Sector-Level Frictions and Aggregate Productivity

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Abstract

This paper examines the extent to which sector-level frictions affect aggregate productivity. I show that a broad range of sector-level frictions can be summarized as an implication of sector-level taxes on factor inputs. I introduce and analyze a simple multisector model with sector-specific taxes on factor inputs; based on this model, I define allocational efficiency indexes, which measure how the distribution of frictions affects aggregate productivity. I construct these indexes using sector-level data and find that the productivity gap caused by the sector-level frictions is large and explains a sizable amount of the differences in aggregate productivity across developed countries.

JEL Codes: E23, O11, O41, O47

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1 Introduction

There are large disparities in income across developed countries. Prescott (2002) reports that there is approximately a 30 to 40% difference in per capita income between the US and France, Japan, and the UK. One source of this disparity is the differences in aggregate productivity. Prescott argues that aggregate productivity is the most important factor in accounting for the income differences among developed countries. If we accept this position, the next question that subsequently arises is: "What accounts for the differences in aggregate productivity?"

One candidate is sector- or firm-specific frictions. Several theoretical models show that such frictions affect aggregate productivity. For example, Restuccia and Rogerson (2003) demonstrate that idiosyncratic firm-specific taxes and subsidies affect resource allocation and the level of aggregate productivity. Hayashi and Prescott (2006) analyze a model in which barriers to labor mobility across sectors affect aggregate productivity. Additionally, Kiyotaki and Moore (1997) analyze firm-specific borrowing constraints that act to constrain the access of high productivity firms to resources and instead allocate resources to unproductive firms, thereby affecting aggregate productivity.

The goal of my paper is to build an accounting framework that can be used to measure the importance of sector-level frictions for aggregate total factor productivity (TFP). For this purpose, I follow the approach of Chari et al. (2003). In their paper, they show that a wide range of models with various frictions is equivalent to a prototype aggregate one-sector growth model with time-varying aggregate TFP, labor taxes, and investment taxes. By employing their model from national account data, they measure frictions in the forms of aggregate TFP and taxes and assess which of these frictions accounts for aggregate business cycle fluctuations. While they focus on the effect of aggregate-level frictions on business cycle fluctuations, this paper focuses on the effect of heterogeneous frictions on the level of aggregate TFP. I show that several types of sector- or firm-level frictions in the above examples can be represented as sector- or firm-level taxes on factor inputs. I measure the friction effect by employing the model from sector-level data and assess the effect on aggregate TFP. An advantage of this approach is that it is possible to capture the effect of several types of frictions simultaneously.

In the following, I introduce the contents of this paper more precisely.

In this study, I develop a static multisector general equilibrium model with sector-specific taxes (or subsidies) on factor inputs. This multisector model with taxes is an extension of the one presented in Chari et al. (2003) and shares a structure similar to the computable general equilibrium model of Melo (1977). I demonstrate that in the model, it is not the overall level but the distribution of sector-specific taxes that affects resource allocation across sectors and aggregate TFP.

Using the model, I propose two allocational efficiency indexes that link the effect of the distribution of sector-specific frictions in the form of taxes on aggregate TFP. One is the translog allocational efficiency index (TAE), which measures the productivity gap caused by the distribution of sector-specific frictions. This index has the following notable relationships with the previous literature: The growth rate of TAE is equal to the total reallocation effect index in the growth accounting literature and TAE is closely connected to Theil's (1967) entropy measure, which measures the degree of income inequality. The other index is the relative TAE (RTAE), which measures the productivity gap between two countries caused by the distribution of frictions in the two countries.

I use this framework to measure the importance of sector-level frictions for the differences in the levels of aggregate TFP and cross-country aggregate TFP. I measure the allocational efficiency indexes, TAE and RTAE, using sector-level data of developed countries (West Germany (until 1991), Germany (from 1991), France, Italy, Japan, the UK, and the US, mainly from 1987 to 1993).[1] In general, the more subdivided is the definition of sector classification, the greater is the effect of frictions on aggregate TFP. Therefore, I adjust the data to maintain the consistency of sector classification between the countries. I find that the sector-level frictions reduce the level of aggregate TFP by 2% in the US and as much as 8% in Italy and Japan around 1990 and that the frictions account for 20 to 30% of the differences in aggregate TFP between the US and France, Japan, and Italy.[2] Further, I analyze the causes of the friction effect for each country in more depth.

This paper deals with a vast amount of research in the growth accounting literature, which

[1] I use data that is collected mainly from the STAN database and ISDB provided by OECD.

[2] Approximately 20% for France and 30% for Italy and Japan, whereas West Germany is an exception (the difference cannot be explained whatsoever).

Since the UK and the US data needed for the calculation do not overlap in any year, the results for the UK are not available for cross-country comparison.

measures the effect of factor reallocation on aggregate TFP growth (see Syrquin 1986, and Basu and Fernald 2002, among others). As compared with the literature, an advantage of this paper's approach is that we can measure not only the rate of change of the effect of frictions on resource allocation over time, which the literature actually measures, but also the size of the effect of the frictions, which the literature does not measure. This paper also provides microfoundations for the reallocation effects.

There is a growing literature focused on measuring frictions on resource allocation using the general equilibrium framework, and my work belongs to this literature. To the extent of my knowledge, the earliest work in the literature is that of Melo (1977). He uses a computable multisector general equilibrium model, which incorporates international trade, and applies it to the Columbian economy to calculate the effect when frictions on sector-level resource allocation were removed. There are three differences between his and my paper. First, my model is more analytically tractable. Second, while my paper explicitly analyzes how sector-level frictions affect resource allocation and aggregate productivity, his paper does not. Third, I measure the allocational efficiency of several countries over several years, while de Melo only measures it for one country for one year. Recently, researchers have begun measuring the effect of frictions on resource allocation and on aggregate productivity by using the general equilibrium framework. Restuccia et al. (2004) and Vollrath (2005) use a two-sector model to measure the barriers of resource allocation between the old agricultural and modern manufacturing sectors. While their papers focus on these two sectors and on developing countries, my paper focuses on a multisector environment and on developed countries. In addition, while their papers mainly focus on computation or measurement, this paper also analyzes how several types of sector- and firm-level frictions are theoretically described in a uniform manner and analyzes the properties of these frictions (e.g., the theoretical connections to other literature) more closely.

The remainder of the paper is organized as follows. Section 2 presents the examples of sector- or firm-level frictions and postulates how these frictions can be reformulated as taxes on factor inputs. Section 3 sets up and analyzes a static multisector general equilibrium model with sector-specific taxes on factor inputs. Section 4 introduces a translog bilateral aggregate TFP index (TTFP) to measure aggregate productivity, which is consistent with the model introduced in the previous section. Using the model framework and the TTFP, this section defines the allocational efficiency indexes. Based on these results, section 5 measures these indexes based on the data in order to note the effect of the sector-level frictions on aggregate productivity. Section 6 contains the concluding remarks.

2 Examples of Sector- or Firm-Level Frictions

In later sections, I assume that firms are price-takers and that the frictions that they face appear as taxes imposed on their factor inputs. While these two assumptions allow me to measure the effect of frictions, they might appear as the strong restrictions of this paper's analysis. Therefore, in this section, I provide the examples that illustrate how several types of sector- or firm-level frictions can be described by taxes on factor inputs under the price-taker assumption.

2.1 The prototype firm's problem

First, consider the prototype firm's problem. The firm produces a good by using capital and labor. The firm is a price-taker in both the good and the factor markets. The firm pays taxes on capital and labor. Thus, the firm's maximization problem can be written as

$$\max_{K_i L_i} \; p_i F_i(K_i, L_i) - (1+\tau_{Ki})K_i - (1+\tau_{Li})L_i, \tag{1}$$

where K_i, L_i denote the capital and labor used by the firm; p_i is the output price, and p_K and p_L are the factor prices; and τ_{Ki} and τ_{Li} are taxes on capital and labor inputs, respectively. Note that these notations are used in the following examples. The first-order conditions (FOCs) of the problem are

$$p_i \frac{\partial F_i(K_i, L_i)}{\partial K_i} = (1+\tau_{Ki})p_K, \tag{2}$$

$$p_i \frac{\partial F_i(K_i, L_i)}{\partial L_i} = (1+\tau_{Li})p_L. \tag{3}$$

In the following examples, I demonstrate that the firm's problem or the FOCs of the firm's problem under several types of frictions are isomorphic to those in the above equations.

2.2 Output tax

Consider a firm that pays a tax τ_i on output. The firm's maximization problem will therefore be

$$\max_{K_i L_i} (1-\tau_i)p_i F_i(K_i, L_i) - p_K K_i - p_L L_i. \tag{4}$$

This firm's problem can be rewritten as follows:

$$\max_{K_i L_i} p_i F_i(K_i, L_i) - \frac{p_K}{1-\tau_i} K_i - \frac{p_L}{1-\tau_i} L_i. \tag{5}$$

This problem is isomorphic to that in (1). Therefore, the output tax affects factor demands in the same manner as the taxes on factor inputs.

2.3 Imperfect competition

I demonstrate that frictions caused by imperfect competition such as monopoly, oligopoly, or monopolistic competition can also be expressed as taxes on factor inputs.

Let us consider the following firm's problem: the firm is a price-taker in the factor market but a price-setter in the output market. Accordingly, the firm's cost minimization problem is

$$\min_{K_i, L_i} p_K K_i + p_L L_i \tag{6}$$

$$\text{s.t.} V_i = F_i(K_i, L_i). \tag{7}$$

The FOCs of the problem are

$$p_i \frac{\partial F_i(K_i, L_i)}{\partial K_i} = \frac{p_i}{\gamma_i} p_K, \tag{8}$$

$$p_i \frac{\partial F_i(K_i, L_i)}{\partial L_i} = \frac{p_i}{\gamma_i} p_L, \tag{9}$$

where γ_i is the Lagrange multiplier and p_i is the price of the good that the firm produces. Since γ_i is equal to marginal cost, p_i/γ_i is the markup and is equal to unity when the firm is a price-taker in output market.

Suppose the prototype firm's problem in which $1 + \tau_{Ki}$ and $1 + \tau_{Li}$ are equal to the equilibrium values of p_i/γ_i. Thus, the FOCs of the prototype firm's problem given by (2) and (3) are the same as those in (8) and (9) in equilibrium. In this example, imperfect competition functions in the same way as taxes on the factor input in that they reduce factor demands and output. Therefore, frictions caused by imperfect competition can be written as taxes on factor inputs.[3]

2.4 Constraints

Kiyotaki and Moore (1997) show that a firm-specific borrowing constraint can affect resource allocation and aggregate productivity. Hayashi and Prescott (2006) argue that barriers to labor mobility from the agricultural sector to the manufacturing sector were one of the causes of stagnation in prewar Japan. I demonstrate that such constraints can also be expressed as the taxes on factor inputs.

I take the borrowing constraint case as an example. Constraints on labor mobility can be treated similarly. Suppose that a firm faces a borrowing constraint; the firm's problem can be written as follows:

$$
\begin{aligned}
\max_{K_i, L_i, B_i} \quad & \pi_i + \frac{1}{1+r} V_i(K_i, B_i) \\
\text{s.t.} \quad & \pi_i = p_i F_i(K_i, L_i) - p_L L_i - q_K (K_i - (1-\delta) K_{i,-1}) \\
& \qquad + \frac{B_i}{1+r} - B_{i,-1}, \\
& B_i \leq \theta q_{K,+1} K_i \\
\text{given} \quad & K_{i,-1}, \; B_{i,-1},
\end{aligned}
$$

where r is interest rate, B_i is the volume at which the firm borrows, θ is a collateral constraint parameter and is between zero and one, q_K is the value of capital, $V_i(K_i, B_i)$ is the next-period value function of owning K_i and B_i, and subscripts -1 and $+1$ indicate the previous and next

[3] A strict argument is as follows. Suppose the following two economies: in one economy, firms follow the prototype firm's problem given by (1), and in the other economy they follow the firm's problem of the section given by (7). Further suppose that the other environment is the same as the model in the next section. Therefore, if $1 + \tau_{Ki}$ and $1 + \tau_{Li}$ in the former firm's problem are equal to the equilibrium values of p_i/γ_i in the latter firm's problem, the equilibrium allocation is the same in both the economies.

periods. As the FOCs of the problem, we obtain:

$$
\begin{aligned}
p_i \frac{\partial F_i(K_i, L_i)}{\partial K_i} &= q_K - \frac{1}{1+r}\frac{\partial V_i(K_i, B_i)}{\partial K_i} \\
&\quad - \frac{1}{1+r}\theta q_{K,+1}\left(1 + \frac{\partial V_i(K_i, B_i)}{\partial B_i}\right), \qquad (10) \\
p_i \frac{\partial F_i(K_i, L_i)}{\partial L_i} &= p_L. \qquad (11)
\end{aligned}
$$

Suppose the prototype firm's problem in which $(1+\tau_{Ki})p_K$ is equal to the equilibrium values of the right-hand side of (10), and $1+\tau_{Li}$ is equal to unity. Accordingly, the FOCs of the prototype firm's problem given by (2) and (3) are the same as those in (10) and (11) in equilibrium.[4] In this example, the borrowing constraint functions in the same way as a tax on capital input in that it affects the factor demands and output. Therefore, the constraints can be described as the taxes on factor inputs.

3 A Model of the Production Economy

As shown in the previous section, several types of sector-level frictions can be summarized as the implications of taxes on factor inputs. Based on this result, in this section, I develop a static multisector competitive equilibrium model with sector-specific taxes (or subsidies) on factor inputs.

3.1 I Industrial sectors

There are I industrial sectors in the economy. Each sector produces a different good by using two factor resources: capital and labor. I assume that a sector corresponds to a firm. Thus, in the following, I identify a sector with a firm. Each sector's (i.e., firm's) problem is basically equal to the prototype firm's problem defined in the previous section. I assume that firms are price-takers in both the good and factor markets and that they pay taxes on capital and labor inputs. I denote τ_{Ki} and τ_{Li} as the capital and labor taxes, respectively, faced by sector i. Thus, the factor costs that the sector incurs become $(1+\tau_{Ki})p_K$ and $(1+\tau_{Li})p_L$, where p_K and p_L are the common factor prices across sectors. Because they produce different goods, the goods price p_i can vary across sectors in equilibrium (even if there are no taxes). On the other hand, because capital and

[4]Strict argument follows as in footnote 3.

labor are homogeneous across sectors, if $\tau_{Ki} = 0$ and $\tau_{Li} = 0$, the factor costs incurred by firms become the same, if not, firms face different factor costs.

The firms have Cobb-Douglas production technology exhibiting constant returns to scale. Therefore, firm i's production function can be written as follows:

$$V_i = F_i(K_i, L_i) \equiv A_i K_i^{\alpha_i} L_i^{1-\alpha_i},$$

where V_i is output and A_i is productivity of the firm. The capital intensity α_i can vary by sector.

The FOCs under the Cobb-Douglas assumption are as follows:

$$\frac{\alpha_i p_i V_i}{K_i} = (1 + \tau_{Ki}) p_K, \tag{12}$$

$$\frac{(1-\alpha_i) p_i V_i}{L_i} = (1 + \tau_{Li}) p_L. \tag{13}$$

Note that from the FOCs, we also attain the unit cost function:

$$p_i = \frac{1}{\alpha_i^{\alpha_i}(1-\alpha_i)^{1-\alpha_i}} \frac{\{(1+\tau_{Ki})p_K\}^{\alpha_i}\{(1+\tau_{Li})p_L\}^{1-\alpha_i}}{A_i}. \tag{14}$$

3.2 Aggregate production sector

I assume that an aggregate production sector produces an aggregate good V from the I goods produced by the industrial sectors. Additionally, I assume that the firm in the aggregate good sector is also a price-taker. The aggregate production function is a Cobb-Douglas one:

$$V = V(V_1, \ldots, V_I) \equiv \prod_{i=1}^{I} V_i^{\sigma_i}, \tag{15}$$

where $\sum \sigma_i = 1$.

The firm's problem in the aggregate good sector is as follows

$$\max_{V_i} \; p \prod_i V_i^{\sigma_i} - \sum_i p_i V_i,$$

where p is the price of the aggregate good. This form of the aggregate production function implies that the expenditure shares are constant:

$$\frac{p_i V_i}{pV} = \frac{p_i V_i}{\sum_j p_j V_j} = \sigma_i. \tag{16}$$

In addition, the FOCs imply that p is proportional to the geometric average of p_i.

The Cobb-Douglas assumption for the aggregate production function is used to derive the property that expenditure share is constant. The reason for the assumption is that it is difficult to know the manner in which expenditure share responds to changes in taxes. Provided that we regard σ_i as expenditure share and not a constant parameter, results in this section, except for these in section 3.5 are valid with more general assumptions (section 3.5 analyzes the effect of the taxes to the aggregation function V). Basic methods explained in subsequent sections would also be useful with more general assumptions, although some modifications need to take into account the changes in expenditure shares brought about by the changes in taxes, and the aggregation function in the more general assumptions need to satisfy required conditions for the TTFP index introduced later. On the other hand, the generalization of the production function in the industrial sectors makes it difficult to solve the model.

3.3 Resource constraints

Finally, I assume that aggregate capital and labor supply are exogenous. Thus, the following resource constraints apply.

$$\sum_i K_i = K, \tag{17}$$

$$\sum_i L_i = L, \tag{18}$$

where K and L are aggregate capital and labor supply.

3.4 Equilibrium

A competitive equilibrium of this economy is defined in the following way.

Definition. A static industrial equilibrium given productivities and taxes of I goods sectors $\{A_i\, 1+\tau_{Ki}, 1+\tau_{Li}\}$, and aggregate capital and labor K, L is a set of output, capital, labor, and prices of

I goods sectors $\{V_i, K_i, L_i, p_i\}$, aggregate output V, aggregate price p, and common factor prices p_K, p_L that satisfy the following conditions:

1. FOCs of firms in I goods sectors (12) and (13),
2. FOCs of firms in aggregate good sector (16),
3. resource constraints (17) and (18).

In what follows, I derive equilibrium values from the equilibrium conditions.

First, I calculate the equilibrium values of K_i and L_i. By substituting the FOC of capital (12) into the capital market condition (17), we get

$$\frac{1}{p_K} = \frac{K}{\sum_i \alpha_i p_i V_i/(1+\tau_{Ki})}. \tag{19}$$

By substituting this equation into (12), we obtain

$$K_i = \frac{\alpha_i p_i V_i/(1+\tau_{Ki})}{\sum_j \alpha_j p_j V_j/(1+\tau_{Kj})} K.$$

By substituting the FOC in the aggregate good sector (16), we finally get the equilibrium value of K_i:

$$K_i = \frac{\sigma_i \alpha_i/(1+\tau_{Ki})}{\sum_j \sigma_j \alpha_j/(1+\tau_{Kj})} K.$$

Similarly, from (13), (18), and (16), we get the equilibrium value of L_i:

$$L_i = \frac{\sigma_i (1-\alpha_i)/(1+\tau_{Li})}{\sum_j \sigma_j (1-\alpha_j)/(1+\tau_{Lj})} L.$$

For convenience in subsequent notations, I introduce new symbols $\lambda_{Ki} \equiv 1/(1+\tau_{Ki})$ and $\lambda_{Li} \equiv 1/(1+\tau_{Li})$, which I refer to as the *inverse capital and labor taxes.* Then,

$$K_i = \frac{\sigma_i \alpha_i \lambda_{Ki}}{\sum_j \sigma_j \alpha_j \lambda_{Kj}} K, \tag{20}$$

$$L_i = \frac{\sigma_i (1-\alpha_i) \lambda_{Li}}{\sum_j \sigma_j (1-\alpha_j) \lambda_{Lj}} L. \tag{21}$$

Other equilibrium values can be easily obtained once the equilibrium values of K_i and L_i are obtained. From the equilibrium values of K_i and L_i, we can also derive the equilibrium values of V_i and V. Suppose that p is the numeraire. Then, other equilibrium prices can also be obtained as follows. From (16) and (19), we obtain

$$p_K = \frac{pV}{K}\sum_i \sigma_i \alpha_i \lambda_{Ki}, \tag{22}$$

where the equilibrium value V has previously been obtained. Similarly, we can obtain equilibrium p_L. Once p_K and p_L are obtained, equilibrium p_i is obtained from (14).

Further, I define three symbols. First, I define α as the weighted average of α_i:

$$\alpha \equiv \sum_i \sigma_i \alpha_i. \tag{23}$$

Second, I define $\tilde{\lambda}_{Ki}$ and $\tilde{\lambda}_{Li}$ as

$$\tilde{\lambda}_{Ki} \equiv \frac{\lambda_{Ki}}{\sum_j \left(\frac{\sigma_j \alpha_j}{\alpha}\right) \lambda_{Kj}}, \tag{24}$$

$$\tilde{\lambda}_{Li} \equiv \frac{\lambda_{Li}}{\sum_j \left(\frac{\sigma_j (1-\alpha_j)}{1-\alpha}\right) \lambda_{Lj}}. \tag{25}$$

Since the sum of $\sigma_i \alpha_i / \alpha$ and $\sigma_i(1-\alpha_i)/(1-\alpha)$ are equal to unity, denominators of $\tilde{\lambda}_{Ji}$ (J is K or L) can be considered as the weighted averages of λ_{Ji}. Thus, $\tilde{\lambda}_{Ji}$ represents the ratio of inverse tax λ_{Ji} and the average of inverse taxes λ_{Jj}. Therefore, I refer to $\tilde{\lambda}_{Ki}$ and $\tilde{\lambda}_{Li}$ as the *relative capital and labor wedges.*

By substituting the definitions above into (20) and (21), we can rewrite equilibrium K_i and L_i as

$$K_i = \frac{\sigma_i \alpha_i}{\alpha} \tilde{\lambda}_{K_i} K, \tag{26}$$

$$L_i = \frac{\sigma_i (1-\alpha_i)}{1-\alpha} \tilde{\lambda}_{L_i} L. \tag{27}$$

Equations (26) and (27) uncover several findings on the effect of taxes on resource allocation of capital and labor. First, taxes affect the allocation of resources through $\tilde{\lambda}_{Ji}$. Second, the absolute magnitude of the taxes does not affect the resource allocation among sectors. Suppose, for instance, that tax on capital is identical across sectors. Then, λ_{Ki} is the same across sectors. Because the denominator of $\tilde{\lambda}_{Ki}$ is the weighted average of λ_{Kj}, $\tilde{\lambda}_{Ki}$ becomes unity, irrespective of the absolute magnitude of taxes. Thus, identical taxes do not affect the resource allocation across sectors. Third, the distribution of taxes across sectors affects resource allocation. If λ_{Ki} is smaller than the weighted average of λ_{Kj}, that is, if sector i's capital is taxed more, $\tilde{\lambda}_{Ki}$ becomes less than unity. Thus, less capital is allocated to sector i than to the level with no frictions.

3.5 Relation with standard one-sector model

In this section, let us examine the relation between this model and the standard aggregate one-sector model. Here I show that this model can be reduced to a one-sector model with wedges on aggregate productivity, aggregate capital supply, and aggregate labor supply and that the distribution of taxes among sectors changes wedges on aggregate productivity while the absolute magnitude of the taxes changes wedges on the aggregate capital and labor supply.[5]

First, V can be rewritten as follows:

$$V = \tilde{A} K^{\alpha} L^{1-\alpha}, \tag{28}$$

where

$$\tilde{A} = \prod_i A_i^{\sigma_i} (\tilde{\lambda}_{Ki}^{\alpha_i} \tilde{\lambda}_{Li}^{1-\alpha_i})^{\sigma_i} (\sigma_i (\alpha_i/\alpha)^{\alpha_i} ((1-\alpha_i)/(1-\alpha))^{1-\alpha_i})^{\sigma_i}. \tag{29}$$

Equation (28) can be viewed as an aggregate one-sector production function, where the relative wedges affect aggregate productivity. Note that it is not the absolute magnitude of the taxes but the distribution that affects wedges on aggregate productivity because of the property of $\tilde{\lambda}_{Ji}$ discussed in the previous section.

Next, we consider the aggregate real return of capital (since the same things can be argued for labor, I focus on capital). From (22), the aggregate real return of capital p_K/p in the economy can

[5] This section's results are slight extensions of Chari, Kehoe and McGrattan's (2003) results. Chari et al. conduct the same analysis using a two-sector, one-factor input model, while this paper uses a multisector, two-factor input model.

be written as follows:

$$\frac{p_K}{p} = \frac{V}{K}\sum_i \sigma_i \alpha_i \lambda_{Ki}. \tag{30}$$

When $\lambda_{Ki} = 1$, the real return becomes $\alpha V/K$. It is the same as the result in the standard one-sector Cobb-Douglas production function. Instead, suppose that some sectors are taxed on capital input so that $\lambda_{Ki} < 1$. Then, the aggregate real return decreases from the case when $\lambda_{Ki} = 1$. If $1 - \tilde{\tau}_K$ and $\tilde{p}_K$ are defined such that

$$\begin{aligned} 1 - \tilde{\tau_K} &\equiv \sum_i \frac{\sigma_i \alpha_i}{\alpha}\lambda_{Ki}, \\ \tilde{p}_K &\equiv \frac{\alpha V}{K}, \end{aligned}$$

equation (30) can be rewritten as

$$\frac{p_K}{p} = (1 - \tilde{\tau}_K)\tilde{p}_K.$$

Then, it is evident that taxes on capital inputs for sectors appear as a tax on aggregate capital in the reduced one-sector model. In addition, since the sum of $\sigma_i \alpha_i/\alpha$ is equal to unity, $(1 - \tilde{\tau}_K)$ can be considered as the weighted averages of λ_{Ki}. Therefore, it also becomes apparent that the absolute magnitude of the taxes affects wedges on the aggregate capital.

To sum up, this paper's multisector setting with taxes can be reduced to a one-sector model, with wedges on aggregate productivity, aggregate capital supply, and aggregate labor supply, when we focus on aggregate variables. An interesting point is that while the tax effect through aggregate productivity $\tilde{A}$ is generated by the distribution of taxes as discussed in the previous section, the tax effect through aggregate capital or labor tax $1 - \tilde{\tau_J}$ (J is K or L) is generated by the absolute magnitude of taxes.

Although taxes on factor inputs would affect the aggregate capital and labor particularly in a dynamic framework through $1 - \tilde{\tau_J}$, in the following sections, I focus on the tax effect on aggregate productivity. One reason for this is that while the tax effect on aggregate productivity can be calculated from relative wedges $\tilde{\lambda}_{Ji}$, which can be measured from data, measuring the tax effect on aggregate capital or labor would require us to measure inverse taxes λ_{Ji}, which are difficult to obtain.

4 Aggregate Productivity and Allocational Efficiency Indexes

We now measure how sector-level frictions in the form of taxes affect aggregate productivity. For this purpose, one might simply use $\tilde{A}$ found in (28) and (29) in order to measure the effect of the frictions. However, it might be problematic for a cross-country comparison to simply measure the difference of $\ln \tilde{A}$, which is calculated for each of different economic structures.

In order to deal with this problem, I use the translog bilateral aggregate TFP (TTFP) index, which is developed by by Christensen and Jorgenson (1970) and Caves et al. (1982), for measuring the differences in aggregate productivity. There are three reasons to use this index. First, this index is widely used for cross-country comparisons under a multisector environment. Second, my model in the previous section satisfies the assumptions of the TTFP, as demonstrated in appendix A. Third, as I mention later, the TTFP is a natural extension of $\tilde{A}$ given in (28).

Using the TTFP index, I define two types of allocational efficiency indexes. One is the TAE, which measures the productivity gap caused by the distribution of sector-specific frictions. The other index is the RTAE, which measures the productivity gap between two countries caused by the distribution of frictions in the countries.

4.1 Translog bilateral aggregate TFP index (TTFP)

In this section, I introduce the TTFP index proposed by Christensen and Jorgenson (1970) and Caves et al. (1982). In appendix A, I demonstrate that my model is consistent with the assumptions of the index. TTFP with capital and labor inputs can be expressed as follows:

$$\begin{aligned} \mathrm{TTFP}(s,t) \quad \equiv \quad & \sum_i \frac{1}{2}\{\sigma_i^s + \sigma_i^t\} \ln(V_i^s/V_i^s) \\ & -\frac{1}{2}\{\alpha^s + \alpha^t\} \ln(K^s/K^t) \\ & -\frac{1}{2}\{(1-\alpha^s) + (1-\alpha^t)\} \ln(L^s/L^t), \end{aligned} \tag{31}$$

where s and t are states (for example, states can represent different time periods or countries).[6] This TTFP measures the differences in aggregate productivity between s and t. Note that if $\sigma_i^s = \sigma_i^t$ and $\alpha^s = \alpha^t$ between state s and t, the TTFP becomes equal to the difference of $\ln \tilde{A}$ given in (28).

4.2 Translog allocational efficiency index (TAE)

Employing the model described in the previous section and the TTFP index, I now define TAE for TTFP. Let t^a be time t's actual state, and t^n be t's hypothetical state where all the frictions are removed from the actual state (i.e., $\tilde{\lambda}_{Ki} = \tilde{\lambda}_{Li} = 1$ for all sectors, but other conditions are the same as t^a). Then, I define TAE as TTFP for the two states:

$$\mathrm{TAE}(t) \equiv \mathrm{TTFP}(t^a, t^n) = \sum_i \sigma_i \alpha_i \ln \tilde{\lambda}_{Ki} + \sum_i \sigma_i (1 - \alpha_i) \tilde{\lambda}_{Li}. \tag{32}$$

Note that this TAE is equal to the difference of $\ln \tilde{A}$ in (29) between states t^a and t^n. I further divide TAE into its capital and labor components. I write the first term of the right-hand side of equation (32) as capital TAE or TAE_K and the second term as labor TAE or TAE_L.

TAE takes its maximum value, zero, when capital and labor frictions are equal across sectors and that otherwise, TAE is negative. These results can be found by solving the following problem:

$$\max_{\{\lambda_{Ki}, \lambda_{Li}\}} \sum_i \sigma_i \alpha_i \ln \tilde{\lambda}_{Ki} + \sum_i \sigma_i (1 - \alpha_i) \tilde{\lambda}_{Li}.$$

TAE has theoretical connections with the previous research (for details, see appendix B). First, the growth rate of TAE is equal to the total reallocation effect in the growth accounting literature (see, for e.g., Syrquin 1986 and Basu and Fernald 2002). Second, labor TAE divided by $-(1 - \alpha)$ is equivalent to Theil's (1967) entropy measure of income inequality.

[6] I slightly abuse the notations in that I do not distinguish expenditure share and capital share from σ_i and α_i.

4.3 Relative TAE (RTAE)

TTFP can be utilized for a cross-country comparison of aggregate TFP. Here, I define another allocational efficiency index based on TTFP and use it for cross-country comparison. Suppose we compare Japan with the US at time t. In this case, we can decompose TTFP as

$$\mathrm{TTFP}(t^a_{\mathrm{jpn}}, t^a_{\mathrm{usa}}) = \mathrm{TTFP}(t^n_{\mathrm{jpn}}, t^n_{\mathrm{usa}}) + \mathrm{RTAE}(t^a_{\mathrm{jpn}}, t^a_{\mathrm{usa}}), \tag{33}$$

where the meanings of t^a and t^n are the same as in TAE. Thus, $\mathrm{TTFP}(t^a_{\mathrm{jpn}}, t^a_{\mathrm{usa}})$ is the actual TTFP between Japan and the US and $\mathrm{TTFP}(t^n_{\mathrm{jpn}}, t^n_{\mathrm{usa}})$ is the TTFP of these countries when there is no friction for both countries. Here, I define $\mathrm{RTAE}(t^a_{\mathrm{jpn}}, t^a_{\mathrm{usa}})$ as

$$\begin{aligned}\mathrm{RTAE}(t^a_{\mathrm{jpn}}, t^a_{\mathrm{usa}}) &= \sum_i \bar{\sigma}_i \left(\alpha_{\mathrm{jpn},i} \ln \tilde{\lambda}_{\mathrm{jpn},Ki} - \alpha_{\mathrm{usa},i} \ln \tilde{\lambda}_{\mathrm{usa},Ki}\right) \\ &+ \sum_i \bar{\sigma}_i \left((1-\alpha_{\mathrm{jpn},i}) \ln \tilde{\lambda}_{\mathrm{jpn},Li} - (1-\alpha_{\mathrm{usa},i}) \ln \tilde{\lambda}_{\mathrm{usa},Li}\right), \end{aligned} \tag{34}$$

where $\bar{\sigma}_i$ is $(\sigma_{\mathrm{jpn},i} + \sigma_{\mathrm{usa},i})/2$. We term this index RTAE, because it captures relative allocational efficiency of Japan as compared to the US. I use RTAE to measure the effect of allocational efficiency on the differences in cross-country aggregate productivity. I use RTAE divided by $\mathrm{TTFP}(t^a_{\mathrm{jpn}}, t^a_{\mathrm{usa}})$ (hereafter referred to as RTAE/TTFP) as an indicator of the contribution of frictions because as we can see from (33), RTAE/TTFP indicates the percentage of TTFP that can be explained by RTAE. Akin to TAE, I also divide RTAE into its capital and labor components. I write the first term of the right-hand side of (34) as capital RTAE or RTAE_K and the second term as labor RTAE or RTAE_L.

5 Measurement

In this section, I measure the TAE, RTAE, and TTFP defined in the previous section from the sector-level data of developed countries in order to measure the importance of sector-level frictions for aggregate productivity per se and the differences in cross-country aggregate productivity. Further, I analyze the causes of the friction effect for each country in more depth.

5.1 Data and measurement procedure

Before the measurement, I explain the data and measurement procedure.

First, I explain the data.[7] The countries considered are West Germany (until 1991), Germany (from 1991), France, Italy, Japan, the UK and the US. Sector-level data except for capital input are taken from the OECD Structural Analysis (STAN) database for the period from 1970 to 2003. For capital input, I use the OECD International Sectoral Data Base (ISDB) for the period from 1970 to 1993. The other country-level data used for this analysis are also collected from the OECD. All the data are annual. The sectors considered in this study include (1) "Agriculture, Hunting, Forestry and Fishing" (hereafter, agricultural), (2) "Mining and Quarrying" and "Total Manufacturing" (manufacturing), (3) "Electricity, Gas and Water Supply" (electricity), (4) "Wholesale and Retail Trade" (wholesale), (5) "Transport and Storage and Communication" (transport), and (6) "Financial Intermediation" (financial). For the cross-country comparison, I am careful with maintaining the consistency of sector classification between countries because in general, the more subdivided is the definition of sector classification, the bigger is the effect of frictions on aggregate TFP. Note that since the covered data period of a sector differs across countries (and across sectors), the estimation period is different between countries.

Next, the method of measurement is explained. I do not directly measure capital and labor frictions in the form of taxes τ_{Ki} and τ_{Li}. Instead, I use (26) and (27) to derive $\tilde{\lambda}_{Ki}$ and $\tilde{\lambda}_{Li}$, where σ_i and α_i are estimated from the value added and capital shares of sector i for each year. The estimated $\tilde{\lambda}_{Ki}$ and $\tilde{\lambda}_{Li}$ would explain several kinds of frictions that affect the sector-level allocation of capital and labor. Given these variables, we can calculate the TAE and RTAE. The TTFP is measured from (31).[8]

5.2 TAE

In this section, I calculate the TAE of developed countries.

Figure 1 plots the time-series TAE of the countries. The values range from approximately -2% in the US and -8% in Japan and Italy around 1990, to over -14% in Italy around the early 1970s.

[7] I provide a summary of the data here; for details, see data appendix Aoki (2007).

[8] To calculate the TTFP, I use the hours worked times employment for labor input, as is typically done in international comparison. On the other hand, to calculate TAE and RTAE, I use employment for labor input due to data restrictions. For the robustness on labor input, see section D.5.

This signifies that the absence of sector-level frictions causes the aggregate productivity of the to increase by 2-14%.

The TAE can be further broken down into capital and labor components. These are plotted in Figures 2 and 3. The capital TAE is relatively stable over time for all countries. It ranges from around -1% for the US around 1990 to -8% for Italy around 1970s and 1980s and for Japan around the late 1980s. On the other hand, the labor TAE shows a convergence trend. In particular, for 1970, Italy's labor TAE is very low at around -8%, while West Germany has the highest value at more than -1%. However, in 2003, the labor TAE of the countries was between less than -3% (Japan) and around -4% (the UK). A notable characteristic is that the labor TAE of the US decreased during the 1990s. Because the decline of labor TAE indicates the increase in labor income inequality as I mention in section 4.2, this might reflect the recent increase in wage differentials (see, for e.g., Autor, Katz and Kearney 2005).

5.3 RTAE

One motivation to measure the effect of sector-level frictions is to determine whether the frictions can explain the differences in cross-country aggregate productivity. For this purpose, RTAE is an appropriate measure. In this section, I measure the RTAE of developed countries relative to the US. Figure 4 plots the results.[9] From the figure, we find that the rank order of countries in the TAE is preserved in the RTAE, except for West Germany. West Germany's RTAE is near zero, while its TAE is lower than that of the US. The reason for this is that the RTAE's weight share $\bar{\sigma}_i$ is different from that of TAE (in the RTAE, the weight value added share, except for that used for the measurement of $\tilde{\lambda}_{Ji}$, is calculated from the average of West Germany and the US data while in the TAE it is calculated based on West Germany data only). On the other hand, the RTAE of France, Japan, and Italy are low. Italy's RTAE is the lowest and is around -8%.

Similarly to that case of TAE, Figures 5 and 6 illustrate the capital and labor components of RTAE. The basic characteristics of the capital and labor TAE are also preserved in the capital and labor RTAE; although, the difference between Italy and the UK disappears in labor RTAE.

Using the RTAE, we can calculate the contribution of sector-level frictions to aggregate productivity by RTAE/TTFP, as explained in section 4.3. Table 1 presents the TTFP, RTAE, and

[9] The UK is not included in the figure. See footnote 2.

RTAE/TTFP relative to the US for the years 1987, 1990, and 1993. From the RTAE/TTFP, we can observe that the measured relative wedges can explain, on average, 35% of the aggregate productivity difference between the US and Italy, 30% of that between the US and Japan, and 20% of that between the US and France.[10] The measured frictions do not explain the differences in aggregate TFP for West Germany because the country has a high RTAE.

5.4 Which sector and what factors contribute to TAE and RTAE?

In this section, I analyze which sector contributes to the time-series changes in TAE and cross-country differences in RTAE, and whether these are due to frictions or other factors (i.e., value added and capital shares).

I summarize the results of this section.

For TAE, I focus on increase in Italy's labor TAE from 1970 to 2003 because it is the most striking result.[11] The increase is due to changes in the agricultural, financial, and transport sectors. In the agricultural sector, the changes in frictions are weak; other factors (mainly changes in value added share) account for this. In the financial sector, changes in frictions are dominant. In the transport sector, changes in both frictions and other factors affect the changes in labor TAE.

For RTAE, I focus on RTAE between the US and France, Italy, or Japan for the year 1990. The RTAE in France (and the US) is affected by the capital RTAE of the financial sector; this is due to frictions. In Italy, the capital and labor RTAE of the agricultural and financial sectors and the capital RTAE of the wholesale sector mainly affect the overall RTAE. With the exception of the labor RTAE of the agricultural sector, this is mainly due to frictions as well. In Japan, the capital RTAE of the financial, agricultural and construction sectors mainly affect the overall RTAE; this

[10] While in the study conducted by Prescott (2002), the level of France's aggregate productivity is higher than that of the US, the result is the opposite in my study. This is attributed to the differences in data sources (possibly the differences in data coverage).

In what follows, I verify it. Let us assume that aggregate capital intensity α and aggregate capital-output ratio are the same across countries (Prescott probably implicitly assumes the former assumption and demonstrates the latter through data). Then, in his accounting procedure, the cross-country differences in aggregate productivity, $\Delta \ln A$, is equal to $(1-\alpha)$ times the differences in aggregate GDP per hours worked, $\Delta \ln(V/H)$ (V is aggregate value added and H is aggregate hours worked). I verify whether in our dataset, aggregate value added per hours worked of France (adjusted by purchasing power parity) is smaller than that of the US. The result is that the value for France is smaller, which is consistent with the TTFP result.

On the other hand, for the case between Japan and the US, TTFP$/(1-\bar{\alpha})$ is around -30% (where $\bar{\alpha}$ is the average α between the two countries), which is close to the productivity difference in Prescott (2002).

[11] For TAE, I don't analyze which sector affects the level because the level of TAE might be affected by the quality differences of factor inputs between sectors (e.g., the differences in ability). RTAE would eliminate some of these effects if the distribution of the quality is similar across countries.

is also mainly due to frictions.

5.4.1 Causes of labor TAE fluctuations

In this section, I analyze which sector explains the increase of Italy's labor TAE from 1970 to 2003 and whether it is due to changes in frictions.

Before examining the results, I explain how to measure the contribution of a particular sector to the labor TAE and how to measure whether the contributions are made due to changes in frictions or other factors. (Although I only discuss the labor case, the expressions for the capital case are obtained similarly.)

First, I explain how to measure the contribution of a particular sector to the overall labor TAE. I define the sector i's contribution, which I denote by TAE_{Li}, with actual labor TAE, TAE_L, minus counterfactual labor TAE, TAE'_L, where only sector i's labor frictions λ'_{Li} becomes "ineffective" (I denote the counterfactual "ineffective" case by the prime sign). I define "ineffective" such that the inverse labor tax of sector i, λ'_{Li}, is adjusted to achieve $\tilde{\lambda}'_{Li} = 1$ (where sector i's labor allocation is the same as the case with no frictions) while the inverse labor taxes of the other sectors, λ'_{Lj}, are unchanged. TAE_{Li} measures the magnitude of the barriers between sector i and other sectors.[12] Then, appendix C.1 demonstrates that TAE_{Li} can be calculated as follows:

$$\begin{aligned} \mathrm{TAE}_{Li} &\equiv \mathrm{TAE}_L - \mathrm{TAE}'_L \\ &= \sigma_i(1-\alpha_i)\ln\tilde{\lambda}_{Li} + \sigma_{-i}(1-\alpha_{-i})\ln\tilde{\lambda}_{L,-i}, \end{aligned} \tag{35}$$

where

$$\tilde{\lambda}_{L,-i} \equiv \frac{1-\alpha}{\sigma_{-i}(1-\alpha_{-i})}\frac{L-L_i}{L}, \tag{36}$$

$$\sigma_{-i} \equiv \sum_{j\neq i}\sigma_j, \tag{37}$$

$$\alpha_{-i} \equiv \sum_{j\neq i}\left(\frac{\sigma_j}{\sum_{m\neq i}\sigma_m}\right)\alpha_j. \tag{38}$$

The above equation demonstrates that TAE_{Li} corresponds to TAE_L when the economy is divided

[12] TAE_{Li} is not defined as sector i's component of labor TAE, $\sigma_i(1-\alpha_i)\ln\tilde{\lambda}_{Li}$. The reason for this is that $\sigma_i(1-\alpha_i)\ln\tilde{\lambda}_{Li}$ can be positive even if the result is caused by the sector (for example, if there is a labor barrier from the agricultural sector to other sectors, and the agricultural sector holds excess labor inputs).

into two: sector i and all the other. As demonstrated in appendix C.2, the sum of TAE_{Li} is close to TAE_L. Although this is a crude approximation, in the following, I do not distinguish between the sum of TAE_{Li} and TAE_L for convenience of explanation.

Second, I explain how to measure whether the contribution measured by TAE_{Li} is made due to frictions or other factors. Since TAE_{Li} contains $\{\sigma_i, \sigma_{-i}\}$ and $\{\alpha_i, \alpha_{-i}\}$, the change in TAE_{Li} might be due to these. To distinguish the effect of the changes in $\{\sigma_i, \sigma_{-i}\}$ and $\{\alpha_i, \alpha_{-i}\}$ from that of changes in $\{\lambda_{Lm}\}$ (m includes all the sectors), I calculate the counterfactual TAE_{Li}, where I use inverse labor taxes taken from another period; otherwise (i.e., $\{\sigma_i, \sigma_{-i}\}$ and $\{\alpha_i, \alpha_{-i}\}$) I use actual values. Let us take an example that TAE_{Li} increases from 1970 and 2003 and that we would like to know the reason. Thus, I calculate the counterfactual TAE_{Li} for 2003 in a manner in which all variables except for inverse labor taxes are taken from 2003, but inverse labor taxes are taken from 1970. This counterfactual TAE_{Li} measures the TAE_{Li} for 2003 in which labor frictions had not changed since 1970. By comparing the counterfactual TAE_{Li} with the actual TAE_{Li} for 1970 and 2003, the contribution of $\{\sigma_i, \sigma_{-i}\}$, $\{\alpha_i, \alpha_{-i}\}$, and $\{\lambda_{Li}\}$ can be measured. For example, if the counterfactual TAE_{Li} is close to or higher than the actual TAE_{Li} for 2003, the reason of increasing TAE_{Li} between 1970 and 2003 is due to the changes in $\{\sigma_i, \sigma_{-i}\}$ and $\{\alpha_i, \alpha_{-i}\}$. On the other hand, if the counterfactual TAE_{Li} is close to or lower than the actual TAE_{Li} for 1970, this reason is due to the changes in $\{\lambda_{Lm}\}$. To calculate the counterfactual TAE_{Li}, I use the counterfactual relative labor wedges that are measured as follows:[13]

$$\tilde{\lambda}^{2003}_{Li,1970} = \frac{\tilde{\lambda}_{Li,1970}}{\sum_j \{\sigma_{j,2003}(1-\alpha_{j,2003})/(1-\alpha_{2003})\}\tilde{\lambda}_{Lj,1970}}. \tag{39}$$

Other than the relative labor wedges, the counterfactual TAE_{Li} can be calculated in the same way as original TAE_{Li}.[14] I define the counterfactual TAE_{Li} as $\text{TAE}_{Li,2003}(\lambda_{Lm,1970})$.

Based on these tools, I examine the increase in Italy's labor TAE from 1970 to 2003. Columns 1 and 2 of Table 2 reveal that the agricultural sector explains the most causes of increase in labor

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[13] This is because

$$(39) = \frac{\lambda_{Li,1970}}{\sum_j \{\sigma_{j,2003}(1-\alpha_{j,2003})/(1-\alpha_{2003})\}\lambda_{Lj,1970}}.$$

[14] More precisely, for $\tilde{\lambda}_{Li}$, I use $\tilde{\lambda}^{2003}_{Li,1970}$, and in the calculation of $\tilde{\lambda}_{L,-i}$ in (36), I use L_i that is calculated from $\tilde{\lambda}^{2003}_{Li,1970}$ by (26).

TAE. Moreover, the agricultural sector's $\text{TAE}_{Li,2003}(\lambda_{Li,1970})$, which encompasses labor frictions for the year 1970 (given in the first row of column 4), is much higher than $\text{TAE}_{Li,1970}$ (given in the first row of column 1). This suggests that changes in the value added or capital shares account for much of the change in TAE_{Li} in the agricultural sector. In fact, the value added share in the agricultural sector decreases from 12.2% in 1970 to 4.6% in 2003. Other reasons for the increase in labor TAE are the increase in the contributions of the transport and financial sectors (see columns 1 and 2). Both labor frictions and other factors explain the changes in the transport sector, while the majority of changes in the financial sector are explained by labor frictions.

5.4.2 Causes of cross-country capital and labor RTAE differences

This section focuses on which sectors contribute to capital and labor RTAE between the US and France, Italy, and Japan. I focus on France, Italy and Japan, because their RTAE/TTFP are low.

As in the previous section, before considering the results, I explain how to measure the contribution of a particular sector to capital RTAE (I take capital RTAE as an example) and whether the contribution is due to changes in frictions or other factors. These are extensions of the contributions mentioned in the previous section. In the following, In take an example that we compare Japan with the US.

First, I explain how to calculate sector i's contribution to capital RTAE. I define the contribution, which I denote as RTAE_{Ki}, by actual capital RTAE, RTAE_K, minus the counterfactual capital RTAE, RTAE'_K, where only sector i's labor frictions λ'_{Li} becomes "ineffective" for both Japan and the US.[15] The contribution is calculated as follows:

$$\begin{aligned}\text{RTAE}_{Ki} &\equiv \text{RTAE}_K - \text{RTAE}'_K \\ &= \bar{\sigma}_i \alpha_{\text{jpn},i} \ln \tilde{\lambda}_{\text{jpn},Ki} - \bar{\sigma}_i \alpha_{\text{usa},i} \ln \tilde{\lambda}_{\text{usa},Ki} \\ &+ (\bar{\alpha}_{\text{jpn}} - \bar{\sigma}_i \alpha_{\text{jpn},i}) \ln \tilde{\lambda}_{\text{jpn},K,-i} - (\bar{\alpha}_{\text{usa}} - \bar{\sigma}_i \alpha_{\text{usa},i}) \ln \tilde{\lambda}_{\text{usa},K,-i},\end{aligned} \tag{40}$$

where $\bar{\alpha}_c \equiv \sum \bar{\sigma}_i \alpha_{c,i}$ and $\ln \tilde{\lambda}_{c,K,-i}$ is calculated according to (36) for both the countries (c denotes country).[16] Since the sum of RTAE_{Ki} is close to RTAE_K as in the previous section, they are not

[15] For the meaning of "ineffectiveness," see the previous section.

[16] In contrast to the TAE case, RTAE_{Ki} does not exactly correspond to RTAE_K when the economy is divided into two sectors, sector i and all the other.

distinguished. If capital inputs are replaced with labor inputs, and $\bar{\alpha}$ and α_i with $(1-\bar{\alpha})$ and $(1-\alpha)$ respectively, the labor RTAE version of contribution is obtained.

Second, I explain how to measure whether this contribution is due to changes in frictions or other factors. Since RTAE_{Ki} contains $\{\sigma_m\}$ and $\{\alpha_m\}$ for Japan and the US, they might affect RTAE_{Ki}. To distinguish the effect by the differences in $\{\sigma_m\}$ and $\{\alpha_m\}$ from that by the differences in $\{\lambda_{Km}\}$ between the two countries, I calculate the counterfactual RTAE_{Ki}, where the $\{\lambda_{Km}\}$ of Japan is the same as that of the US. If the counterfactual RTAE_{Ki} is much higher than the actual RTAE_{Ki}, the difference is due to capital frictions of Japan. On the other hand, if the counterfactual RTAE_{Ki} is close to or lower than the actual RTAE_{Ki}, the low RTAE_{Ki} is due to the $\{\sigma_m\}$ and $\{\alpha_m\}$ of Japan. I calculate the counterfactual relative capital wedge as

$$\tilde{\lambda}^{\mathrm{jpn}}_{\mathrm{usa},Ki} = \frac{\tilde{\lambda}_{\mathrm{usa},Ki}}{\sum_j (\sigma_{\mathrm{jpn},j}\alpha_{\mathrm{jpn},j}/\alpha_{\mathrm{jpn}})\tilde{\lambda}_{\mathrm{usa},Kj}}.$$

Using these variables, I calculate the counterfactual capital RTAE.[17] I denote the counterfactual capital RTAE as $\mathrm{RTAE}^{\mathrm{jpn}}_{Ki}(\lambda_{\mathrm{usa},Km})$.

Table 3 displays the results for 1990 achieved by utilizing the above tools.

First, I consider France's RTAE. I focus on capital RTAE because the labor RTAE is not very low. The low level of the capital RTAE is considerably explained by the financial sector; the contribution can be explained by capital frictions caused by the sector (compare columns 1 and 2 of Table 3 under France).

Next, I examine Italy's capital and labor RTAE. It becomes evident from column 1 of Table 3 under Italy, that the low value of the capital RTAE is explained by the agricultural, wholesale, and financial sectors. From column 2 of Table 3 under Italy, we find that the low RTAE_{Ki} are also explained by capital frictions. On the other hand, Italy's low labor RTAE can be mainly explained by the agricultural and financial sectors. The contribution for the agricultural sector is mainly explained by factors other than labor frictions, because the agricultural sector's counterfactual labor RTAE provided in column 4 of Table 3 is close to the actual one.[18] On the other hand, the cause for the financial sector is mainly explained by labor frictions in this sector.

[17] I calculate $\tilde{\lambda}_{\mathrm{jpn},K,-i}$ in the same manner as in footnote 14.

[18] The value added share σ_i is the probable cause because the value added share of the agricultural sector is higher in Italy than in the US (I do not report the value).

Finally, considering Japan's case, I do not examine the labor RTAE because the labor RTAE is not very low. Most of Japan's capital RTAE is explained by the agricultural and financial sectors. Capital frictions explain the capital RTAE of these sectors (see columns 1 and 2 of Table 3 under Japan).

5.5 Robustness

In this section, I summarize the robustness check of the above results by changing the details of the measurements. The details are listed in appendix D. The robustness checks used in this study test whether (1) the measurement of TAE depends on its functional form; (2) the indirect tax data of sectors contained in the STAN database accounts for TAE; (3) the changes in capital share α_i affect measurement; (4) the inclusion or exclusion of some sectors affects results; (5) the results change if we take into account the differences in the hours worked between sectors.

Roughly speaking, when we treat each sectors' capital intensity α_i to be constant, or when we include the real estate sector, which was excluded in the previous section's analysis, the RTAE/TTFP decreases (i.e., the effects explained by frictions becomes insignificant).[19] However, for the former case, the procedure described in the previous section, where α_i is measured from the capital share in each sector during each period, seems more reasonable. For the latter case, in the previous sections, I exclude the real estate sector because it contains a large number of owner-occupied dwellings. The values of the variables would be distorted because labor input and the value added from the labor input of the owner-occupied dwellings are not measured. In addition, the share of owner-occupied dwellings is different across countries (for example, Japan is said to have a high share, and the US a low share).

Therefore, in sum, I deem the results given in the previous sections to be basically reliable.

6 Concluding Remarks

In this paper, I propose a simple multisector accounting framework to analyze the effect of several types of sector-level frictions on aggregate productivity. Using this framework, I estimate the extent to which the sector-level frictions affect aggregate productivity and the extent to which

[19] I treat the former case in appendix D.3 and the latter case in appendix D.4.

they account for the aggregate productivity differences among developed countries. We find that the sector-level frictions have a significant effect on aggregate productivity; in particular, they account for approximately 30% of the productivity differences between the US and Italy or Japan around 1990.

There are also some caveats in the analysis. The first pertains to the data issue. The definition of data (e.g., the definition of sector classification) is somewhat different between countries. Moreover, the capital stock data from the ISDB might not be very reliable. Second, the tax approach in this paper cannot take into account some dynamic effects. For example, suppose that there are adjustment costs for factor inputs. In this case, the return from these inputs in a sector can be higher than other sectors due to the adjustment costs. This effect is captured as frictions by this paper's approach even though resource allocation is efficient. Third, the method of measurement proposed in this paper cannot distinguish between frictions and the quality differences of factor inputs. For example, the return from labor input might be different between sectors because a sector with higher return might have more able employment. This effect is also captured as frictions by this paper's approach. These effects in the second and third points might be cancelled out in a cross-country comparison, if adjustment process is similar or the quality difference between sectors is equal across countries. However, it is not certain whether these assumptions are appropriate. To deal with these problems, more direct approaches might be needed. For example, building general equilibrium models that incorporate either a specific type of frictions, dynamic effects, or quality differences, measuring these effects directly, and checking if these models explain the magnitude of measured relative wedges of this paper might provide some insight. Further research is needed to resolve these issues.

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A Conditions for TTFP

This appendix explains the conditions that the translog bilateral indexes introduced in section 4 assume in the relation between outputs and inputs, and it briefly shows that our model introduced in section 3 satisfies these conditions.

The first condition is that the multiproduct, multifactor transformation function

$$F(\ln V^s, \ln X^s, s) = 1,$$

where V^s is output vector and X^s is input vector, has translog a functional form:

$$\begin{aligned}&\gamma_0^s + \sum_i \gamma_i^s \ln V_i^s + \sum_n \beta_n^s \ln X_n^s + \frac{1}{2}\sum_i \sum_j \gamma_{ij} \ln V_i^s \ln V_j^s \\ &+\frac{1}{2}\sum_n \sum_m \beta_{nm} \ln X_n^s \ln X_m^s + \sum_i \sum_m \xi_{in} \ln V_i^s \ln X_n^s = 1,\end{aligned}$$

where $\gamma_{ij} = \gamma_{ji}$ and $\beta_{ij} = \beta_{ji}$.

The second condition is that the translog function is restricted to constant returns to scale:

$$\begin{aligned}-\sum_i \gamma_i^s = \sum_n \beta_n^s = 1, \sum_i \gamma_{ij} = 0\\ \sum_n \beta_{nm} = 0, \sum_i \xi_{in} = \sum_n \xi_{in} = 0.\end{aligned}$$

The third condition is that the following FOCs are satisfied:[20]

$$\begin{aligned}
\frac{\partial F(\ln V^s, \ln X^s, s)}{\partial \ln V_i^s} &= \sigma_i^s, \\
\frac{\partial F(\ln V^s, \ln X^s, s)}{\partial \ln K^s} &= \alpha^s, \\
\frac{\partial F(\ln V^s, \ln X^s, s)}{\partial \ln L^s} &= 1-\alpha^s.
\end{aligned}$$

We can easily show that if we impose $\ln V^s = (\ln V_1^s, \ldots, \ln V_I^s)$ and $\ln X^s = (\ln K^s, \ln L^s)$, these conditions are satisfied in this paper's model that is introduced in section 3. (These conditions are verified by taking the log of both sides of (28) and comparing it with these conditions.)

B Theoretical Relationships with the Previous Research

B.1 TAE and total reallocation effect index (TRE)

I show theoretically that the growth rate of TAE is equal to TRE in the growth accounting literature (see, for e.g., Syrquin 1986 and Basu and Fernald 2002).

Let us consider an instantaneous change of actual TTFP in my model. Let $D\text{TTFP}^a \equiv \lim_{\Delta t \to 0} \text{TTFP}((t+\Delta t)^a, t^a)/\Delta t$. Then, $D\text{TTFP}^a = d\tilde{A}$, where d is the operator of growth rate and $\tilde{A}$ is given in (29). Therefore,

$$D\text{TTFP}^a = \sum_i \sigma_i dA_i + d\text{TAE},$$

where dTAE is given by $\sum_i \sigma_i \{\alpha_i d\tilde{\lambda}_{Ki} + (1-\alpha_i) d\tilde{\lambda}_{Li}\}$.

On the other hand, the growth accounting literature decomposes the aggregate Divisia productivity growth index dP ($\equiv dV - \alpha dK - (1-\alpha)dL$) as follows:

$$dP = \sum_i \sigma_i dP_i + \text{TRE},$$

where dP_i is sector i's Divisia productivity growth index ($\equiv dV_i - \alpha_i dK_i - (1-\alpha_i)dL_i$).[21] TRE

[20] As in footnote 6, I slightly abuse the notations.
[21] As in footnote 6, I slightly abuse the notations.

is further decomposed as TRE $= R_K + R_L$, where

$$\begin{aligned} R_K &\equiv \sum_i \sigma_i \alpha_i \left[\frac{p_{Ki} - p_K}{p_{Ki}} \right] dK_i, \\ R_L &\equiv \sum_i \sigma_i (1-\alpha_i) \left[\frac{p_{Li} - p_L}{p_{Li}} \right] dL_i, \end{aligned}$$

where $p_{Ji} \equiv (1 + \tau_{Ji}) p_J$.

Since $dA = dP$ and $dA_i = dP_i$, we find that dTAE $=$ TRE. More precisely, we can show that $d\text{TAE}_K = R_K$ and that $d\text{TAE}_L = R_L$, as follows.

$$\begin{aligned} R_K &= \sum_i \left(\frac{p_{Ki} K_i}{pV} - \frac{p_K K_i}{pV} \right) dK_i \\ &= \sum_i \frac{p_{Ki} K_i}{pV} dK_i - \frac{p_K K}{pV} \sum_i \frac{K_i}{K} dK_i \\ &= \sum_i \sigma_i \alpha_i d\left(\sigma_i \frac{\alpha_i}{\alpha} \tilde{\lambda}_{Ki} K \right) - \frac{p_K K}{pV} dK \\ &= \sum_i \sigma_i \alpha_i d\tilde{\lambda}_{Ki}, \end{aligned}$$

where (26) is used from the second to third equation and the relation that $\sum_i (K_i/K) dK_i = dK$, $\sum_i \sigma_i \alpha_i d\alpha = \alpha d\alpha$ and $\sum_i \sigma_i \alpha_i d(\sigma_i \alpha_i) = \alpha d\alpha$ is used in the third to the last equation. We can show the labor component in a similar manner.

B.2 TAE and Theil's entropy measure

I show that labor TAE is equivalent to Theil's (1967) entropy measure of income inequality.

Labor TAE divided by $-(1-\alpha)$ can be rewritten as

$$-\frac{\text{TAE}_L}{1-\alpha} = \sum_i \frac{(1-\alpha_i) p_i V_i}{(1-\alpha) pV} \ln \left(\frac{(1-\alpha_i) p_i V_i}{(1-\alpha) pV} \bigg/ \frac{L_i}{L} \right). \tag{41}$$

$(1-\alpha_i) p_i V_i$ is sector i's labor income, such that $(1-\alpha_i) p_i V_i / \{(1-\alpha) pV\}$ is the labor income share. L_i/L can be interpreted as employment share (here, I assume L_i and L as employment). Thus, the left hand side of (41) is equal to Theil's entropy measure (see Theil 1967, equation (3.2) in p.102).

C TAE_{Li}

C.1 Derivation of TAE_{Li}

In this section, given the settings in section 5.4.1, I demonstrate equation (35):

$$\text{TAE}_{Li} \quad = \quad \sigma_i(1-\alpha_i)\ln\tilde{\lambda}_{Li} + \sigma_{-i}(1-\alpha_{-i})\ln\tilde{\lambda}_{L,-i}.$$

The case of RTAE can be demonstrated in a similar manner.

From the assumption that $\tilde{\lambda}'_{Li} = 1$, we obtain

$$\lambda'_{Li} = \frac{1}{1-\sigma_i(1-\alpha_i)/(1-\alpha)}\sum_{m\neq i}\frac{\sigma_m(1-\alpha_m)}{1-\alpha}\lambda_{Lm},$$

where as in section 5.4.1, I denote the counterfactual case by the prime sign. By substituting this result into the definition of $\tilde{\lambda}'_{Lj}$ $(j \neq i)$, we obtain

$$\tilde{\lambda}'_{Lj} = \frac{\lambda_{L_j}}{\left\{\left(\frac{1}{1-\sigma_i(1-\alpha_i)/(1-\alpha)}\right)\sum_{m\neq i}\frac{\sigma_m(1-\alpha_m)}{1-\alpha}\lambda_{Lm}\right\}}$$

Using $\{\tilde{\lambda}_{Lm}\}$ and $\{\tilde{\lambda}'_{Lm}\}$, we obtain

$$\begin{aligned}
\text{TAE}_{Li} \quad &= \quad \sum_m \sigma_m(1-\alpha_m)\ln\tilde{\lambda}_{Lm} - \sum_{j\neq i}\sigma_j(1-\alpha_j)\ln\tilde{\lambda}'_{Lj} \\
&= \quad \sigma_i(1-\alpha_i)\ln\tilde{\lambda}_{Li} + \sum_{j\neq i}\sigma_j(1-\alpha_j)\ln\left(\frac{1-\alpha}{\sigma_{-i}(1-\alpha_{-i})}\frac{\sum_{m\neq i}\frac{\sigma_m(1-\alpha_m)}{1-\alpha}\lambda_{Lm}}{\sum_m\frac{\sigma_m(1-\alpha_m)}{1-\alpha}\lambda_{Lm}}\right) \\
&= \quad \sigma_i(1-\alpha_i)\ln\tilde{\lambda}_{Li} + \sigma_{-i}(1-\alpha_{-i})\ln\left(\frac{1-\alpha}{\sigma_{-i}(1-\alpha_{-i})}\sum_{m\neq i}\frac{L_m}{L}\right).
\end{aligned}$$

We can easily find that the last equation is equal to (35).

C.2 Relationship between TAE_{Li} and TAE_L

In this appendix, I explain the relation $\sum_i \text{TAE}_{Li} \approx \text{TAE}_L$, which I use in section 5. We can show similar relationships between TAE_{Ki} and TAE_K and between RTAE_{Ji} and RTAE_J $(J \in \{K, L\})$ in a similar manner.

From (35), we can find that

$$\sum_i \mathrm{TAE}_{Li} = \mathrm{TAE}_L + \sum_i \{(1-\alpha) - \sigma_i(1-\alpha_i)\} \ln\left(\frac{1-\sigma_{Li}}{1-\sigma_i(1-\alpha_i)/(1-\alpha)}\right), \tag{42}$$

where $\sigma_{Li} \equiv L_i/L$.

In what follows, I show that if σ_{Li} and $\sigma_i(1-\alpha_i)/(1-\alpha)$ are small, the second term of equation (42) is close to zero. In addition, if $\sigma_i(1-\alpha_i)$ is approximately equal across sectors, the difference between the second term and zero becomes much smaller.

By taking the first-order approximation of the second term of (42) around $\sigma_{Li} = 0$ and $\sigma_i(1-\alpha_i)/(1-\alpha) = 0$, we obtain

$$\begin{aligned}
& \sum_i \{(1-\alpha) - \sigma_i(1-\alpha_i)\} \ln\left(\frac{1-\sigma_{Li}}{1-\sigma_i(1-\alpha_i)/(1-\alpha)}\right) \\
\approx & \sum \{(1-\alpha) - \sigma_i(1-\alpha_i)\}(-\sigma_{Li} + \sigma_i(1-\alpha_i)/(1-\alpha)) \\
= & \sum \sigma_i(1-\alpha_i)(\sigma_{Li} - \sigma_i(1-\alpha_i)/(1-\alpha)) \\
= & \sum \left(\sigma_i(1-\alpha_i) - \frac{1-\alpha}{I}\right)\left(\frac{\sigma_{Li}}{\tilde{\lambda}_{Li}}\right)(\tilde{\lambda}_{Li} - 1),
\end{aligned} \tag{43}$$

where I is the number of sectors. By taking the first-order approximation of TAE_L around $\tilde{\lambda}_{Li} = 1$, we get

$$\mathrm{TAE}_L \approx \sum_i \sigma_i(1-\alpha_i)(\tilde{\lambda}_{Li} - 1). \tag{44}$$

By comparing (43) and (44), we find that (43) is smaller by the order of σ_{L_i}. From (43), we also find that if $\sigma_i(1-\alpha_i) \approx (1-\alpha)/I$ (i.e. if $\sigma_i \alpha_i$ is about the same across sectors), (43) approaches zero more closely.

Note that this is a crude approximation. In the empirical analysis, we have seven sectors. Thus, $\sigma_{L_i} \approx 1/7$. Therefore, taking into account the effect of $(\sigma_i(1-\alpha_i) - (1-\alpha)/I)$, roughly speaking, the error would be 1/7 of TAE_L or lower. In addition, the difference between the sum of TAE_{Li} and TAE_L is about 0.5% in the data.

D Robustness

This appendix provides robustness checks for the measurements in section 5.

The robustness checks in this appendix test whether (1) the measurement of TAE depends on its functional form; (2) the indirect tax of sectors contained in the STAN database accounts for TAE; (3) the changes in capital share α_i affect measurement; (4) the inclusion or exclusion of some sectors affects results; (5) the results change if we take into account the differences in the hours worked between sectors.

Roughly speaking, when we change the procedure related to the measurement of capital intensity or when we add the real estate sector, which was excluded in the above analysis, RTAE/TTFP decreases. I treat the former case in appendix D.3 and the latter case in appendix D.4.

D.1 Other Indexes

The measured TAE might be dependent on its functional form. To check its dependency, I calculate the Laspeyres and Fisher index versions of allocational efficiency (LAE and TAE).[22] The calculated results of TAE, LAE, and FAE for 1990 are listed in Table 4. We can find that LAE is qualitatively similar to TAE. However, quantitatively, LAE is downward biased compared with TAE. Diewert (1976) demonstrates that indexes that are superlative take similar values and that translog indexes are superlative while Laspeyres are not. On the other hand, Hill (2006) shows that even among superlative indexes, there could be large differences in quantitative terms. To check these points, I also measure FAE, which is a superlative index. We can find that the values are similar to that of TAE.

D.2 Indirect taxes

As analyzed in section 2.2, a sector-specific output tax administered by governments can be the source of the sector-level frictions. Since the STAN database contains indirect tax data for each sector, in this appendix, I analyze whether indirect tax affects aggregate productivity when the indirect tax is assumed to be in the form of output tax.

I examine the effects of indirect tax in two ways. First, I calculate counterfactual TAE,

[22] For the details in calculation, see appendix E.

$\mathrm{TAE}_{\text{indirect tax}}$, where the only sources of frictions are indirect taxes among sectors. In this case, relative capital wedge can be calculated as follows:[23]

$$\tilde{\lambda}_{\text{indirect tax},Ki} = \frac{(1 - \tau_{\text{indirect tax},i})}{\sum_j (\sigma_j \alpha_j / \alpha)(1 - \tau_{\text{indirect tax},j})}.$$

Relative labor wedge can be calculated in a similar manner. By substituting these relative capital and labor wedges into the equation of TAE, we obtain $\mathrm{TAE}_{\text{indirect tax}}$. Second, I calculate another counterfactual TAE, $\mathrm{TAE}_{-\text{indirect tax}}$, where the effects of indirect tax are eliminated from relative wedges. Thus, relative capital wedge can be calculated using the following equation:

$$\tilde{\lambda}_{-\text{indirect tax},Ki} = \frac{\tilde{\lambda}_{Ki} / (1 - \tau_{\text{indirect tax},i})}{\sum_j (\sigma_j \alpha_j / \alpha) \tilde{\lambda}_{Kj} / (1 - \tau_{\text{indirect tax},j})}. \tag{45}$$

Table 5 shows the results. It illustrates that the effects of indirect tax are negligible for the countries. $\mathrm{TAE}_{\text{indirect tax}}$ is near zero for all countries. It also shows that $\mathrm{TAE}_{\text{indirect tax}}$ is the lowest in the US. In addition, $\mathrm{TAE}_{-\text{indirect tax}}$ does not deviate much from the actual TAE.

D.3 Value of capital intensity α_i

Throughout the empirical analysis, labor share is utilized in order to calculate capital intensity α_i for each sector. Basically, labor share is calculated from the compensation of employees (CE) over the value added. However, because CE does only contain labor cost for employees, the labor cost of the self-employed and unpaid family workers is excluded from CE. In order to adjust for this factor, an adjusted CE may be calculated for each sector by dividing CE by the number of employees and multiplying it by the total employment; labor share can thus be calculated from the adjusted CE over value added.

It is not particularly matter to use this procedure for any sector except for the agricultural sector. Thus, throughout the empirical analysis I use the labor share calculated in this manner except for the agricultural sector. However, when the labor share of the agricultural sector is calculated in this manner, it exceeds unity in some countries (West Germany, Germany, and Japan). This might be due to the fact that there are many part-time farmers who are self-employed

[23] This result is obtained from (5) (as mentioned above, I assume that the indirect tax is in the form of output tax).

or unpaid family workers in the agricultural sector. In order to deal with this issue, I assume that in the agricultural sector, each self-employed or unpaid family worker works β percentage of the time that an employee works. Under this assumptions, I calculate the adjusted employment and adjusted CE of the agricultural sector as follows:

$$
\begin{aligned}
(\text{adjusted EN}_{\text{agr}}) &= \text{EE}_{\text{agr}} + \beta(\text{EN}_{\text{agr}} - \text{EE}_{\text{agr}}), && (46)\\
(\text{adjusted CE}_{\text{agr}}) &= R_{\text{agr}} \cdot (\text{adjusted EN}_{\text{agr}}), && (47)
\end{aligned}
$$

where R_{agr} is the CE per employee of the sector and EN and EE denote total employment and employees, respectively. Although it is ad hoc, I assigned $\beta = 0.5$ throughout the empirical analysis in section 5 (a value more than 0.6 cannot be chosen because in such a case, labor share exceeds unity).

In this appendix, for the robustness check, I change the value of β to be 0 and 0.25. This change affects the values of labor input and capital intensity in the agricultural sector. Under these values, I calculate the RTAE/TTFP for 1990. Columns 1–3 of Table 6 exhibit the results. RTAE/TTFP changes somewhat; however, the basic characteristics are preserved. For further investigation, I also examine other cases where the adjusted CE is calculated by (47) not only for the agricultural sector but also the other sectors. The results are listed in column 4 of Table 6. In this case, RTAE/TTFP exceeds the values of the normal case for Italy and Japan.

Next, I examine a case in which capital intensity α_i is equal across sectors and countries although the case in which α_i is measured from the capital share in each sector during each period seems more reasonable. Here, EN in the agricultural sector is not adjusted. Column 5 of Table 6 displays the results. The table demonstrates that the magnitude of RTAE decreases and does not explain the differences in aggregate productivity whatsoever: for Italy and Japan, the RTAE only explains approximately 10% of the productivity differences; for West Germany and France, the differences are not explained.

In order to attain possible explanations, I also calculate the contribution of each sectors to RTAE (Table 7). To avoid the issue where changes in α_i affect the allocation between capital and labor RTAE, it is appropriate to consider their sum, RTAE_i $(= \text{RTAE}_{Ki} + \text{RTAE}_{Li})$. The upper row of Table 7 exhibits the normal case; the lower row of the table, the case for $\alpha_i = 0.3$. By comparing the upper row with lower row, we can find similar tendencies across countries: in the agricultural sector, RTAE_i is rather low in the case where $\alpha_i = 0.3$. However, in the electricity, construction, wholesale, and financial sectors, RTAE_i increases in the $\alpha_i = 0.3$ case. In particular, RTAE_i becomes highly positive for the electricity, construction, and wholesale sectors, which means that these countries are more allocationally efficient in these sectors than the US.

D.4 Inclusion and exclusion of some sectors

In the above analysis, I excluded the real estate sector. The reason for this is that this sector contains many owner-occupied dwellings, and the share of owner-occupied dwellings is different across countries (for example, Japan is said to have a high share and the US is said to have a low share). Including this sector would distort the values of the variables, because for owner-occupied dwellings, labor input and value added from the labor input are not measured. In this section, I include the real estate sector and calculate RTAE/TTFP. The results are listed in Table 8. When introducing the real estate sector, the RTAE/TTFP of Italy and Japan decreases by 10–20%.

Next, I exclude electricity sector. Electricity sector is a public undertaking, and it might be better to exclude it from the analysis. Aligned with the expectation that this sector's contribution to capital and labor RTAE is not significant as is demonstrated in Table 3, RTAE/TTFP does not change much by excluding this sector in 1990 (I do not report the result).

D.5 Hours worked

Until now, for labor input, I have used total employment adjusted for the agricultural sector, except for calculating TTFP (as for TTFP, I use hours worked per employee times total employment adjusted for labor input). Thus, until now, I implicitly assumed that the hours worked are different between countries but are the same across sectors. However, the magnitude of the measured relative wedges might not become substantial if the differences in hours worked per employment between sectors are taken into account. In order to examine the effect, I calculate the RTAE/TTFP where labor input data for RTAE incorporates the hours worked for each sector (as I discuss below, I retain the previous procedure used to calculate TTFP).[24]

[24] Because as I explain later, the manner how the hours worked is incorporated into labor input data is different between countries, and because West Germany and Germany do not provide sector-level labor input data adjusted for hours worked, I have used total employment for labor input throughout the analysis, except for TTFP.

There are two kinds of sector-level data in the STAN database for labor input which are adjusted for hours worked. Some countries (France, Italy, and the US) provide full-time equivalent jobs data, which is defined as the total hours worked divided by average annual hours worked in full-time jobs for each sector. Japan provides hours worked by employees for each sector. (West Germany and Germany does not provide such data, so I exclude them.) I directly use these data as labor input for RTAE for France, Italy, and the US. For Japan, since the hours worked by employees does not take into account for the self-employed and unpaid family workers, an adjustment is needed. I adjust it in the same manner that the compensation of employees was adjusted in appendix D.3. For TTFP, I keep the previous procedure to calculate it. The reason for this is that although in order to calculate TTFP, it is needed to calculate the ratio of labor inputs between two countries, it is impossible if these two countries use the different definitions for labor input.

Table 9 exhibits the results. It is evident that the effect of the relative wedges becomes stronger.

E LAE and FAE

In this appendix, I define the Laspeyres and Fisher index versions of allocational efficiency index, LAE and FAE, which I use in appendix D.1 and explain how to calculate them.

I define TAE as the log of the ratio of the actual aggregate productivity to the aggregate productivity when all frictions are removed. I define LAE and FAE in the same way. Since we compare actual aggregate productivity with that of no frictions in the same period, the aggregate factor inputs are the same between these two states (i.e., actual and counterfactual with no frictions). Therefore, the comparison of aggregate productivity is equal to the comparison of the value added for the two states.

LAE is defined as follows:

$$\mathrm{LAE} = \ln \left\{ \frac{\sum_i V_i^a}{\sum_i V_i^n} \right\}, \tag{48}$$

where V_i^a is the actual real value added and V_i^n is real value added when there is no sectoral frictions, and is equal to $A_i(\sigma_i \alpha_i/\alpha K)^{\alpha_i}(\sigma_i(1-\alpha_i)/(1-\alpha_i)L)^{1-\alpha_i}$. Both are real value in that they are adjusted to base year price level.

When the Laspeyres index is calculated, it is calculated under the condition that price level does not change. Thus, LAE might overestimate the frictions. When barriers to high productivity sectors disappear, production in that sector would increase, but at the same time, the sector's price level would also decline. LAE does not take into account the latter effect. FAE is defined below considers this price effect.

FAE is defined as follows:

$$\text{FAE} = \ln\left(\frac{p^a V^a}{p^a V^n}\frac{p^n V^a}{p^n V^n}\right)^{0.5}, \tag{49}$$

where $p^s V^t$ is the aggregate value added when price level is that of state s and the value added is that of state t (state a implies actual and state n means no frictions). To calculate it, we need to employ another method. Without a loss of generality, we can assume that $p^a V^a = p^n V^n$. Then

$$\begin{aligned} \text{FAE} &= \ln\left(\frac{p^n V^a}{p^a V^n}\right)^{0.5} \\ &= \ln\left(\frac{\sum_i (p_i^n/p_i^a) p_i^a V_i^a}{p^a V^n}\right)^{0.5}. \end{aligned}$$

Here,

$$\begin{aligned} \frac{p_i^n}{p_i^a} &= \frac{p_i^n V_i^n}{p_i^a V_i^a}\frac{V_i^a}{V_i^n} \\ &= \frac{\sigma_i p^a V^a}{\sigma_i p^n V^n}\frac{p_i^a V_i^a}{p_i^a V_i^n} \\ &= \frac{p_i^a V_i^a}{p_i^a V_i^n}. \end{aligned}$$

Thus,

$$\text{FAE} = \ln\left(\frac{\sum_i (p_i^a V_i^a)^2/(p_i^a V_i^n)}{p^a V^n}\right)^{0.5}. \tag{50}$$

$p_i^a V_i^n$ can be calculated from data by employing the relations that

$$p_i^a A_i = \frac{p_i^a V_i^a}{K_i^{\alpha_i} L_i^{1-\alpha_i}},$$

Moreover, $K_i^n = \sigma_i \alpha_i/\alpha K$, and $L_i^n = \sigma_i(1-\alpha_i)/(1-\alpha)L$. Therefore, the variables in (50) can be calculated from data.

	1987	1990	1993
TTFP			
West Germany	−18.2%	−11.3%	n.a
France	−15.1%	−10.3%	−14.7%
Italy	−20.6%	−19.5%	−23.8%
Japan	−30.2%	−18.8%	−24.0%
RTAE			
West Germany	−0.2%	0.0%	n.a
France	−2.6%	−2.8%	−2.9%
Italy	−6.8%	−7.7%	−7.9%
Japan	−6.8%	−6.8%	−7.6%
RTAE/TTFP			
West Germany	0.9%	−0.1%	n.a
France	17.0%	27.1%	20.0%
Italy	32.9%	39.4%	33.3%
Japan	22.5%	36.1%	31.5%

Table 1: TTFP, RTAE, and RTAE/TTFP of West Germany, France, Italy, and Japan. The values are the percentage differences relative to the US.

	(1) $\text{TAE}_{Li,1970}$	(2) $\text{TAE}_{Li,2003}$	(3) (2)−(1)	(4) $\text{TAE}_{Li,2003}(\lambda_{Lm,1970})$
Agricultural	−4.3%	−0.9%	3.4%	−1.7%
Manufacturing	−0.2%	0.0%	0.2%	0.0%
Electricity	−0.4%	−0.1%	0.3%	−0.2%
Construction	−0.1%	−0.2%	0.0%	−0.3%
Wholesale	0.0%	−0.2%	−0.2%	−0.1%
Transport	−1.0%	−0.3%	0.7%	−0.6%
Financial	−2.2%	−1.0%	1.2%	−2.9%
Total	−8.2%	−2.7%	5.5%	−5.8%

Table 2: Labor TAE decomposition of Italy from 1970 to 2003. Columns 1 and 2 are calculated according to (35). Column 3 exhibits the differences between columns 2 and 1. Column 4 is a counterfactual TAE_{Li} for 2003, which is the same as the actual TAE_{Li} for 2003, except for the frictions which are from 1970 data.

France	(1) $\text{RTAE}_{Ki}^{\text{fra}}$	(2) $\text{RTAE}_{Ki}^{\text{fra}}(\lambda_{\text{usa},Km})$	(3) $\text{RTAE}_{Li}^{\text{fra}}$	(4) $\text{RTAE}_{Li}^{\text{fra}}(\lambda_{\text{usa},Lm})$
Agricultural	0.5%	0.6%	−0.8%	−1.3%
Manufacturing	0.1%	0.1%	0.1%	0.1%
Electricity	−0.2%	0.2%	−0.1%	−0.1%
Construction	0.1%	0.0%	−0.2%	−0.1%
Wholesale	−0.1%	−0.3%	0.7%	0.7%
Transport	−0.5%	0.0%	0.1%	0.0%
Financial	−2.0%	0.3%	−0.3%	−0.1%
Total	−2.2%	0.9%	−0.6%	−0.9%
Italy	(1) $\text{RTAE}_{Ki}^{\text{ita}}$	(2) $\text{RTAE}_{Ki}^{\text{ita}}(\lambda_{\text{usa},Km})$	(3) $\text{RTAE}_{Li}^{\text{ita}}$	(4) $\text{RTAE}_{Li}^{\text{ita}}(\lambda_{\text{usa},Lm})$
Agricultural	−1.5%	0.4%	−1.4%	−1.0%
Manufacturing	0.3%	0.3%	0.2%	0.1%
Electricity	0.0%	0.3%	−0.3%	−0.3%
Construction	−0.5%	−0.1%	−0.2%	−0.1%
Wholesale	−1.2%	−0.2%	0.4%	0.4%
Transport	−0.5%	0.0%	−0.4%	−0.1%
Financial	−1.6%	0.1%	−1.5%	−0.1%
Total	−5.1%	0.7%	−3.1%	−0.9%
Japan	(1) $\text{RTAE}_{Ki}^{\text{jpn}}$	(2) $\text{RTAE}_{Ki}^{\text{jpn}}(\lambda_{\text{usa},Km})$	(3) $\text{RTAE}_{Li}^{\text{jpn}}$	(4) $\text{RTAE}_{Li}^{\text{jpn}}(\lambda_{\text{usa},Lm})$
Agricultural	−2.5%	0.1%	−0.7%	−0.5%
Manufacturing	0.3%	0.4%	0.2%	0.2%
Electricity	0.1%	−0.1%	0.0%	0.0%
Construction	−1.1%	0.3%	−0.2%	−0.3%
Wholesale	−0.1%	−0.2%	0.8%	0.8%
Transport	0.0%	0.1%	−0.3%	−0.3%
Financial	−3.2%	0.5%	−0.5%	−0.2%
Total	−6.2%	1.1%	−0.6%	−0.2%

Table 3: Capital and labor RTAE decomposition of France, Italy, and Japan relative to the US for 1990. Columns 1 and 3 are calculated according to (40). Columns 2 and 4 list the counterfactual capital and labor RTAE that are the same as the actual capital and labor RTAE of each country; however, the frictions of each country are substituted for that of the US.

	TAE_{1990}	LAE_{1990}	FAE_{1990}
West Germany	−4.5%	−6.7%	−4.3%
France	−4.7%	−7.4%	−5.0%
Italy	−8.4%	−13.9%	−8.6%
Japan	−9.0%	−14.4%	−9.0%
US	−1.9%	−2.1%	−1.9%

Table 4: TAE, LAE, and TAE of West Germany, France, Italy, Japan, and the US for 1990.

Country Name	normal TAE	$\text{TAE}_{\text{indirect tax}}$	$\text{TAE}_{-\text{indirect tax}}$
West Germany	−4.5%	−0.1%	−4.0%
France	−4.7%	0.0%	−4.6%
Italy	−8.4%	0.0%	−8.2%
Japan	−9.0%	−0.1%	−9.1%
US	−1.9%	−0.4%	−2.6%

Table 5: Effects of indirect tax on aggregate productivity: normal TAE, $\text{TAE}_{\text{indirect tax}}$ and $\text{TAE}_{-\text{indirect tax}}$ for 1990.

$(\text{RTAE/TTFP})_{1990}$	(1) $\beta = 0$	(2) $\beta = 0.25$	(3) $\beta = 0.5$ (normal case)	(4) $\beta = 0.5$ (all sectors use adjusted CE)	(5) $\alpha_i = 0.3$ (total employment) is not adjusted)
West Germany	−5.1%	−3.0%	−0.1%	−1.2%	−23.5%
France	31.8%	28.6%	27.1%	25.8%	−9.1%
Italy	34.8%	37.1%	39.4%	82.4%	11.1%
Japan	28.5%	31.6%	36.1%	40.3%	11.2%

Table 6: Changes in capital intensity and RTAE/TTFP in 1990. For details, see the main text.

$\mathrm{RTAE}_{i,1990}$ Normal case	West Germany	France	Italy	Japan
Agricultural	−0.3%	0.5%	−2.9%	−3.2%
Manufacturing	2.2%	0.0%	0.5%	0.6%
Electricity	−0.1%	−0.4%	−0.4%	0.1%
Construction	−1.1%	0.8%	−0.7%	−1.2%
Wholesale	1.2%	0.0%	−0.8%	0.9%
Transport	−0.2%	−0.8%	−0.9%	−0.3%
Financial	−0.5%	−2.0%	−3.0%	−3.7%
Total	1.1%	−2.0%	−8.2%	−6.9%
$\mathrm{RTAE}_{i,1990}$ $\alpha_i = 0.3$ case	West Germany	France	Italy	Japan
Agricultural	−1.3%	−1.5%	−4.5%	−5.6%
Manufacturing	2.2%	0.3%	0.4%	0.2%
Electricity	0.8%	0.8%	1.2%	0.8%
Construction	1.2%	2.5%	2.6%	2.6%
Wholesale	1.9%	1.4%	1.2%	1.4%
Transport	−0.5%	−0.8%	−0.6%	−0.1%
Financial	−0.1%	−1.2%	−2.6%	−2.1%
Total	4.3%	1.5%	−2.3%	−2.6%

Table 7: RTAE_i ($= \mathrm{RTAE}_{Ki} + \mathrm{RTAE}_{Li}$) for 1990; the upper section consists of the results obtained by through the normal case; the lower section, through the $\alpha_i = 0.3$ case. For details, see the main text.

$(\mathrm{RTAE/TTFP})_{1990}$	normal case	real estate included
West Germany	−0.1%	−1.5%
France	27.1%	28.0%
Italy	39.4%	21.7%
Japan	36.1%	29.2%

Table 8: Comparison of RTAE/TTFP in 1990 between the normal case and the case where real estate is included.

$(\mathrm{RTAE/TTFP})_{1990}$	normal case	hours worked incorporated
France	27.1%	41.5%
Italy	39.4%	54.6%
Japan	36.1%	41.3%

Table 9: Comparison of RTAE/TTFP in 1990 between the normal case and the case where hours worked is incorporated in the calculation of RTAE.

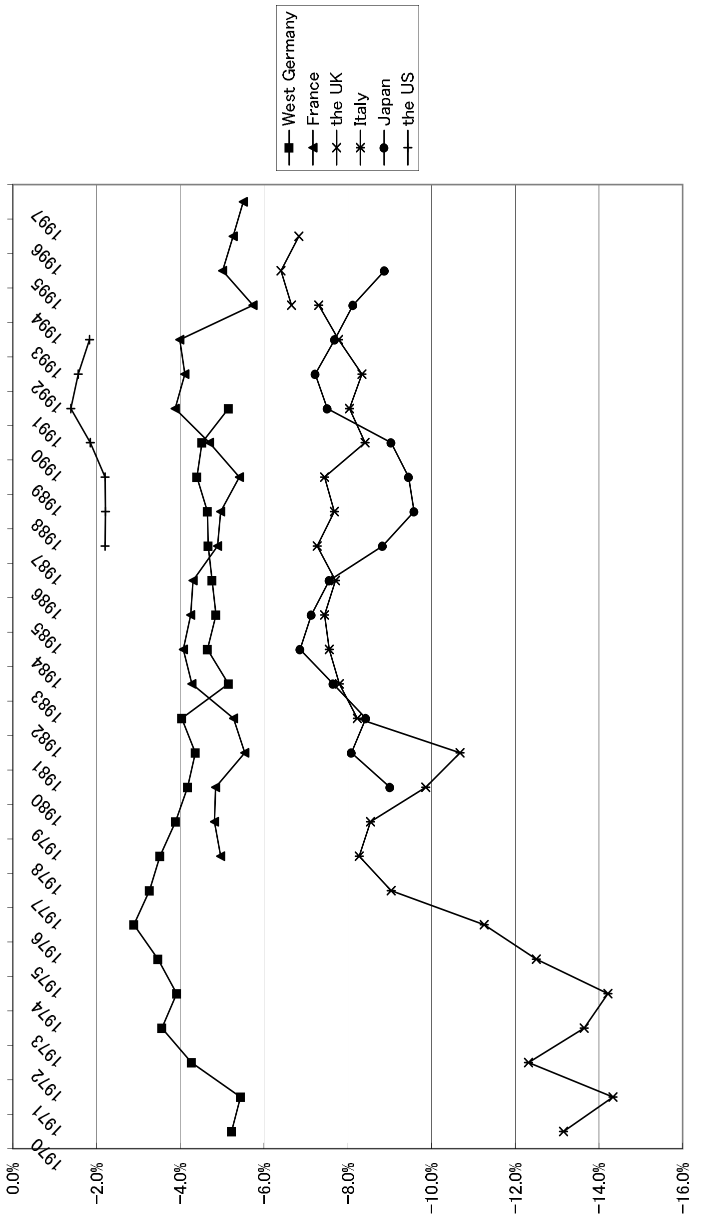

Figure 1: TAE of West Germany (before 1991), France, Italy, Japan, the UK, and the US.

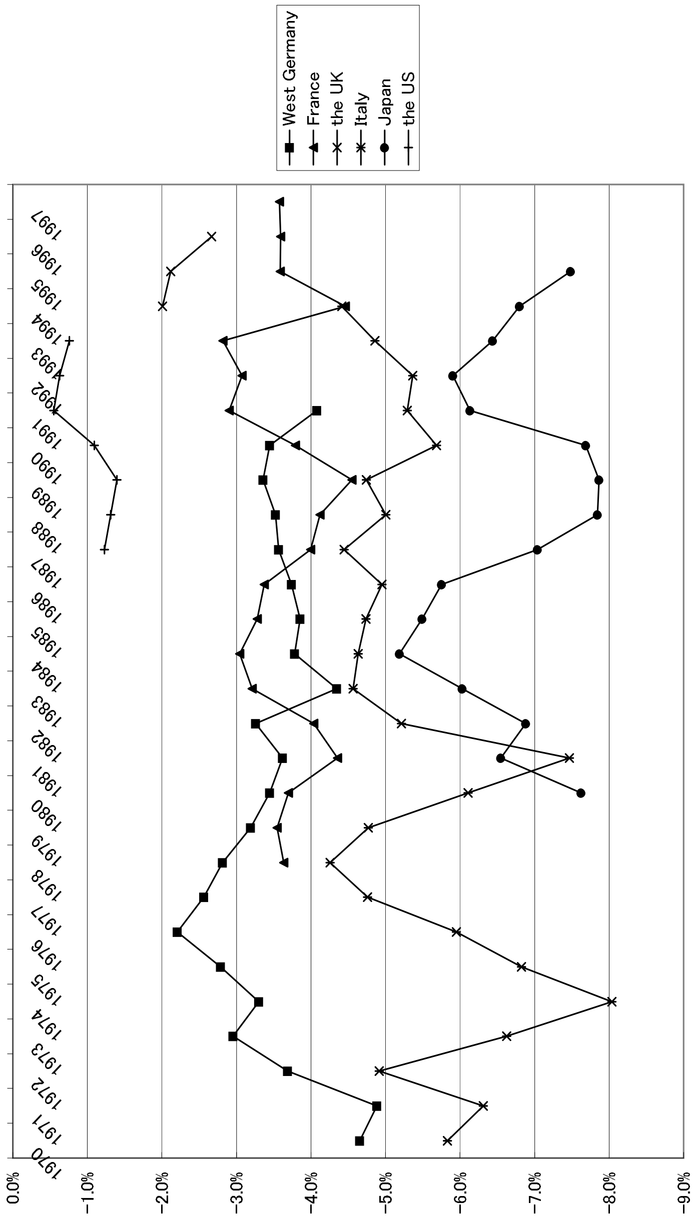

Figure 2: Capital TAE, TAE_K of West Germany (before 1991), France, Italy, Japan, the UK, and the US.

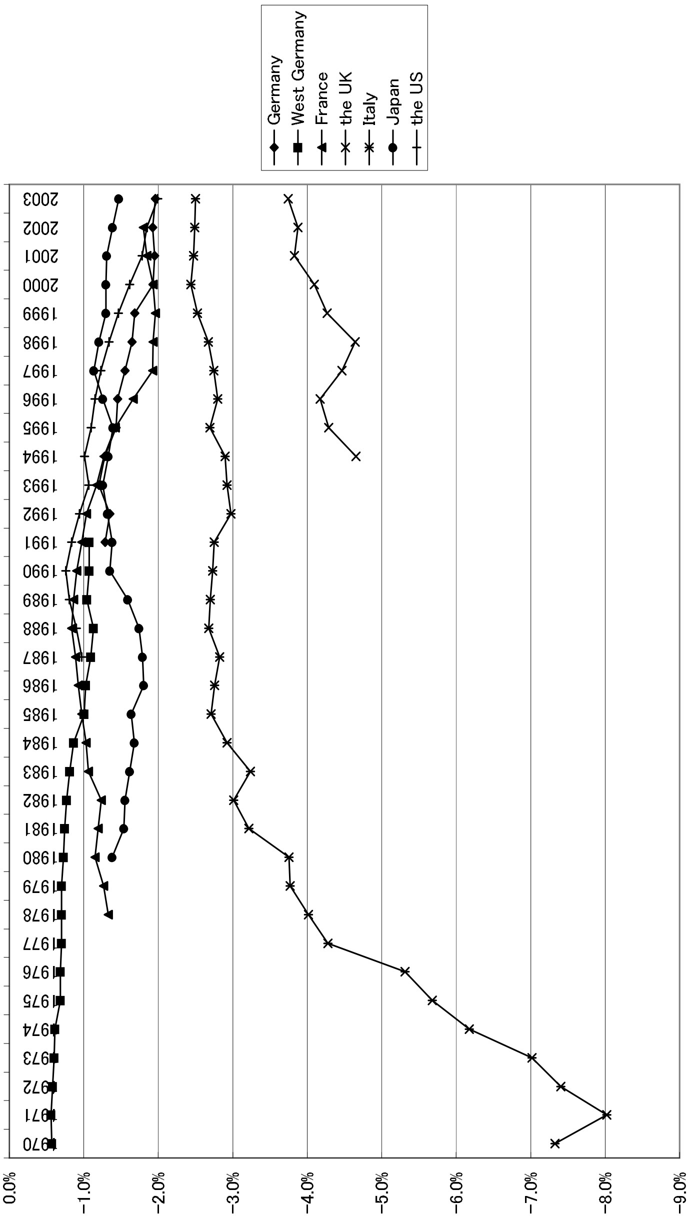

Figure 3: Labor TAE, TAE_L of West Germany (before 1991), Germany (from 1991), France, Italy, Japan, the UK, and the US.

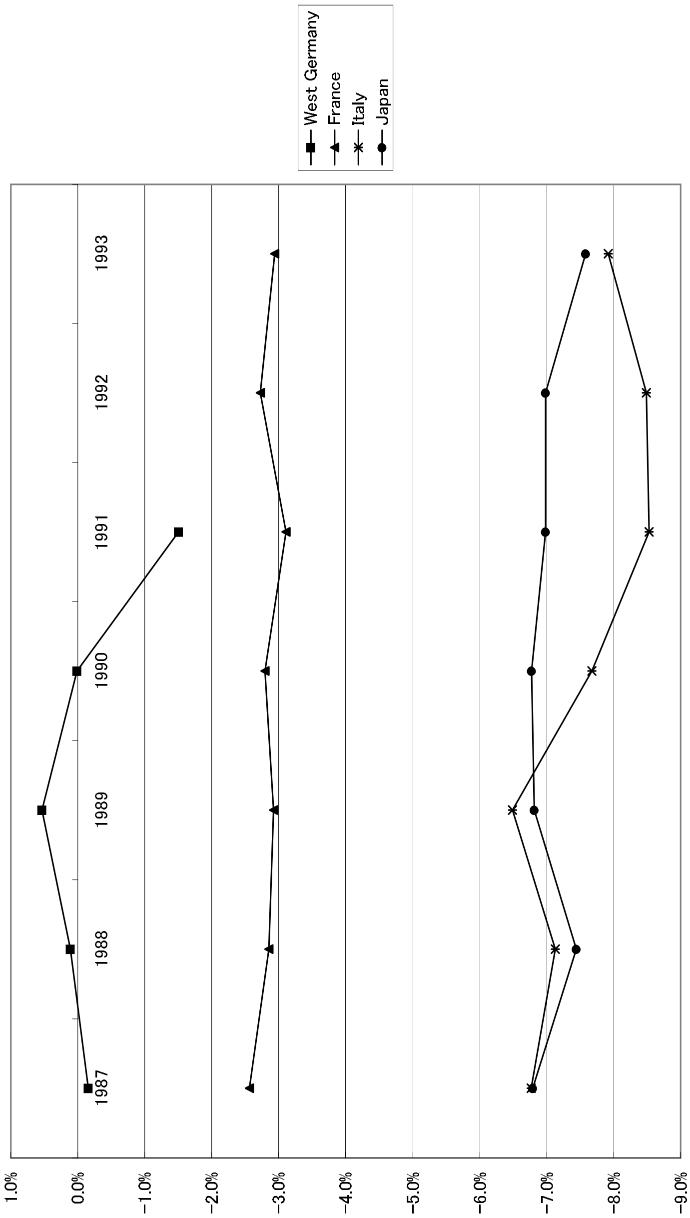

Figure 4: RTAE of West Germany (before 1991), France, Italy, and Japan. The values are the percentage differences relative to the US.

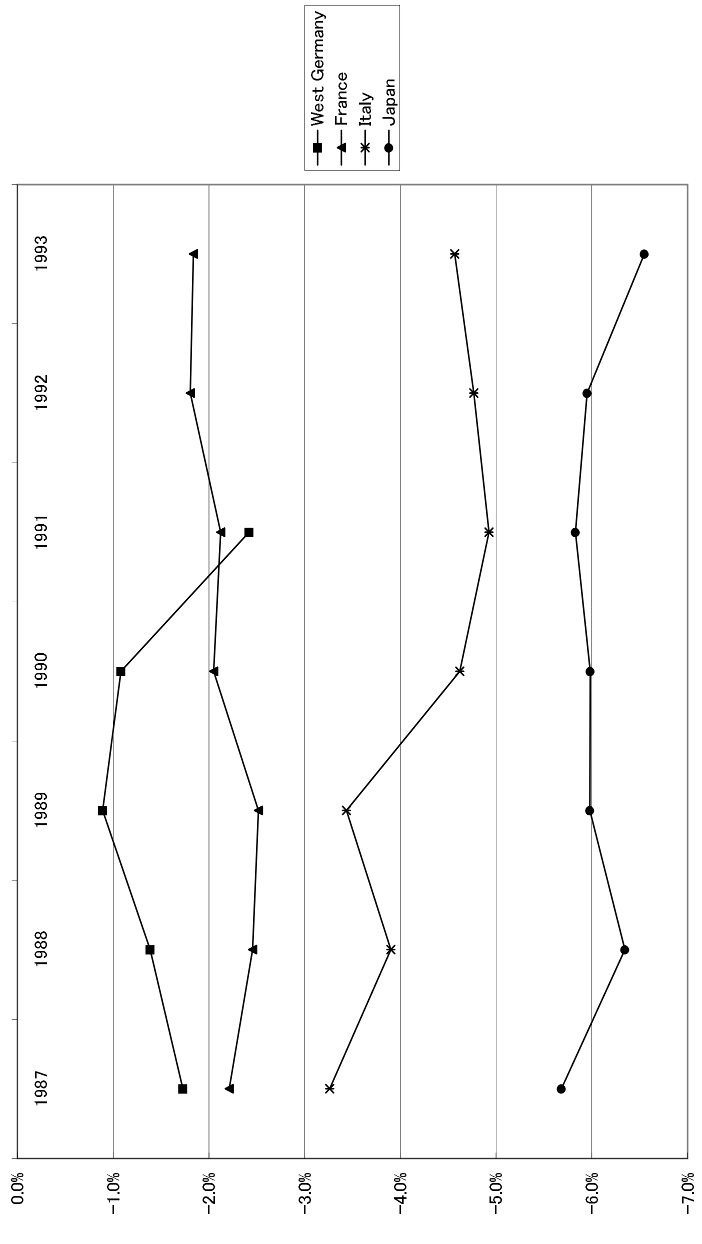

Figure 5: Capital RTAE, RTAE_K of West Germany (before 1991), France, Italy, and Japan relative to the US.

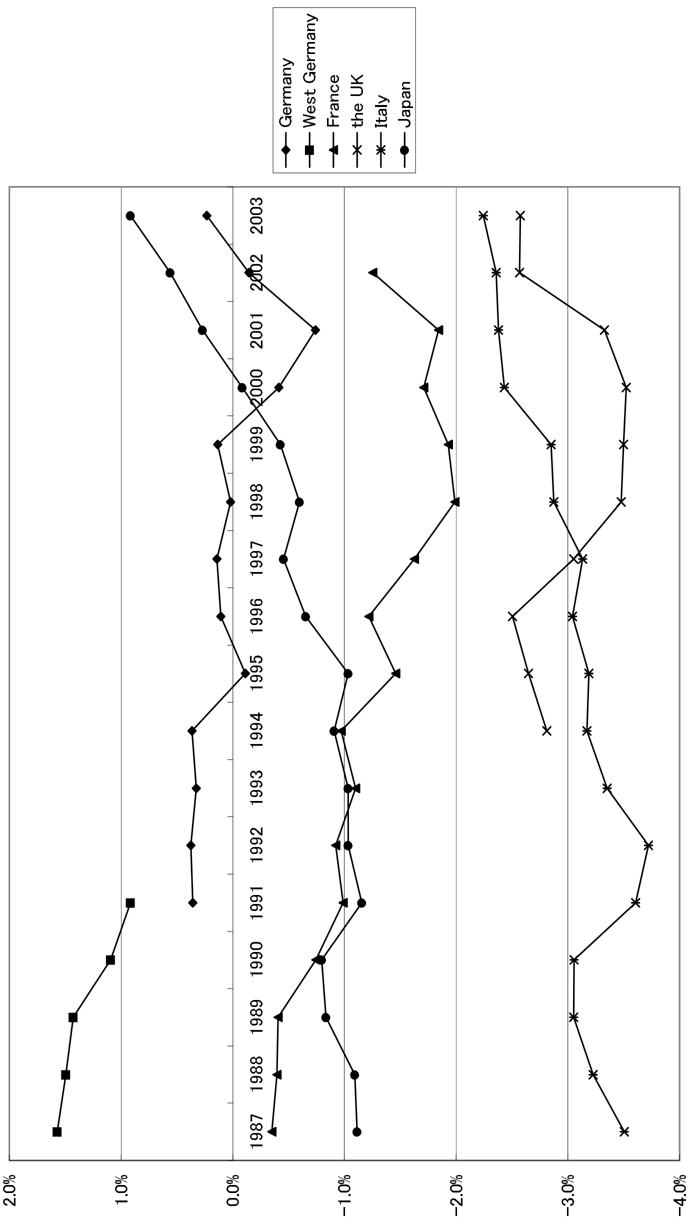

Figure 6: Labor RTAE, RTAE_L of West Germany (before 1991), Germany (from 1991), France, Italy, Japan, and the UK relative to the US.