, v_t) \leq \min\{s_t, v_t\}. \quad (2)$$

For simplicity, I assume that condition (2) holds with strict inequality. Since there are two generations in each period—young and adult—the amount of job-searching workers  $s_t$  can be written as

$$s_t = s_t^Y + s_t^O$$

where  $s_t^i$  is the number of the searching workers of generation  $i$  ( $i \in \{Y, O\}$ ). Since all workers are born unemployed,  $s_t^Y = 1$  for all periods  $t$ . At any period  $t$ , an unfilled vacancy meets a generation- $i$  worker with probability  $\frac{s_t^i}{s_t}q(\theta_t)$ , and each worker contacts a firm with probability  $\theta_t q(\theta_t)$ .

After meeting, every job-worker pair realizes its specific productivity. The job-worker pair  $j$  has its specific productivity  $\alpha_j$ , which is randomly drawn from a known distribution function  $G(\cdot)$ , with finite range  $[0, \bar{\alpha}]$ . The productivity  $\alpha_j$  is assumed to remain constant until job destruction. Given the pair-specific productivity  $\alpha_j$ , the firm and worker decide whether or not to produce together. If the worker and the firm decide not to work together, they continue to search for a new partner. In posting a vacancy, the firm has to bear the fixed cost  $c$ . During the search, I assume that the worker does not enjoy any real return, i.e., his or her utility is zero.

If the worker and the firm agree to begin their job, production occurs and the surplus is shared between them. For simplicity, the production function  $y(\cdot)$  is assumed to be a simple product of three elements: the general productivity  $p_t$ , the pair-specific productivity  $\alpha_j$ , and labor input.

$$y(p_t, \alpha_j) = p_t \alpha_j. \quad (3)$$

Note that the labor input is normalized to one, because I assume an inelastic labor supply in this model. I assume that all agents are perfectly aware of the whole series of the general productivity  $\{p_t\}_{t=0}^{\infty}$ . After the production,

the worker takes the wage that corresponds to a fraction  $\delta \in (0, 1)$  of the matching surplus while the firm retains the remainder.

At the beginning of the next period, all job-matches with adult workers are terminated because adult workers die. On the other hand, young workers grow adult and job-matches with the new adult workers are randomly terminated with an exogenous probability  $\lambda \in (0, 1)$ . In the case of separation, the firm incurs a fixed flow cost denoted by  $\Psi$ , or the separation cost. This is a key assumption. The separated firms and workers return to the labor market to seek a new partner. They continue their job with probability  $(1 - \lambda)$ .

Let  $J_t^i(\cdot)$  be the present-discounted expected value of expected profit from an occupied job with a generation- $i \in \{Y, O\}$  worker and  $V_t$ , the present-discounted expected value of expected profit from a vacant job at time  $t$ . Then,  $V_t$  satisfies the Bellman equation

$$\begin{aligned} V_t = & -p_t c + \frac{s_t^Y}{s_t} q(\theta_t) \int_0^{\bar{\alpha}} \max\{J_t^Y(\alpha), \beta V_{t+1}\} dG(\alpha) \\ & + \frac{s_t^O}{s_t} q(\theta_t) \int_0^{\bar{\alpha}} \max\{J_t^O(\alpha), \beta V_{t+1}\} dG(\alpha) + \beta(1 - q(\theta_t))V_{t+1}, \end{aligned} \quad (4)$$

where  $\beta \in (0, 1)$  is the time discounting factor. Posting a vacancy incurs the flow cost  $p_t c$  in the current period; however, it can meet a worker with a certain probability in the next period. Since I assume “indirect search,” the vacancy meets a young worker with probability  $\frac{s_t^Y}{s_t} q(\theta_t)$  and an adult worker with probability  $\frac{s_t^O}{s_t} q(\theta_t)$ . With probability  $(1 - q(\theta_t))$ , the vacancy cannot meet any of the unemployed workers and will remain vacant. In equilibrium, in order to close the model, it is assumed that  $V_t = 0$  for all  $t$ , which is called the “free entry assumption.”

Using a similar logic,  $J_t^Y(\cdot)$  and  $J_t^O(\cdot)$  satisfy the following Bellman equations:

$$J_t^Y(\alpha_j) = p_t \alpha_j - w_t^Y(\alpha_j) + \beta \lambda (V_{t+1} - \Psi) + \beta(1 - \lambda) J_{t+1}^O(\alpha_j) \quad (5)$$

$$J_t^O(\alpha_j) = p_t \alpha_j - w_t^O(\alpha_j) + \beta V_{t+1} \quad (6)$$

where  $w_t^i(\alpha_j)$  stands for the wage and  $\Psi$  denotes the separation cost. Separation from a worker who worked for more than one period entails a separation cost.

Let  $U_t^I$  be the present-discounted expected value of the expected income stream of newborn workers, and  $U_t^i$  and  $W_t^i$ , the present-discounted expected

value of the expected income stream of an unemployed and an employed generation- $i$  worker, respectively. Then, each of these values satisfies the following Bellman equation.

$$U_t^I = \theta_t q(\theta_t) \int_0^{\bar{\alpha}} \max\{W_t^Y(\alpha), U_t^Y\} dG(\alpha) + (1 - \theta_t q(\theta_t)) U_t^Y \quad (7)$$

$$U_t^Y = \beta \theta_{t+1} q(\theta_{t+1}) \int_0^{\bar{\alpha}} \max\{W_{t+1}^O(\alpha), U_{t+1}^O\} dG(\alpha) + \beta(1 - \theta_{t+1} q(\theta_{t+1})) U_{t+1}^O \quad (8)$$

$$U_t^O = 0 \quad (9)$$

$$W_t^Y(\alpha_j) = w_t^Y(\alpha_j) + \beta \lambda U_{t+1}^O + \beta(1 - \lambda) W_{t+1}^O(\alpha_j) \quad (10)$$

$$W_t^O(\alpha_j) = w_t^O(\alpha_j) \quad (11)$$

The wage function  $w_t^i(\cdot)$  is determined to be the solution of an asymmetric Nash bargaining problem. Let  $\delta$  be the bargaining power of the worker; then, the Nash problem is defined as

$$\max_{w^i(\alpha_j)} \{W_t^i(w_t^i(\alpha_j)) - U_t^i\}^\delta \{J_t^i(w_t^i(\alpha_j)) - V_t\}^{1-\delta}, \quad (12)$$

for all time  $t$  and generation of worker  $i$ . The first order condition is

$$(1 - \delta)(W_t^i(\alpha_j) - U_t^i) = \delta(J_t(\alpha_j) - V_t).$$

Let  $S_t^i(\alpha_j)$  be the surplus of the matching of the specific productivity  $\alpha_j$  with generation- $i$  worker:

$$S_t^i(\alpha_j) = J_t^i(\alpha_j) - V_t + W_t^i(\alpha_j) - U_t^i.$$

Thus, we obtain

$$W_t^i(\alpha_j) - U_t^i = (1 - \delta) S_t^i(\alpha_j), \quad (13)$$

$$J_t(\alpha_j) - V_t = \delta S_t^i(\alpha_j). \quad (14)$$

Equations (13) and (14) imply that the wage is determined such that the surplus is shared optimally based on the bargaining power  $\delta$ .

Firms and all generations of workers decide which job-match to begin. The decision rule for each agent exists uniquely and has a reservation property, because  $J_t^i(\cdot)$  and  $W_t^i(\cdot)$  are monotonically increasing in  $\alpha$  and because  $V_t$  and  $U_t^i$  are constant. The reservation productivity is defined as

$$J_t^i(\alpha_{r,f,t}^i) = \beta V_{t+1}, \quad (15)$$

for firms that have contacted a generation- $i$  worker, and

$$W_t^i(\alpha_{r,w,t}^i) = U_t^i \quad (16)$$

for generation- $i$  workers. Since the wage is based on the Nash sharing rule, the firms and workers always agree on which job-matches to accept and which to reject. Therefore, we have  $\alpha_{r,t}^i \equiv \alpha_{r,f,t}^i = \alpha_{r,w,t}^i$  for all  $t$  and  $i$ .

Using the reservation productivities, the probability of matching for a generation- $i$  worker can be written as<sup>10</sup>

$$q_{w,t}^i \equiv [1 - G(\alpha_{r,t}^i)] \frac{m(s_t, v_t) \frac{s_t^i}{s_t}}{s_t^i} = [1 - G(\alpha_{r,t}^i)] \theta_t q(\theta_t). \quad (17)$$

In a similar manner, the probability of matching each generation of workers to a firm is

$$q_{f,t}^i \equiv \frac{s_t^i}{s_t} [1 - G(\alpha_{r,t}^i)] \frac{m(s_t, v_t)}{v_t} = \frac{s_t^i}{s_t} [1 - G(\alpha_{r,t}^i)] q(\theta_t). \quad (18)$$

We can now present the flows among the states of the workers. In each generation,  $Y$  and  $O$ , there are two states for each worker-unemployment and employment. Let  $u_t^i$  be the generation- $i$  unemployment population in period  $t$ . Since the population of each generation  $i$  is normalized to one,  $u_t^i$  also represents the generation- $i$  unemployment rate. The flow conditions can be written as follows:

$$u_t^Y = s_t^Y - q_{w,t}^Y s_t^Y, \quad (19)$$

$$u_t^O = s_t^O - q_{w,t}^O s_t^O, \quad (20)$$

$$\text{where } s_t^Y = 1, \quad (21)$$

$$s_t^O = u_{t-1}^Y + \lambda(1 - u_{t-1}^Y) = \lambda + (1 - \lambda)u_{t-1}^Y, \quad (22)$$

for all  $t$ . Note that  $s_t^i$  denotes the generation- $i$  searching population.

The competitive equilibrium (CE) in this model can be defined as follows.

Definition of CE:

Given a separation rate  $\lambda \in (0, 1)$ , a separation cost  $\Psi \in \mathcal{R}_{++}$ , a bargaining power of workers  $\delta \in (0, 1)$ , a meeting technology  $m(s_t, v_t)$ , a distribution

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<sup>10</sup>This model also has the standard relationship:  $\theta(q_{f,t}^Y + q_{f,t}^O) = q_{w,t}$ , for all  $t$ .

of job-worker specific productivity  $G(\alpha)$ , and an initial condition  $s_0^O$ , a CE in this economy is the unemployment rate  $u_t^i$ , the labor market tightness  $\theta_t \equiv \frac{v_t}{u_t}$ , the searching population  $s_t^i$ , the number of the posted vacancies  $v_t$ , the wage functions  $w_t^i(\alpha)$ , and the policy function  $\alpha_{r,t}^i$  of each generation  $i \in \{Y, O\}$  and period  $t$ , which satisfy

- (Optimization)
  - $\alpha_{r,t}^Y$  and  $\alpha_{r,t}^O$  solve the agents' problem (4) - (11) in each period  $t$ .
  - $w_t^i(\alpha)$  solves the generation- $i$  Nash bargaining problem (12) for all  $i$  and  $t$ .
  - The free entry condition holds, i.e.,  $V_t = 0$  for all  $t$ .
- (Resource Constraint)
  - $u_t^i$  and  $s_t^i$  satisfy the flow conditions (19) - (22).

Since the policy and wage functions have their closed-form expressions, we can consolidate equilibrium conditions. The closed-form solutions also aid our understanding of mechanisms and intuitions. In the following section, I will explain closed-form solutions and how to pin down the equilibrium.

Assuming the free entry assumption,  $V_t = 0$  for all  $t$ , we can obtain the closed-form expressions for the wage functions

$$w^Y(\alpha_j) = \delta p_t \alpha_j + \beta \delta (1 - \delta) \theta_{t+1} q(\theta_{t+1}) p_{t+1} E[\alpha] - \beta \delta \lambda \Psi \quad (23)$$

$$w^O(\alpha_j) = \delta p_t \alpha_j, \quad (24)$$

where  $E[\cdot]$  denotes the mathematical expectation.

The closed-form expressions for reservation productivities are

$$\alpha_{r,t}^Y = \frac{\beta \lambda \Psi}{p_t + \beta(1 - \lambda)p_{t+1}} + \frac{\beta \delta \theta_{t+1} q(\theta_{t+1}) p_{t+1}}{p_t + \beta(1 - \lambda)p_{t+1}} E[\alpha] \quad (25)$$

$$\alpha_{r,t}^O = 0. \quad (26)$$

The reservation productivity of the job in the case of youth (25) consists of two terms—the expected separation cost for the firm and the opportunity cost for the young worker to accept the offer. In other words, the reservation productivity reflects the cost for both the firm and the worker to begin working. This is because the surplus of job-matching is shared based on the Nash

bargaining problem, which is a cooperative game. Note that the opportunity cost does not include the current one owing to the assumption that there is no utility in being unemployed. Therefore, the reservation productivity for the job in the case of the adult—who do not have a long future ahead—is equal to zero.

Since the free entry assumption is always satisfied in equilibrium, (4) and (5) provide the “job creation” (JC) condition. Using the closed-form expressions of the wage functions and the reservation productivities, the JC condition in this model can be written as

$$\frac{c + s_t^O \{c - q(\theta_t)(1 - \delta)E[\alpha]\}}{(1 - \delta)q(\theta_t)} = \frac{p_t + \beta(1 - \lambda)p_{t+1}}{p_t} \int_{\alpha_{r,t}^Y}^{\bar{\alpha}} \{\alpha - \alpha_{r,t}^Y\} dG(\alpha). \quad (27)$$

The left-hand side of (27) corresponds to the expected cost of posting a vacancy, and its right-hand side represents the expected benefit. Therefore, from this JC condition, we can observe that the number of job vacancies posted in period  $t$  is determined such that the expected cost of posting a vacancy is equal to its expected return.

Consequently, the equilibrium conditions can be summarized in the following two equations:

$$\alpha_{r,t}^Y = \frac{\beta\lambda\Psi}{p_t + \beta(1 - \lambda)p_{t+1}} + \frac{\beta\delta\theta_{t+1}q(\theta_{t+1})p_{t+1}}{p_t + \beta(1 - \lambda)p_{t+1}} E[\alpha] \quad (28)$$

$$\frac{c + s_t^O \{c - q(\theta_t)(1 - \delta)E[\alpha]\}}{(1 - \delta)q(\theta_t)} = \frac{p_t + \beta(1 - \lambda)p_{t+1}}{p_t} \int_{\alpha_{r,t}^Y}^{\bar{\alpha}} \{\alpha - \alpha_{r,t}^Y\} dG(\alpha) \quad (29)$$

where

$$s_t^O = \lambda + (1 - \lambda)\{1 - \theta_{t-1}q(\theta_{t-1})(1 - G(\alpha_{r,t-1}^Y))\} \quad (30)$$

These two equations jointly pin down the equilibrium market tightness  $\theta_t$  and the reservation productivity of youth  $\alpha_{r,t}^Y$ . The other equilibrium variables can be expressed as functions of  $\theta_t$  and  $\alpha_{r,t}^Y$ .

## 4 Analysis

In this section, I argue that the existence of separation costs causes a variety of age-heterogeneities concerning employment. Propositions 1 and 2 summarize the main findings.

The level fact  $u_t^Y > u_t^O$  appears in this model equilibrium. Substituting (17), (21), and (22) into (19) and (20), we obtain

$$u_t^Y = 1 - [1 - G(\alpha_{r,t}^Y)]\theta_t q(\theta_t) \quad (31)$$

$$u_t^O = (1 - \theta_t q(\theta_t))[\lambda + (1 - \lambda)u_{t-1}^Y] = (1 - \theta_t q(\theta_t))s_t^O < 1. \quad (32)$$

Note that  $[1 - G(\alpha_{r,t}^Y)] \in (0, 1)$ , because the youth reservation productivity  $\alpha_{r,t}^Y$  is strictly positive as clearly indicated by its closed-form expression (26). Then,

$$u_t^Y > 1 - \theta_t q(\theta_t) > u_t^O. \quad (33)$$

In the last inequality, I took into account the fact that the adult job-searching population  $s_t^O$  is strictly less than one, the total adult population. Therefore, the youth employment rate is lower than the adult population's employment rate.

One of the main factors that induce the level fact is that youth have a larger incentive to reject the offer because they have other vocational choices. Although the adult consider the current period only, youth have to consider the current and the subsequent period. Since the job-match specific productivity  $\alpha$  remains constant until job destruction, the youth do not want to accept an offer with  $\alpha$ , considering their option value of drawing a new  $\alpha$  in the subsequent period. This aspect of the model is consistent with the existing literature, such as Jovanovic (1979) and Neal (1999). I will now investigate the generational and international gaps in the volatility of employment.

The dynamics of  $(\alpha_{r,t}^Y, \theta_t)$  along with the change in the general productivity  $p_t$  reveals the characteristics of the volatility. In order to enable the analytical investigation, I focus on the transitional dynamics from one steady state to the other.<sup>11</sup> For example, I will discuss the characteristics of the transitional dynamics, when a permanent increase in the general productivity occurs. However, such transitional dynamics of  $(\alpha_{r,t}^Y, \theta_t)$  is trivial, because both  $\alpha_r^Y$  and  $\theta$  are jumping variables. Therefore, a comparative steady state analysis will suffice for the discussion.

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<sup>11</sup>The simple setting such as a two-period overlapping generation structure also enables analytical examination. If N-period overlapping generation structure were to be used, one would still have the same mechanism but a numerical method to solve the model may be desired.

Assuming that general productivity  $p_t$  takes a constant value for all  $t$ , we have a steady state equilibrium. Using an approach similar to the one I adopted in the previous section, the steady-state equilibrium conditions can be also consolidated in two equations:

$$\alpha_r^Y = \frac{\beta\lambda\Psi}{\{1 + \beta(1 - \lambda)\}p} + \frac{\beta\delta\theta q(\theta)}{1 + \beta(1 - \lambda)} E[\alpha] \quad (34)$$

$$\frac{(1 + s^O)c - s^O q(\theta)(1 - \delta)E[\alpha]}{q(\theta)(1 - \delta)} = \{1 + \beta(1 - \lambda)\} \int_{\alpha_r^Y}^{\bar{\alpha}} \{\alpha - \alpha_r^Y\} dG(\alpha) \quad (35)$$

where

$$s^O = \lambda + (1 - \lambda)\{1 - \theta q(\theta)(1 - G(\alpha_r^Y))\} \quad (36)$$

This system of equations pins down the equilibrium market tightness  $\theta$  and youth reservation productivity  $\alpha_r^Y$ . Other equilibrium variables can be calculated from them.

These two equations are illustrated graphically in Figure 4. The equation for the youth reservation productivity, (34), is linear with intercept  $\frac{\beta\lambda\Psi}{\{1 + \beta(1 - \lambda)\}p}$  and is denoted by RP. The JC curve, (35), is a downward-sloping curve under reasonable conditions and is labeled JC. In the Appendix, I present a proof showing that the JC curve has a negative slope in  $\theta$ - $\alpha_r^Y$  plane, under mild conditions. Equilibrium  $(\alpha_r^Y, \theta)$  is given by the intersection of the two curves and it is unique.

In equilibrium, a positive separation cost creates the employment volatility gap between generations. According to the data facts in the second section, the youth employment rate fluctuates over business cycles more than the adult employment rate does. In this model, the change in general productivity  $p$  represents business cycles. The following lemma summarizes how the change in  $p$  influences equilibrium  $(\alpha_r^Y, \theta)$ .

**Lemma 1:** If there exist a positive separation cost,  $\Psi > 0$ , higher general productivity  $p$  decreases the youth reservation productivity  $\alpha_r^Y$  and increases the market tightness  $\theta$ .

Figure 4 demonstrates how Lemma 1 holds. A higher general productivity, shown by higher  $p$ , shifts the RP line downward. On the other hand, as is evident in (35), the JC curve is not influenced by the increase in  $p$ . Therefore,

the higher general productivity decreases the youth reservation productivity  $\alpha_r^Y$  but increases the market tightness.

The intuition behind Lemma 1 is straightforward. Higher general productivity  $p$  lowers the expected value of the separation cost, the first term of the right-hand side of (34). This means that the opportunity cost of job-matching for the firm decreases. Consequently, the pairs of youth can begin working at lower  $\alpha$ , i.e., the youth reservation productivity  $\alpha_r^Y$  decreases. Since lower youth reservation productivity  $\alpha_r^Y$  increases the right-hand side of (35), i.e., the expected return from posting a vacancy, job creation increases. As a result, market tightness  $\theta$  also increases, despite the negative influence that the increase in  $\alpha_r^Y$  has on it.

Age-heterogeneity in the job-match-rate volatility follows Lemma 1.

Proposition 1: If there exists a positive separation cost,  $\Psi > 0$ , general productivity  $p$  causes a greater decrease in the youth job-match probability  $q_w^Y$  than in the job-match probability of the adult  $q_w^O$ .

**Proof:** From Lemma 1, higher general productivity  $p$  decreases the youth reservation productivity  $\alpha_r^Y$ . On the other hand, since the adult reservation productivity  $\alpha_r^O = 0$ , its response to  $p$  is also zero. From Lemma 1, higher general productivity  $p$  also leads to higher market tightness  $\theta$ . Thus, it follows that the probability of contact with an unfilled job  $\theta q(\theta)$  increases for all workers uniformly. However, the age-specific job-matching probability  $q_w^i$  reacts differently to the increase in the general productivity, because the age-specific job-acceptance rate  $(1 - G(\alpha_r^i))$  does so too. As is evident in (26), the reservation productivity of the adult  $\alpha_r^O$  is independent of  $p$ , whereas the youth reservation productivity  $\alpha_r^Y$  is decreasing with  $p$ . Hence, the adult job-acceptance rate does not change in response to an increase in  $p$ . Hence, the adult job-acceptance rate does not change in response to an increase in  $p$  more than the adult probability does. (Q.E.D.)

Proposition 1 states that the separation cost  $\Psi$  is the main source of age-heterogeneity in employment volatility. The key factor is that firms face much difficulty in separation from long-serving workers. In fact, there is no gap in the employment volatility between generations if  $\Psi = 0$ . If the separation cost assumes a multiplicative form with respect to  $p$ , the age-gap in the employment volatility also disappears. Costs that prevent the long-

tenured workers' separation possibly include multiplicative parts and also include independent parts in  $p$ . The number and effectiveness of employment protection legislations are examples of independent costs in  $p$ . Thus, it is reasonable to model the relative difficulty of long-serving workers' separation as a non-multiplicative form with respect to  $p$ . If a separation cost that is in a non-multiplicative form with respect to  $p$  exists, youth employment fluctuates much more than the adult rate does over business cycles.

Next, I examine a mechanism inducing the gap in the magnitude of the youth employment volatility between countries. Existing theories do not provide a convincing explanation for international gaps. They consider that youth have a larger incentive to reject job offers as they have more vocational choices. This certainly produces the level-gap but does not necessarily induce the volatility-gap. Additionally, considering the relative difficulty of separation by age-group, we can understand why the magnitude of the youth employment volatility differs across countries.

The following lemma summarizes the influence of separation cost on equilibrium  $(\alpha_r^Y, \theta)$ . According to the data presented in the second section, the youth employment rate fluctuates over business cycles more than the adult rate does. In this model, the change in general productivity  $p$  represents business cycles. The following lemma summarizes how the change in  $p$  influences equilibrium  $(\alpha_r^Y, \theta)$ .

Lemma 2: Higher  $\Psi$  increases the youth reservation productivity  $\alpha_r^O$  and decreases the market tightness  $\theta$ .

Figure 5 illustrates how the difference in  $\Psi$  affects equilibrium  $(\alpha_r^Y, \theta)$ . A higher separation cost, represented by higher  $\Psi$ , shifts the RP line in Figure 5 upward. On the other hand, as is evident in (35), the JC curve is not influenced by the increase in  $\Psi$ . Therefore, the higher separation cost increases the youth reservation productivity  $\alpha_r^Y$  but decreases the market tightness  $\theta$ .

As in the case of Lemma 1, the intuition behind the result is simple. Higher separation cost  $\Psi$  increases the cost of possible separation, denoted by the first term on the right-hand side of (34). This implies that a job-mach with a young worker offers a higher opportunity cost to the firm. Consequently, the pairs of youth become more reluctant to work at lower  $\alpha$ , i.e., the youth reservation productivity  $\alpha_r^Y$  increases. The higher youth reservation productivity  $\alpha_r^Y$  decreases in the right-hand side of (35), i.e., the expected

return from posting a vacancy, and leads to less job creation. As a result, market tightness  $\theta$  decreases, despite the increase in  $\alpha_r^Y$  having the opposite influence on it.

In this model, the separation cost  $\Psi$  also has an implication on the level of the unemployment rate. From Lemma 2, the youth reservation productivity  $\alpha_r^Y$  is increasing in  $\Psi$ . It follows that higher  $\Psi$  decreases the youth job-acceptance rate  $(1 - G(\alpha_r^Y))$ . Higher  $\Psi$  also decreases the probability of contact with an unfilled job  $\theta q(\theta)$ . Therefore, higher separation cost  $\Psi$  decreases the job-matching probability  $q_w^i$  for all generations ( $i \in \{Y, O\}$ ). As a result, as is evident in the steady-state version of (31) and (32), higher  $\Psi$  increases  $u^Y$  and  $u^O$ , and thus decreases the equilibrium employment level.

The gap in the country-specific separation cost also creates the gap in the youth job-acceptance-rate volatility across countries.

Proposition 2: The youth job-acceptance rate  $(1 - G(\alpha_t^Y))$  is more volatile to the change of  $p_t$  in an economy with higher  $\Psi$  than in an economy with lower  $\Psi$ .

**Proof:** Partially differentiating (34) with respect to  $p$ , we have

$$\frac{\partial \alpha_r^Y}{\partial p} = \frac{-\beta \lambda \Psi}{p^2 \{1 + \beta(1 - \lambda)\}^2} < 0. \quad (37)$$

From this derivative, the RP line shifts downward when  $p$  increases. In addition, the magnitude of the downward shift is increasing to  $\Psi$ .

If the JC curve is a monotonically decreasing and non-convex function as in Figures 4 and 5, the increase in the equilibrium  $\alpha_r^Y$  due to the increase in  $p$  is increasing in  $\Psi$ . In other words, the equilibrium youth job-acceptance rate  $(1 - G(\alpha_r^Y))$  fluctuates more widely along with the change in  $p$  in an economy with higher  $\Psi$ . (Q.E.D.)

The intuition of proposition 2 is as follows. Higher separation cost  $\Psi$  amplifies the change in the expected cost due to the change in general productivity  $p$ . Since  $\Psi$  is a fixed cost, the expected separation cost is low if  $p$  is high and vice versa. The job-match decision reflects the change in the expected separation cost due to the change in general productivity. In this sense, the gap in country-specific separation cost is important for understanding gap in the youth employment volatility across countries.

Age-heterogeneity in the employment volatility emerges if there exists a positive separation cost  $\Psi$ . In addition, different country-specific separation costs produce the gap in the magnitude of the youth employment volatility between countries. Thus, the separation cost is a key factor determining labor market performance.

## 5 Concluding Remarks

OECD countries commonly experience larger cyclical fluctuations in the youth employment rate. However, the magnitude of the fluctuations differs significantly across countries. In this paper, I argued that these two facts can be explained by the separation costs that increase with the workers' tenure. In the analysis, I formulated a search and matching model with an OLG structure, assuming higher separation costs for tenured workers.

The main contribution of this paper is that it provides a model that explains the age-heterogeneous volatility of employment by separation costs. Separation costs of highly attached workers render the youth employment more volatile than that of the adult. In addition, this model illustrates that larger separation cost increases the magnitude of youth employment volatility. Using the mechanisms described in this paper, we can also understand the cross-country differences in the magnitude of the youth employment volatility as differences in country-specific separation costs.

Although this paper provides a mere analysis of the accession rate, this rate is closely associated with the fluctuations in the age-specific employment rates. Shimer (1998) showed empirically that the probability of finding a job is procyclical and the separation probability is nearly non-cyclical with respect to the unemployment rate. Hall (2000) also argued that the main source of the employment fluctuation is the accession rate. Even though further research is required to directly examine whether the separation cost structure determines the characteristics of age-specific employment fluctuations, the study on accession rate in this paper is sufficient for addressing the purpose of discussing employment fluctuations.

This paper has policy implications and could lead to several researches on new topics. Although this paper only provides a theoretical analysis, it would be useful to measure the separation costs implied by this model. Many OECD countries are lately facing youth employment issues and are seeking effective policies to combat them. The specific values of the separation costs would be

helpful for policy decision making because they have a structural relationship with age-heterogeneous employment fluctuations. Another research topic is the quantitative analysis of business cycles, considering age-heterogeneous employment fluctuations. As Gomme et al. (2004) suggested, capturing the patterns of cyclical fluctuations at the disaggregated level would significantly improve the explanatory power of RBC models.

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## Appendix

In this appendix, I analytically show that the job creation condition at the steady state (35) becomes a downward-sloping curve in the  $\theta$ - $\alpha_r^Y$  plane under reasonable assumptions. Under the assumptions, we can demonstrate that the JC curve at the steady state has a negative slope everywhere in the  $\theta$ - $\alpha_r^Y$  plane. If not, we have to focus on the local analysis near the steady state.

The assumptions are as follows. First, I normalize the flow cost for a vacancy,  $c$ , to be equal to the expected value of the pair-specific productivity  $E[\alpha]$ .

Assumption 1:  $c = E[\alpha]$ .

This assumption implies that the cost is equal to the expected return. Second, I specify the functional form of the matching function  $m(s_t, v_t)$ .

Assumption 2: The meeting technology  $m(s_t, v_t)$  is specified as follows.

$$m(s_t, v_t) = As_t^\gamma v_t^{1-\gamma}, \quad (38)$$

where  $A$  is small enough to ensure that condition (2) holds with strict inequality.

Third, I impose a restriction on the range of its exponent  $\gamma$ .

Assumption 3:  $\gamma \geq 1/2$ .

Note that empirical research reports that under the restriction of constant returns to scale, the exponent  $\gamma$  is estimated to be between 0.70 and 0.81 (Petrongolo and Pissarides (2001)), which is certain to satisfy this restriction.

Before discussing the algebraic details, I define  $n^Y$ , which represents the number of young workers who continue their jobs.

$$n^Y \equiv \theta q(\theta) \{1 - G(\alpha_r^Y)\} (1 - \lambda) = 1 - s^O < 1. \quad (39)$$

The second equality means that continued workers are those who do not search for jobs. I extensively use this notation in the following analysis.

Now, let us begin to analyze the slope of (35) in the  $\theta$ - $\alpha_r^Y$  plane. As a reminder, I restate here the same equation as (35).

$$\frac{c + s^O \{c - q(\theta)(1 - \delta)E[\alpha]\}}{(1 - \delta)q(\theta)} = \{1 + \beta(1 - \lambda)\} \int_{\alpha_r^Y}^{\bar{\alpha}} \{\alpha - \alpha_r^Y\} dG(\alpha), \quad (40)$$

where

$$s^O = \lambda + (1 - \lambda)\{1 - \theta q(\theta)(1 - G(\alpha_r^Y))\}. \quad (41)$$

**Proof:** Replacing  $E[\alpha]$  by  $c$  in light of Assumption 1, (40) becomes

$$\frac{c + s^O c \{1 - q(\theta)(1 - \delta)\}}{(1 - \delta)q(\theta)} - \{1 + \beta(1 - \lambda)\} \int_{\alpha_r^Y}^{\bar{\alpha}} \{\alpha - \alpha_r^Y\} dG(\alpha) = 0. \quad (42)$$

Taking the total differentiation of (42), we obtain

$$D_1 d\alpha_r^Y + D_2 d\theta = 0 \quad \Leftrightarrow \quad \frac{d\alpha_r^Y}{d\theta} = -\frac{D_2}{D_1}, \quad (43)$$

where

$$D_1 = \left( \frac{\partial s^O}{\partial \alpha_r^Y} \right) \left( \frac{c \{1 - q(\theta)(1 - \delta)\}}{(1 - \delta)q(\theta)} \right) + \{1 + \beta(1 - \lambda)\}(1 - G(\alpha_r^Y)), \quad (44)$$

$$\begin{aligned} D_2 &= \frac{c}{1 - \delta} \frac{\partial}{\partial \theta} \left\{ \frac{1 + s^O \{1 - q(\theta)(1 - \delta)\}}{q(\theta)} \right\} \\ &= \left( \frac{c}{(1 - \delta)\{q(\theta)\}^2} \right) \left[ \left( \frac{\partial s^O}{\partial \theta} \right) \{1 - q(\theta)(1 - \delta)\}q(\theta) \right. \\ &\quad \left. - s^O q'(\theta)(1 - \delta)q(\theta) - [1 + s^O \{1 - q(\theta)(1 - \delta)\}]q'(\theta) \right]. \quad (45) \end{aligned}$$

$D_1$  is positive because  $\frac{\partial s^O}{\partial \alpha_r^Y} > 0$  holds. With regard to  $D_2$ , we only have to focus on the sign of the value between the square brackets in (45). Let  $D_3$  denote this value.

$$D_3 \equiv \left[ \left( \frac{\partial s^O}{\partial \theta} \right) \{1 - q(\theta)(1 - \delta)\}q(\theta) \right. \\ \left. - s^O q'(\theta)(1 - \delta)q(\theta) - [1 + s^O \{1 - q(\theta)(1 - \delta)\}]q'(\theta) \right]. \quad (46)$$

The goal of this proof is to demonstrate that  $D_3$  is positive, which ensures that the slope  $\frac{d\alpha_r^Y}{d\theta}$  in (43) is negative.

Using the notation  $n^Y$ , we can transform  $D_3$  as follows.

$$\begin{aligned}
D_3 &= \left[ - \left( \frac{\partial \theta q(\theta)}{\partial \theta} \right) (1 - G(\alpha_r^Y))(1 - \lambda) \{1 - q(\theta)(1 - \delta)\} q(\theta) \right. \\
&\quad \left. - (1 - n^Y) q'(\theta)(1 - \delta) q(\theta) - [1 + (1 - n^Y) \{1 - q(\theta)(1 - \delta)\}] q'(\theta) \right] \\
&= \left[ - (\theta q'(\theta) + q(\theta)) \frac{n^Y}{\theta} \{1 - q(\theta)(1 - \delta)\} \right. \\
&\quad \left. - (1 - n^Y) q'(\theta)(1 - \delta) q(\theta) - q'(\theta) - (1 - n^Y) \{1 - q(\theta)(1 - \delta)\} q'(\theta) \right].
\end{aligned}$$

Arranging this equation by expanding parentheses, we finally obtain

$$D_3 = \left[ - q(\theta) \frac{n^Y}{\theta} \{1 - q(\theta)(1 - \delta)\} + n^Y q'(\theta)(1 - \delta) q(\theta) - 2q'(\theta) \right]. \quad (47)$$

By Assumption 2, we can derive

$$q'(\theta) = \frac{\partial q(\theta)}{\partial \theta} = \frac{\partial}{\partial \theta} A\theta^{-\gamma} = -\gamma A\theta^{-\gamma} = -\gamma \frac{q(\theta)}{\theta}. \quad (48)$$

Substituting (48) into (47), we obtain

$$D_3 = \frac{q(\theta)}{\theta} \left[ -n^Y + (1 - \gamma)n^Y(1 - \delta)q(\theta) + 2\gamma \right]. \quad (49)$$

By Assumption 3,  $2\gamma \geq 1$  holds. Then, we obtain

$$D_3 \geq \frac{q(\theta)}{\theta} \left[ 1 - n^Y + (1 - \gamma)n^Y(1 - \delta)q(\theta) \right] > 0. \quad (50)$$

Hence,  $D_3 > 0$  holds. From (43) and (45), I have proved that the JC condition (40) is a downward-sloping curve in the  $\theta$ - $\alpha_r^Y$  plane. (Q.E.D.)

Table 1: Relative Volatility of Employment Rate by Age Group

	Male		Female	
	15-24	25-64	15-24	25-64
JPN	4.2	0.8	2.6	0.7
USA	2.3	0.9	1.5	0.6
EU15	35.4	7.2	54.6	12.7
OECD	9.1	2.3	9.3	2.3

(Unit: 10,000)

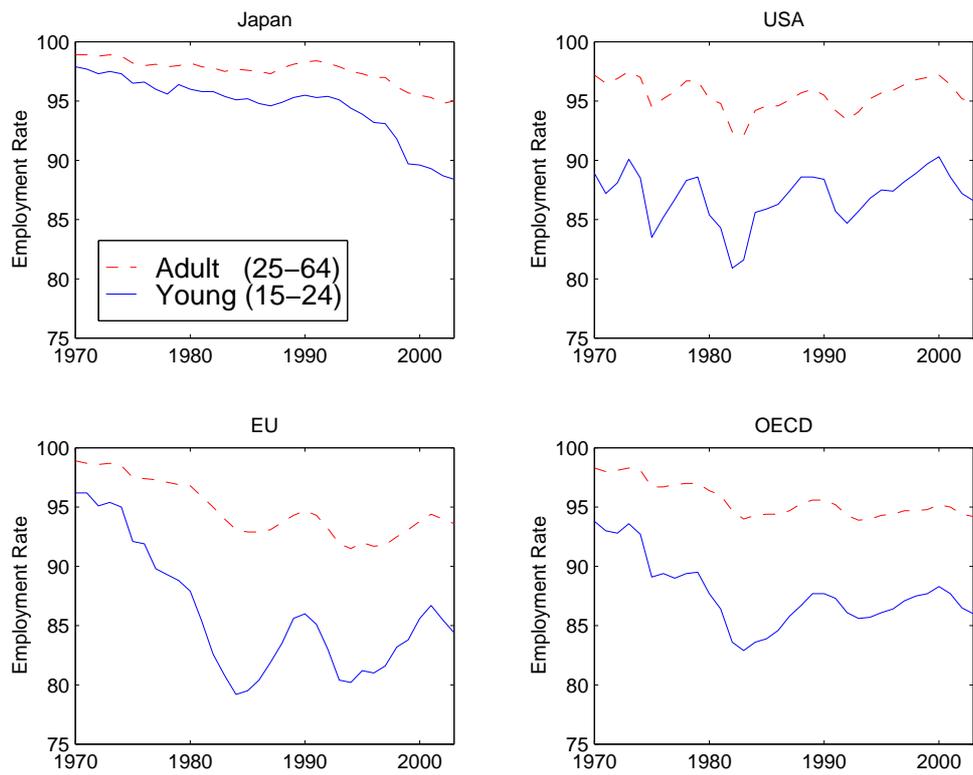


Figure 1: Time-Series of Employment Rate (Male)

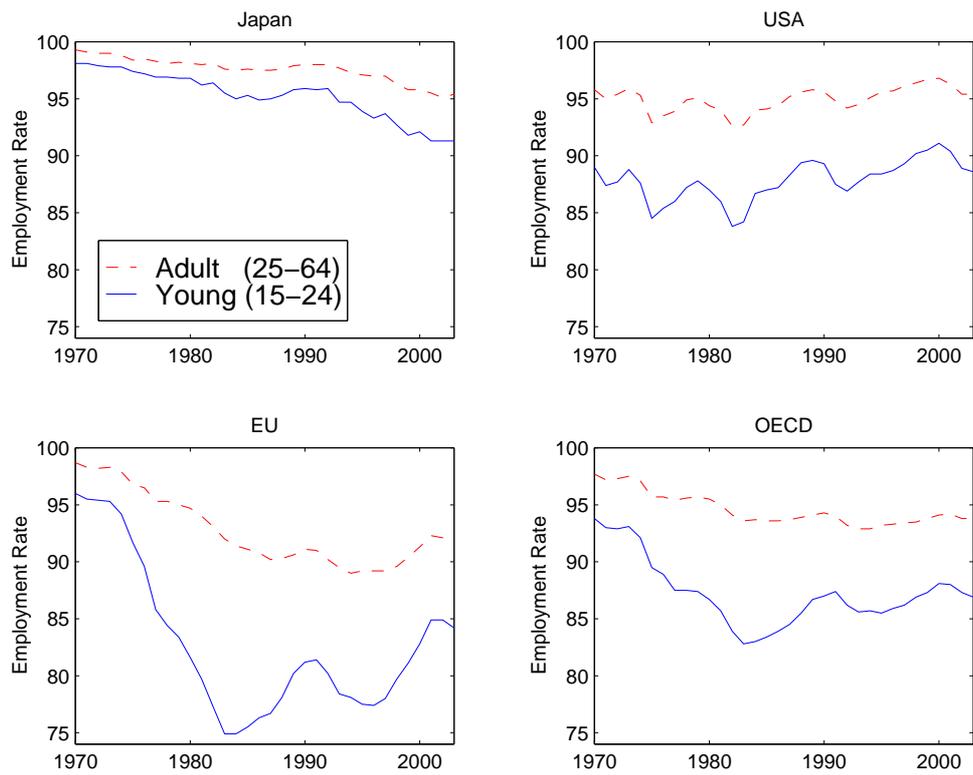


Figure 2: Time-Series of Employment Rate (Female)

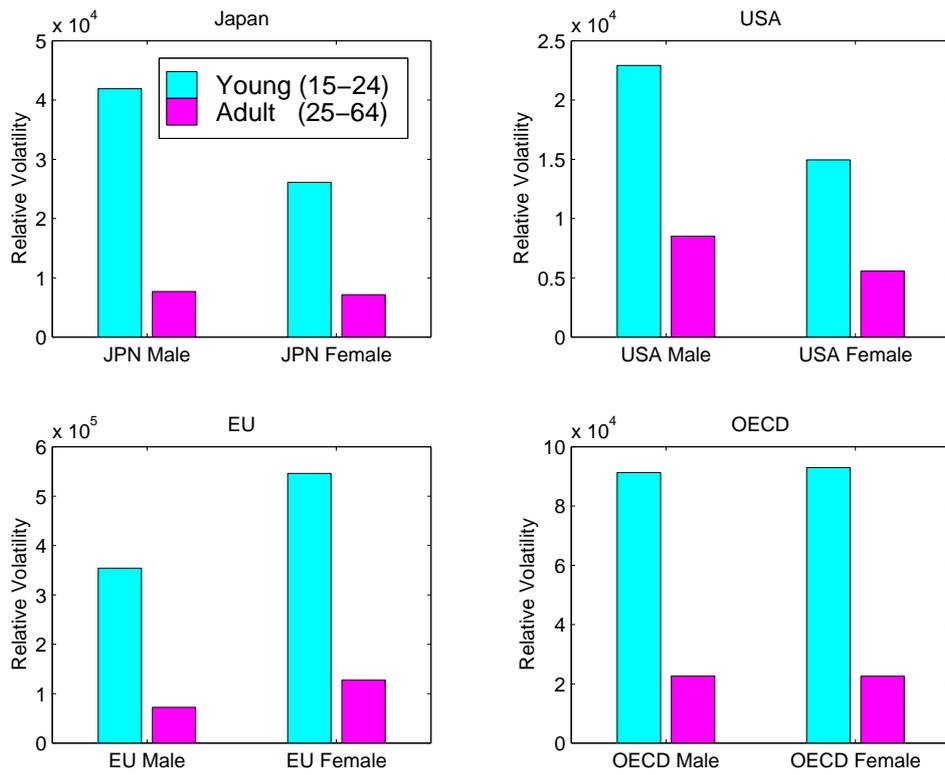


Figure 3: Relative Volatility of Employment Rate by Age Group

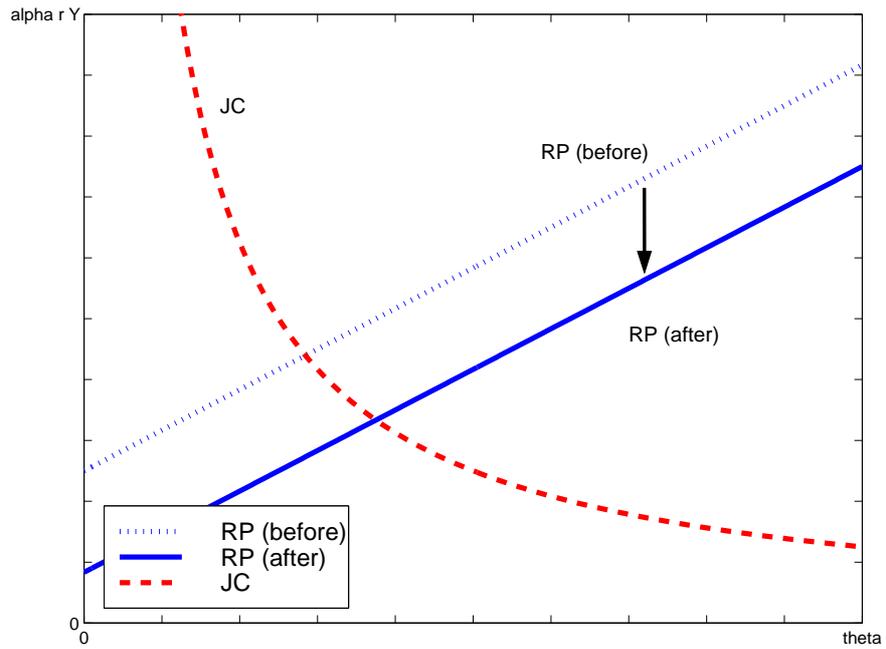


Figure 4: Influence of higher general productivity on equilibrium  
 Note: The vertical axis takes the youth reservation productivity  $\alpha_r^Y$ , and the horizontal axis takes market tightness  $\theta$ . This figure demonstrates the effect on the JC curve and the RP line corresponding with an increase in general productivity  $p$ .

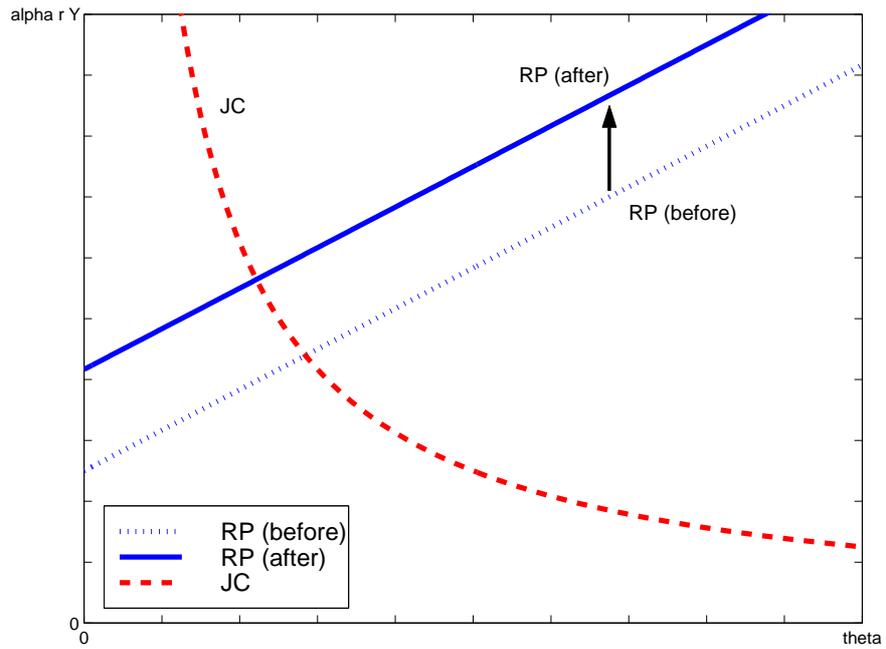


Figure 5: Influence of higher separation cost on equilibrium  
 Note: The vertical axis takes the youth reservation productivity  $\alpha_r^Y$ , and the horizontal axis takes market tightness  $\theta$ . This figure demonstrates the effect on the JC curve and the RP line corresponding with an increase in the separation cost  $\Psi$ .