Security Provision and Exchange Systems

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Abstract

The workforce of commerce may shift from peddlers to shop keepers as security measures become available. The author constructs a model of a decentralized exchange economy with the risk of raiding by armed groups to study such a transition. Merchants have signboards as advertising instruments, by which they attract the attention of both other merchants and armed groups. In an equilibrium, one of the armed groups expels others and become a governor to increase its income. The provision of security, on the other hand, makes trade with advertisements a more reasonable system for merchants to utilize. The author derives the existence conditions of equilibrium which highlight the difficulty of this transition: a required range for the level of power of the governor. A redistribution of taxation rights mitigates this difficulty.

JEL Classification Number : D60, D83, E42
Key words : security provision, exchange system, armed group, confiscation, governor.

1. Introduction

> "Unless the government was both capable and willing to defend its neutrality and to enforce law and impartial justice, foreign merchants had to avoid places occupied by military power." — Polanyi, K. (1966), "Dahomey and the Slave Trade".

Regional peace has been essential for the emergence of periodic markets with settled shops. This prospect itself is based on two historical tendencies. First, periodic markets appeared only in regions where some armed authority provided security. Klengel (1983) describes Godin-Tepe relics where he found a trade center surrounded by a stronghold, which suggests the necessity of an armed force for commercial activities. Amino (1987) desrcibes marketplaces in the middle ages of Japan as independent and religiously authorized areas with assured local security.[2] Second, in a periodic market there usually appeared a type of shop. In Japan, at Heijo-kyo in the 8th century, some laws on market trade described conditions for setting up shops in official periodic markets (Nakamura (2005)). These historical facts endow the author with an insight on the evolution of monetary exchange system.

As can be seen above, the workforce of commerce may shift from peddlers to shop keepers as security measures become available. The tradeoff between the efficiency and sustainability of exchange systems is key to understanding this transition. Suppose that peddlers are faced with the risk of confiscation by several armed groups. They want enhanced security in order to utilize shops as a more efficient exchange system, which makes them more vulnerable to confiscation than the current system. One of the armed groups, on the other hand, may expel others and provide the peddlers with security. It exercises such a state-making instrument for its own income gain. Under the state-provided security, peddlers voluntarily settle down to become shop keepers.

The author constructs a model of a decentralized exchange economy with the risk of confiscation by armed groups in order to study the impact of security on merchants' behavior. The model

[1]The author appreciates the constructive advice of Akihiko Matsui. The author also wishes to acknowledge Toshihiro Matsumura and Hidehiko Ishihara for creative comments, and Roger Smith for careful reviewing. The remaining errors are the author's own.

[2]In Japan, periodic markets spread in the nation when a military government supplied nationwide security. More generally, sustainable peace was important as a background for the evolution of periodic trade. See Polanyi (1966) for Guinea in 17-18 century, Rostovzeff (1932, 1957) for Palmyra in the Roman Empire, Steensgaard (1973) for South-east Asia of 17th century, and Ishihara (1987) for geographic study of ancient Japan.

has a random matching market where the merchants search for their trading partners. The market is subject to friction in matching. Multiple armed groups rove in the economy to occasionally confiscate commodities from merchants.

Merchants hold signboards as advertising instruments, by which they attract the attention of both other merchants and armed groups. Whether they search or advertise is their choice. By hoisting a signboard and setting up a shop, each merchant advertises his/her inventories and the goods that he/she wants. However, the signboard also attracts armed groups.

One of the armed groups, on the other hand, may monopolize the economy by providing the market with security by expeling other groups at an additional cost. This provider of security becomes a governor and collects tax from the merchants. The security provision itself, however, incurs additional manpower cost for the governor. Therefore, the governor emerges only if he/she can expect more revenue from taxation than from raiding.

If the total population of armed groups as a whole is too small, no governor will emerge. This is because, if each armed group can confiscate as much as possible even when all of the armed groups raid the economy, the candidate group for governor has no incentive to monopolize. In other words, he/she will gain no additional revenue by paying the cost of security provision. In the ungoverned area the merchants keep peddling.

If the population of armed groups is relatively large, there is an equilibrium where one of the armed groups expels the others and becomes a governor. This equilibrium stands if the merchants use signboards only when the governor provides security. The governing armed group increases its income by taxation, since it can more easily administrate (and tax) the shop keepers holding signboards than it can peddlers. The provision of security, on the other hand, makes the shops more convenient tools for the merchants. Furthermore, fiat money is superior to commodity money with respect to welfare performance in the shops. This is because the shops-utilizing market lacks search externality, which relatively enhances the commodity money's welfare performance.

The author explicitly derives two kinds of conditions that highlight the difficulty of endogenous security provision. First, the security provision must be necessary as a precondition for merchants to utilize the shops. Second, the power level of the governor must be in an appropriate range: not too small and not overwhelmingly large.

A political adjustment on taxation mitigates the difficulty of transition. If the governor and its nations (merchants) can collaborate to adjust the tax rate into an appropriate range, the possibility of the security provision and the transition of the workforce does not depend on the level of the governor's power. Non-economic motivations for the governor also contribute to the stability of the state.

This paper is related to both the politics of state making and the economic decentralized market models. Olson (1993) points out the possibility of endogenous security provision by armed bandits, which may occur as a solution to the tragedy of commons.[3] The paper depicts an improvement of an exchange system triggered by the provision of security. Burdett, Shi, and Wright (2001) pioneeringly modelize a directed search process: trade utilizing signboards. Also, the shops in the model are derived from the marketplaces of Matsui and Shimizu (2005). While their focus is on a fiat money economy, my model allows for the existence of commodity money. Focusing on shops as market infrastructures, Clower and Howitt (2000) and Howitt (2005) construct a shopping market model as a generalization of the trading posts model by Shapley and Shubik (1987). On institutional change, Greif (1998, 2006) and Greif and Laitin (2004) present a description of endogenous institutional transition through an evolutionary framework. The author's main interest is in the security provision's impact on the economic environment, rather than evolutionary

[3]For a survey on this topic, see McGuire and Olson (1996). The tragedy of commons is defined and explained in Hardin (1968).

processes.

The organization of the paper is as follows. In section 2 the author describes the mathematical features of the model. Section 3 examines stationary equilibria and their welfare results. In section 4 the author discusses two topics: the importance of taxation right re-allocation and some interdisciplinary annotations on the results. Section 5 concludes the paper with a few comments on future research.

2. Model

2.1. Environment

2.1.1. Productive agents and physical environment

There are three types of continuum of agents, each type of which have a measure of 1. The agents are infinitely lived. For all i in $\{1, 2, 3\}$, a type i agent can produce one unit of an indivisible good of type $i + 1$ ($i + 1 = 1$ when $i = 3$) and consume good of type i. Each type i agent gains utility of u when he/she consumes one unit of type i good. There is also one commodity without any intrinsic value, which is fiat money. The fiat money is also indivisible and denoted as a type f good. The aggregate amount of type f goods is Π. The author defines the set of inventory types $I = \{0, 1, 2, 3, f\}$.

There are some assumptions on carrying. Each agent can hold only one unit of good at a time. The author assumes free disposal. When an agent carries type $1, 2,$ or 3 good to some trading place, it incurres an inventory cost. Each type of agent has a special technique to preserve his/her own production good. When a type i agent carries type $i+1$ good, it costs a utility of w (≥ 0) per period for him/her. If a type i agent carries type j good ($j \neq i + 1$), it costs utility of w_+ ($\geq w$) per period. Type f good incurres no inventory cost. The production cost is normalized to be zero for simplification.

Time is discretely infinite. Let $T = \{1, 2, 3, ...\}$ be the set of period numbers. Among four types of goods, only good 1 and good f is storable. Type 2 and type 3 goods rot away at the end of each period.[4]

2.1.2. Market structure : search, advertisement, and outside

2.1.2.1. Search market

As a basic structure, the economy has a pairwise random matching market a la Kiyotaki and Wright (1989). During each period, the agents wandering in the market pairwise-randomly match with each other. The search intensity is 1; that is, each wandering agent definitely matches with someone. Whom he/she matches with is purely random. Suppose there are size Z of agents in the market and Z_{ij} of them are type i agents holding type j goods. Each agent in the market matches with a type i agent holding type j good with probability of $\frac{Z_{ij}}{Z}$.

2.1.2.2. Advertising and shops

Each agent may hoist a signboard in order to advertise his/her inventory and desired goods. An agent who holds a type k good can hoist a signboard (k, A). A is a subset of $\{1, 2, 3, f\}$ which he/she arbitrarily selects. The selling signal k indicates his/her inventory which he/she sells. The acceptance signal A indicates what types of goods he/she accepts in exchange for his/her inventory. An agent who has nothing cannot hoist a signboard.[5] Utilizing a signboard, the agent constructs his/her small shop. Agents without signboards can visit those shops, instead of randomly searching for their partners.[6]

The shop-keepers match with only visitors holding their desired goods. In other words, each

[4] The readers will find that type 1 good is the only candidate for commodity money.
[5] This assumption precludes possibilities of frauds.
[6] For another search theoretic model with a signboard technology, see Iwai (1996).

agent with a signboard screens his/her visitors with respect to their inventories. The criterion of the screening appears in his/her acceptance signal A. Shops with the same signs gather to construct a shopping zone, where the shop-keepers and visitors are pariwise-randomly matched. The shop-keepers, hoisting the signboards, cannot move from their zone but wait for visitors. For an arbitrary zone (k, A), only the shop-keepers holding type k goods and visitors holding the desired types of goods defined in A can enter there. Matchings between the keepers and the visitors are efficient.

When either of the shop-keepers or the visitors are smaller in their size than their counterparts, there occurs efficient rationing. Suppose there are size Z_k of the shop-keepers and size Z_A of the visitors in a shopping zone (k, A). Then each of the shop-keepers match with one of the visitors (randomly chosen) with probability of $\min(Z_A/Z_k,\ 1)$.

2.1.2.3. Outside

In each period, the agents without inventory have to stay in a space called outside, where the agents cannot do anything but consume their consumable inventories (if any) and wait for the next period to come. Note that those who are outside never exhibit any kind of crowding out effect of the market, since the outside always isolates them from the market. Readers may regard the outside as a representation of the agents' home.

2.1.2.4. Locating and matching

Let L be the set of locating actions, $L = \{o,\ m,\ \{(k, A)\},\ \{a(k, A)\},\ \ (k \in \{1, 2, 3, f\},\ A \subset \{1, 2, 3, f\})\}$. A locating action indicates a way in which an agent looks for his/her trading partner. o implies staying in the outside for a period. m indicates trying random matching with those similarly wandering in the market. An arbitrary (k, A) indicates hoisting a signboard of (k, A). If A is a singleton, that is, $A = \{j\}$ $(j \in \{1, 2, 3, f\})$, the author denotes the shopping zone (k, A) simply as (k, j) (not $(k, \{j\})$). $a(k, A)$ implies aiming for the shopping zone (k, A); in other words, becoming a visitor to the zone.

Each agent must take one and only one locating action in one period. Those who aim for some shopping zone cannot try the random matching process in the same period, and vice versa. Those who choose to stay in the outside cannot return to the market in the same period. By this assumption, no agent gains multiple opportunities of trade in one period.

2.2. Game structure : players and events flow

There are two kinds of players in the model: agents and multiple tribes of bandits. The agents are merchants who produce commodities by themselves. The tribes of bandits are groups who confiscate commodities from the agents. During each period, each player (an agent or a tribe) behaves to maximize his/her objective function. There sequentially occur four kinds of events in one period: production, confiscation, trade, and consumption.

2.2.1. Agents

Each agent maximizes his/her expected lifetime utility, whose form is as follows:

$$E\left[\sum_{t=1}^{\infty} \beta^{t-1}\{I_u(t)u - I_w(t)w - I_{w_+}(t)w_+\}\right].$$

$I_u(t)$, $I_w(t)$, and $I_{w_+}(t)$ are indication functions of utility gain (u) and inventory costs (w and w_+) in each period t, respectively. β is a common discount factor which satisfies $\beta \in (0, 1)$. E is an expectation operator based on randomness which the agent faces in his/her life.

Take an arbitrary type i of the agents. For maximization, each type i agent controlls four kinds of policies: production policy, locating policy, trading policy, and consumption policy. The author sequentially explains these policies below.

Production policy $g_i : \{0, 1, f\} \times T \to \{0, 1\}$ determines whether he/she produces his/her production good at the beginning of each period, depending on his/her inventory. $g_i(j, t) = 1$ indicates he/she produces when his/her inventory is j. If $g_i(j, t) = 0$, he/she does not produce anything. Due to inventory constraints, if he/she produces, he/she has to dispose his/her former inventory.

Locating policy $l_i : \{1, f, i+1\} \times T \to \mathcal{L}$ specifies how he/she looks for his/her trading partner. His/her locating action depends on his/her inventory after the production opportunity and period number t. After the agents start locating activities and before trades occur, bandits raid them. Details of such raiding is found in 2.2.2.

Trading policy $\tau_i : \{1, f, i+1\} \times I \times T \to \{0, 1\}$ determines whether he/she accepts a trading opportunity he/she meets in each period. $\tau_i(j, k, t) = 1$ indicates that the type i agent gives away his/her type j good in exchange for type k good. If $\tau_i(j, k, t) = 0$, he/she does not exercise that exchange.

Consumption policy $e_i : T \to \{0, 1\}$ determines whether he/she consumes his/her consumable good when he/she has one at the end of a period. If $e_i(t) = 1$ he/she consumes it, and if $e_i(t) = 0$ he/she does not do so.

Let $[g_i]$, $[l_i]$, $[\tau_i]$, $[e_i]$ be, respectively, the sets of functions g_i, l_i, τ_i, e_i. The author utilizes these notations when he describes transition of the agents' inventories.

Finally, The author assumes a kind of inertia in the agents' behavior. That is, each agent changes his/her inventory only when his/her lifetime utility strictly increases by that change.

2.2.2. Bandits

Multiple tribes of bandits are roving in the market. The number of tribes, B, is a finite positive integer ($B \geq 2$). Each tribe has a number as its name, which is one of $\mathcal{B} = \{1, 2, ..., B\}$. After the agents' production, when they start locating their partners, the tribes raid them as long as there is not a governor (which the author describes in 2.2.2.1.). The tribes' targets are the commodities, namely goods of types $1, 2$ and 3. They simply ignore fiat monies since they simply confiscate the commodities rather than use fiat money for some exchange.

Tribe b has its coverage over the economy, $c_b \in (0, 1)$. Coverage is a representation of a tribe's size. When tribe b raids the agents, each agent whose current locating action is m meets a member of the tribe b with probability of c_b. The author defines the market peace rate p as $p \equiv 1 - c = 1 - \sum_{b=1}^{B} c_b$ and assumes that $c < 1$. If an agent's current locating action is in one of $\{(k, A)\}$ or $\{a(k, A)\}$, on the other hand, he/she meets a tribe b bandit with probability of $\min(1, Dc_b)$, where the detection coefficient D satisfies $D > 1$. The author defines the shop peace rate p' as $p' \equiv \max(0, 1 - Dc)$. If $Dc > 1$ and all the tribes raid on, all the agents in the shopping zones meet bandits. In that case, the tribe b gains $\frac{c_b}{c}$ of the commodities in the shopping zones.

The tribe b's raiding strategy is $r_b \in \{0, 1\}$. $r_b = 1$ indicates raiding and $r_b = 0$ indicates quitting. Then, without a governor, an agent in the random matching process (*resp.* the shopping zones) meets a bandit with probability of $\sum_{b \in \mathcal{B}} r_b c_b$ (*resp.* $\sum_{b \in \mathcal{B}} D r_b c_b$). The author assumes myopicity of the tribes with their objective functions: amounts of confiscation. The tribe b's amount of confiscation at an arbitrary period is:

$$\int_0^3 I_C^b(s) ds$$

$I_C^b(s)$ is an indication function of the agents who have their holding commodities confiscated by the bandits of tribe b.

Each tribe maximizes the amount of commodities it yields during the current period. Their myopicity expresses a situation where a severe degree of the tragedy of the commons occurs. The objective function implies that each tribe of bandits confiscates as much commodities as possible from the economy. In practice, each tribe b chooses $r_b = 1$ at any time, since its coverage is useless for anything else but raiding.

After the raiding, tribes bring their commodities back to the outside to consume them. The author assumes that the tribes are omnivorous, namely they consume all of type $1, 2$, and 3 goods.[7]

2.2.2.1. Settlement

Let the author define the largest and the second-largest tribe of bandits, b^* and b', respectively. The largest tribe b^* has the largest coverage of all, $b^* \in \arg\max_b\{c_b | c_b \in \{c_1, c_2, ..., c_B\}\}$. The author assumes that b^* is unique. Then, $b' \in \arg\max_b\{c_b | c_b \in \{c_1, c_2, ..., c_B\}, c_b < c_{b^*}\}$.

It is assumed that the largest tribe b^* can settle in the market as a governor by paying an additional cost. Tribe b^* decides on settlement strategy $s_{b^*} \in \{0, 1\}$ each period. If $s_{b^*} = 0$, it simply conducts raids in the economy, similarly as other tribes. If $s_{b^*} = 1$, it settles in the trading places instead of raiding.

If tribe b^* settles in, it has to supply security for the market as a whole. In other words, it has to exclude other tribes from the market. It dispatches some of its bandits, namely $c_{b'}$ of its coverage, as guards for the market. The other tribes cannot enter the market during the governor's security provision. Due to the security provision, its coverage decreases to $c_{b^*} - c_{b'}$. Let the author name $c_{b^*} - c_{b'}$ as remaining coverage.

By settling in, the largest tribe becomes a governor who collects tax from the agents. It dispatches its officers, former tribe b^* bandits, into the market in order to collect tax from the agents. The governor can dispatch only its remaining coverage, $c_{b^*} - c_{b'}$. The governor's objective function is the same as that of the largest tribe.[8]

The rule of encounter is the same as before. Each agent in *the random matching process* meets an officer with probability of $c_{b^*} - c_{b'}$. If he/she is in one of *the shopping zones*, he/she meets an officer with probability of $\min[1, D(c_{b^*} - c_{b'})]$. Each officer who meets an agent confiscates that agent's holding commodity, if any.[9] Still the governor does not confiscate fiat money, since it can confiscate commodities without money.

2.2.3. State of the economy

The author defines a state of the economy at period t, $P(t)$. It is a populational distribution of the agents with respect to their types and inventories just after their production opportunities and *before* the bandits' raiding or the governor's confiscation. The mathematical description is as follows:

$$P(t) = \{P_{ij}(t)\}_{(i,j)\in\{1,2,3\}\times I} \quad \text{where} \sum_{j\in I} P_{ij}(t) = 1 \ (i = 1, 2, 3).$$

$P_{ij}(t)$ is the size of type i agents holding type j inventories just after the production opportunity and before raiding at period t. The state is common knowledge for all the agents.

Note that a state depicts the agents' inventories before the confiscation. The whole of produced/preserved goods are in the agents' hands at the time of measurement. After the measurement of state, some armed group surely confiscates a part of their commodities.

[7] Since the author does not analyze the bandits' welfare, he does not specify their utility gain from consumption.

[8] By the myopicity, the governor dispatches all of the remaining coverage. Some readers may suspect that the expelled tribes may disappear from the economy and therefore the governor does not have to pay the manpower cost. However, if the other tribes are armies from local communities in some periphery of the economy, they may still survive. Klengel (1983) points out that armed lords in the ancient Orient were both governors and plunderers in their regions. Vikings in northern Europe were also the case (see Klitgarrd and Svendsen (2003) for example).

[9] For modelization of tax in this style, see Li and Matsui (2005) or Kamiya and Shimizu (2005).

Additionally, the author defines an agent's state at the measurement time. An agent's state at a period is that agent's inventory type just after the production process and before the confiscation. In short, an agent's state is his/her inventory at the measurement time.

2.2.4. Events flow

At each beginning of period, all the players are in the outside. Each agent decides on his/her production according to his/her production policy g_i $(i = 1, 2, 3)$. Simultaneously the largest tribe b^* decides on its settlement according to s_{b^*}.

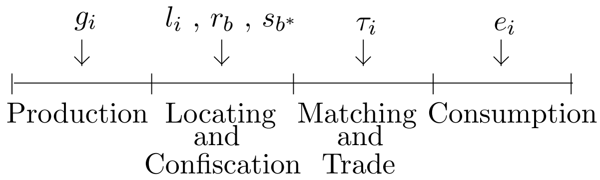
Then each agent's locating policy l_i $(i = 1, 2, 3)$ specifies his/her locating action. During the agents' locating activities, they owe a carrying cost depending on their inventories. Just then, before they trade, the bandits' raiding or the governor's taxation occurs.

Probability of an agent's encounter with a confiscator depends on the tribes' raiding policies $\{r_b\}_{b \in \mathcal{B}}$ and the largest tribe's settling policy s_{b^*}. The explicit calculation rule of the probabilities is in 2.2.3. Note that the myopic tribes will surely conduct raids on the economy. If an agent loses his/her inventory due to the confiscation, he/she goes back to the outside.

Remaining agents in the market randomly match with or be rationed to each other. Once matched, each matched pair of the agents decide on their trade according to their trading policies τ_i $(i = 1, 2, 3)$. Only when both of them agree to trade, the trade realizes.

After the trading process, the agents go back to the outside.[10] Those who have their consumable goods as inventories can consume the goods. Their consumption decisions depend on their consumption policy e_i $(i = 1, 2, 3)$. When the consumption opportunity ends, current period also ends and the next period immediately begins.

Events per period



3. Analysis

3.1. Equilibrium and welfare

3.1.1. Equilibrium notion

Through the analysis the author focuses only on pure strategies. The following is the definition of non-autarky symmetric stationary equilibrium.

Definition: *A profile* $< \{g_i\}_{i\in\{1,2,3\}},\ \{l_i\}_{i\in\{1,2,3\}},\ \{\tau_i\}_{i\in\{1,2,3\}},\ \{e_i\}_{i\in\{1,2,3\}},\ \{r_b\}_{b\in\mathcal{B}},\ s_{b^*},\ P >$ *is a non-autarky symmetric stationary equilibrium if it satisfies the following conditions.*
(1) Each agent consumes his/her consumable good in the long-run with a strictly positive probability. (non-autarky)
(2) Every type i agent takes the same strategy profile $\{g_i,\ l_i,\ \tau_i,\ e_i\}$, $i \in \{1, 2, 3\}$. *(symmetry)*
(3) All the strategies are independent of $t \in T$. *(stationarity)*
(4) The state of the economy P *remains constant across time. (stationarity)*
(5) A strategy profile $\{g_i,\ l_i,\ \tau_i,\ e_i\}$ *maximizes each type i agent's expected lifetime utility.*
(6) For all $b \neq b^*$, r_b *maximizes the tribe b's amount of confiscation.* r_{b^*} *and* s_{b^*} *maximize tribe* b^**'s amount of confiscation.*

By symmetry, the random matching process and the shopping processes cannot coexist. Henceforth the author omits notation of time t from all of the equilibrium strategies.

[10] The author assumes their return to home incurrs no carrying cost.

Let the set of profiles $< \{g_i\}_{i\in\{1,2,3\}},\ \{l_i\}_{i\in\{1,2,3\}},\ \{\tau_i\}_{i\in\{1,2,3\}},\ \{e_i\}_{i\in\{1,2,3\}},\ \{r_b\}_{b\in\mathcal{B}},\ s_{b^*},\ P >$ be $\mathcal{Q} \equiv \{Q\}$. For convenience of the analysis, the author defines transition functions, transition probabilities, and value functions.

Definition: *A type i agent's transition function is $\xi_i : I \times \mathcal{Q} \times [g_i] \times [l_i] \times [\tau_i] \times [e_i] \to I$. ξ_i indicates what his/her state will be at the next period, depending on his/her current state, state of the economy, and his/her strategy profile.*

Henceforth, the author denotes a type i agent's strategy profile $\{g_i, l_i, \tau_i, e_i\}$ as ψ_i, with $[\psi_i]$ as the set of such profiles. Then an equilibrium profile $Q \in \mathcal{Q}$ is $Q =< \{\psi_i\}_{i\in\{1,2,3\}},\ \{r_b\}_{b\in\mathcal{B}},\ s_{b^*},\ P >$.

Note that the value of ξ_i is a random variable. Therefore, the author defines probabilities of shifts in an agent's state; that is, transition probabilities.

Definition: *Given an equilibrium profile Q, a type i agent with state j has transition probabilities $\{x_Q(i,j,k,\psi_i')\}_{k\in I}$, which satisfy $x_Q(i,j,k,\psi_i') = Prob\{\xi_i(j,Q,\psi_i') = k\}$ and $\sum_{k\in I} x_Q(i,j,k,\psi_i') = 1$. $\psi_i' = \{g_i', l_i', \tau_i', e_i'\}$ is the agent's individual strategy profile.*

The equilibrium state of the economy P must satisfy $\forall(i,j) \in I\times I,\ P_{ij} = \sum_{k\in I} P_{ik} x_Q(i,k,j,\psi_i)$.

With the transition probabilities, the author defines a state-wise expectation operator E_Q. For an arbitrary function $a : \{1,2,3\} \times I \to R$, $E_Q(a(i,k)|j,\psi_i') = \sum_{k\in I} x_Q(i,j,k,\psi_i')a(i,k)$.

Furthermore, let the author define a consumption-wise expectation operator E_u somewhat implicitly, to reduce notational complexity. E_u is an exclusive expectation operator for those indication functions I_{iu}'s. I_{iu} is an indication function for a type i agent's consumption in the current period. If he/she consumes his/her consumable, $I_{iu} = 1$. Otherwise, $I_{iu} = 0$. E_u exhibits an expected value of I_{iu} based on the agent's current state j, the equilibrium profile Q, and his/her strategy profile ψ_i'. Namely, $E_u(I_{iu})$ is the probability of his/her success in consumption during the current period.

Now the author defines the value functions.

Definition: *Given an equilibrium profile $Q =< \{\psi_i\}_{i\in\{1,2,3\}},\ \{r_b\}_{b\in\mathcal{B}},\ s_{b^*},\ P >$, the value function for a type i agent at the measurement time of state in a period is $V_i : I \to R$, which satisfies:*

$$V_i(j) = \max_{\psi_i'}[-I_{iw}(j)w + -I_{iw_+}(j)w_+ + E_u\{I_{iu}|j,Q,\psi_i'\}u + \beta E_Q\{V_i(x)|j,\psi_i'\}]$$

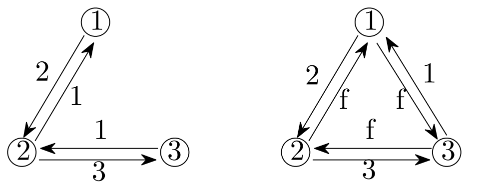
I_{iw} and I_{iw_+} are indication functions which discriminate the commodities from other inventories. $\forall i \in \{1,2,3\}$, $I_{iw}(i+1) = 1$ and $I_{iw}(j) = 0\ \forall j \neq i$. If $j \in \{1,2,3\}\backslash\{i+1\}$, $I_{iw_+}(j) = 1$ and otherwise $I_{iw_+}(j) = 0$.

With the agents' inertia, it must be satisfied that $\forall i \in \{1,2,3\},\ V_i(i+1) > V_i(0) \geq 0$ in each equilibrium, since the agents' production is necessary for non-autarky. Furthermore, with the assumption of free disposal, the agents' incentive compatibility requires that, if $\tau_i(j,k) = 1$ for some i,j,k, either of $(j \neq i \wedge k = i)$ or $(k \in \{1,f\} \wedge V_i(k) > V_i(j))$ must stand.

3.1.2. Equilibria in focus

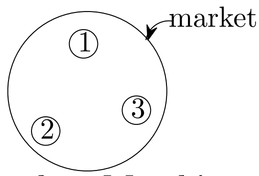
The author analyzes equilibria where there is a unique type of medium of exchange. Then equilibria in focus split into four categories. The categorization is in a two-by-two matrix form. First, as the author noted in the preceding subsection, there are only equilibria where all the agents utilize either of the random matching process or the shopping zones. The author names the former as search equilibria, and the latter as shopping equilibria. Second, there are equilibria with roving banditry ($s_{b^*} = 0$) and those with a governor ($s_{b^*} = 1$). In latter subsections the author analyzes these four kinds of equilibria separately.

Flow of Goods

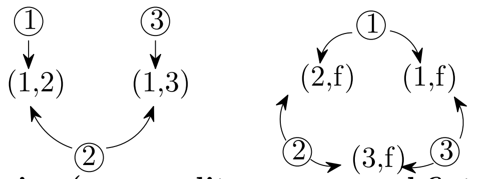


Commodity Money **Fiat Money**

Locating behavior



Random Matching



Shopping (commodity money and fiat money)

3.1.3. Agents' welfare

The agents' welfare is an index of an equilibrium's desirability for the agents. The welfare is measured with utility as its unit. If the agents' welfare is higher in one equilibrium than another, the author assumes that the agents as a group are likely to shift from the latter to the former through some political process.[11]

Because the author analyzes stationary equilibria, it is sufficient to focus on utility flow in one period to evaluate of total agents' welfare. The agent's welfare for an equilibrium Q is:

$$\sum_{i=1}^{3}\sum_{j\in I} P_{ij}[E_u(I_{iu}|j,Q)u - I_{iw}(j)w - I_{iw_+}(j)w_+].$$

3.2. Preparatory analysis

The author derives most of the equilibrium trading policies without specifying an equilibrium. The lemma below summarizes the result.

Lemma 1

Take an arbitrary equilibrium Q.

1. $\forall i \ and \ k,\ \tau_i(k,k) = 0.$
2. $\forall j,k,\ \tau_i(j,k) = 1 \ \Rightarrow \tau_i(k,j) = 0.$
3. $\forall k \neq i,\ \tau_i(k,i) = 1,\ \tau_i(i,k) = 0.$
4. $\tau_3(1,2) = 0.$
5. $\tau_1(2,3) = 0.$
6. $\tau_2(1,3) = 0.$

[11] Along with the results, an emerging governor may provide an opportunity and a place for such a political process. Since the author's interest is political economic one, he does not analyze the bandit's welfare.

Proof : (1) $\tau_i(k,k) = 0$ immediately follows from the inertia of agents. (2) If there exist i, j, k such that $\tau_i(j,k) = 1$, it implies that a type i agent increases his/her expected lifetime utility by that exchange. Then it is clear that $\tau_i(k,j) = 0$. (3) If a type i agent does not accept his/her specialized consumption good, then there exists $k \neq i$ such that $\beta V_i(k) > u + \beta V_i(i+1)$. Since $V_i(i+1) > 0$, $V_i(k)$ should be strictly positive. This implies that he/she consumes his/her consumption good with positive probability in the future. Then the readers will notice that $V_i(k) \leq u + \beta V_i(i+1)$, because he/she does not consume any good at the current period. This incurrs a contradiction such that there exist $\beta_+ \in (0,1)$ which satisfies $\beta_+ V_i(i+1) > V_i(i+1) > 0$. (4) If a type 3 agent accepts good 2 for good 1, he/she cannot consume good 2 and it rots away in that period. Whichever he/she produces or not at the next period, he/she can acquire the same type of inventory at the next period without any trade, since he/she can just dispose of his/her inventory. It leads to $\tau_3(1,2) = 0$. (5) A type 1 agent never accepts good 3, because both of them rot away after the next consumption opportunity. Again, whether he/she trades or not does not affect his/her inventory at the next period. It leads to $\tau_1(2,3) = 0$. (6) Similarly as in (4) and (5), a type 2 agent never accepts good 3 because it rots away at the end of each period. ∎

By the lemma, the author only has to check a fairly small part of the agents' incentive compatibilities to prove the existence of equilibria in focus. Essentially, only values of $\tau_2(3,1)$ and $\tau_i(k,f)$ $(i \in \{1,2,3\},\ k \in \{1, i+1\})$ remain to be determined. $\tau_2(3,1)$ indicates whether type 2 agents mediate the commodity money or not. $\tau_i(k,f)$'s indicate whether fiat money has general acceptability or not.

3.3. Search equilibria

In search equilibria, the agents roam in the market searching for trade partners, without settling in somewhere ($l_i(k) = m,\ k \in \{1, f, i+1\}$). They pairwise randomly match with each other. Intuitively, the situation is such that merchants are wandering in a potential place for market and trading through makeshift stalls which they can quickly clear away. They do not have any predetermined time schedule of market exchange. They can relatively easily escape from the confiscators; on the other hand, however, their matchings inevitably contain some friction.

3.3.1. Search equilibria with roving banditry

Promptly the author proves existence of a commodity money equilibrium.

Proposition 1

If $\frac{p^2\beta}{3(3-p\beta)}u > w + \frac{p\beta}{3-p\beta}(w_+ - w)$, *there exists a commodity money search equilibrium which satisfies* $\tau_2(3,1) = 1,\ \tau_i(k,f) = 0\ \forall k \in \{1, f, i+1\}$. *The equilibrium state satisfies* $P_{12} = P_{31} = 1,\ P_{21} = \frac{p}{3-p},\ P_{23} = \frac{3-2p}{3-p}$.

Proof: When the agents use commodity money, it is clear that $P_{12} = P_{31} = 1$. P_{21} and P_{23} satisfy $\frac{p}{3}P_{21} + (1-p)P_{21} = \frac{p}{3}P_{23}$ since proportion $(1-p)$ of current P_{21} loses their inventories due to the raiding of bandits. It routinely leads to the equilibrum state.

No one accepts fiat money since none of the others will accept it. Then we only have to check whether a type 2 agent has an incentive to accept a type 1 good when he/she has a type 3 good. By holding a type 1 good he/she has a chance to meet a type 1 agent and exchange his/her type 1 good for his/her counterpart's type 2 good. If he/she rejects it, he/she goes into autarky at least for one period. Therefore, if his/her expected utility from such a trade net of expected inventory cost (both appropriately discounted) is strictly positive, he/she will accept a type 1 good.

Practically, the author solves the following equations of value functions:

$$\begin{aligned} V_2(3) &= -w + \frac{p}{3}\beta V_2(1) + (1 - \frac{p}{3})\beta V_2(3), \\ V_2(1) &= -w_+ + \frac{p}{3}(u + \beta V_2(3)) + \frac{2p}{3}\beta V_2(1) + (1-p)\beta V_2(3). \end{aligned}$$

The author derives that $V_2(1) > V_2(3) = \frac{1}{1-\beta}\left(-w - \frac{p\beta}{3-p\beta}(w_+ - w) + \frac{p^2\beta}{3(3-p\beta)}u\right) > 0$ if $\frac{p^2\beta}{3\{3-p\beta\}}u > w + \frac{p\beta}{3-p\beta}(w_+ - w)$. Participation conditions of type 1 and 3 agents prove to be more loose than type 2 agents. ∎

Next the author proves the existence of fiat money search equilibriua.

Proposition 2

For an arbitrary $\pi \in (0,1)$, *if* $\frac{\pi(1-\pi)p^2\beta}{3\{p+(1-p)\pi\}[3\{p+(1-p)\pi\}-\{2p+(3-4p)\pi\}\beta]}u > w$ *and* $w_+ - w > \frac{(1-\pi)p^2}{3\{\pi+(1-\pi)p\}}u$, *there exists a fiat money search equilibrium which satisfies* $\tau_2(3,1) = 0,\ \tau_i(k,f) = 1\ \forall k \in \{1, i+1\}$. *The equilibrium state satisfies* $P_{12} = P_{23} = P_{31} = 1-\pi,\ P_{1f} = P_{2f} = P_{3f} = \pi,\ \Pi = 3\pi$.

Proof: The equilibrium state must satisfy $P_{12}P_{1f} = P_{23}P_{2f} = P_{31}P_{3f}$. The equilibrium state clearly satisfies this condition.

As long as $\tau_i(k,f) = 1$ for all $i \in \{1,2,3\}$ and $k \in \{1, i+1\}$, the general acceptability of fiat money induces the agents individually accept it. Since fiat money has no carrying cost, the agents prefer it to commodity money. Therefore the agents may trade using fiat money if they gain positive value from that trade. The author sets up simultaneous equations of value functions as follows:

$$\begin{aligned} V_2(3) &= -w + \frac{\pi p}{3\{\pi+(1-\pi)p\}}\beta V_2(f) + \left[1 - \frac{\pi p}{3\{\pi+(1-\pi)p\}}\right]\beta V_2(3), \\ V_2(f) &= \frac{(1-\pi)p}{3\{\pi+(1-\pi)p\}}\{u + \beta V_2(3)\} + \left[1 - \frac{(1-\pi)p}{3\{\pi+(1-\pi)p\}}\right]\beta V_2(f). \end{aligned}$$

The author/reader can derive:

$$\begin{aligned} V_2(f) > V_2(3) &= \frac{1}{1-\beta}\left[-\frac{3\{p+(1-p)\pi\} - \{2p+(3-4p)\pi\}\beta}{3\{p+(1-p)\pi\} - \{2p+(3-3p)\pi\}\beta}w \right. \\ &\qquad \left. + \frac{\pi(1-\pi)p^2\beta}{3\{p+(1-p)\pi\}[3\{p+(1-p)\pi\} - \{2p+(3-3p)\pi\}\beta]}u\right]. \end{aligned}$$

$V_2(3)$ is positive if $\frac{\pi(1-\pi)p^2\beta}{3\{p+(1-p)\pi\}[3\{p+(1-p)\pi\}-\{2p+(3-4p)\pi\}\beta]}u > w$. By symmmetry, participation conditions of type 1 and 3 agents are the same to this condition.

With respect to a fiat money search equilibrium, the author also has to verify that each type 2 agent *does not* have an incentive to mediate the commodity money. The author again constructs simultaneous equations, this time for a type 2 agent who deviates to mediate the commodity money.

$$\begin{aligned} V_2(3) &= -w + \frac{\pi p}{3\{\pi+(1-\pi)p\}}\beta V_2(f) + \left[1 - \frac{\pi p}{3\{\pi+(1-\pi)p\}}\right]\beta V_2(3), \\ \bar{V}_2(1) &= -w_+ + \frac{\pi p}{3\{\pi+(1-\pi)p\}}\beta V_2(f) + \frac{(1-\pi)p^2}{3\{\pi+(1-\pi)p\}}\{u + \beta V_2(3)\} \\ &\quad + p \cdot \frac{2}{3}\beta \bar{V}_2(1) + (1-p)\beta V_2(3). \end{aligned}$$

$V_2(3)$ still indicates the equilibrium value of holding type 3 good for a type 2 agent. $\bar{V}_2(1)$ is the value of deviation, namely a type 2 agent's value of holding type 1 good once and for all. If $V_2(3) > \bar{V}_2(1)$ stands, clearly a type 2 agent does not accept the commodity money. The author solves the inequality to derive $w_+ - w > \frac{(1-\pi)p^2}{3\{\pi+(1-\pi)p\}}u$. ∎

The fiat money search equilibrium exists when a type 1 good incurs substantially large carrying cost for type 2 agents. The cost of mediation, w_+, must be sufficiently larger than w.

As a preparation for welfare comparison in 3.5., the author evaluates the agents' welfare.

Lemma 2

Let W_{mc} and $W_{mf}(\pi)$ be the largest levels of the agents' welfares of the commodity money search equilibrium and the fiat money search equilibrium with $P_{if} = \pi$, respectively. Then

$$W_{mc} = \frac{p}{3-p}u - 3w - \frac{p}{3-p}(w_+ - w), \quad \bar{W}_{mf} \equiv \frac{p}{(1+\sqrt{p})^2}u \geq W_{mf}(\pi).$$

Proof: $W_{mc} = \frac{p}{3}(P_{12}P_{21} + P_{21}P_{12} + P_{31}P_{23})u - (3-P_{21})w - P_{21}w_+ = \frac{p}{3}(1+P_{21})u - (3-P_{21})w - P_{21}w_+$, where $P_{21} = \frac{p}{3-p}$. Straightforwardly, the author derives $W_{mc} = \frac{p}{3-p}u - 3w - \frac{p}{3-p}(w_+ - w)$. $W_{mf}(\pi) = \sum_{i=1}^{3} \pi \cdot \frac{1}{3}\frac{(1-\pi)p}{\pi+(1-\pi)p}u - \sum_{i=1}^{3}(1-\pi)w = \frac{\pi(1-\pi)p}{\pi+(1-\pi)p}u - 3(1-\pi)w \leq \frac{\pi(1-\pi)p}{\pi+(1-\pi)p}u$. Considering maximization of $\frac{\pi(1-\pi)p}{\pi+(1-\pi)p}u$ with respect to π, the author derives $\frac{p}{(1+\sqrt{p})^2}u \geq \frac{\pi(1-\pi)p}{\pi+(1-\pi)p}u$. ∎

3.3.2. Search equilibrium with governor

As some readers may expect, as long as the agents utilize the random matching process, the largest tribe b^* never settles in, and therefore there is no search equilibrium with a governor. The author shows the non-existence below.

Proposition 3

There is no search equilibrium with a governor.

Proof: Suppose that the agents utilize the market ($\forall i \in \{1,2,3\},\ \forall k \in \{1, f, i+1\},\ l_i(k) = m$) and the largest tribe settles in ($s_{b^*} = 1$). Then the governor confiscates $(c_{b^*} - c_{b'})(P_{12} + P_{21} + P_{23} + P_{31})$ of the commodities. If the governor deviates to $s_{b^*} = 0$, since $c < 1$, he/she exploits $c_{b^*}(P_{12} + P_{21} + P_{23} + P_{31})$ of the commodities. Clearly the latter is larger than the former. And therefore, it surely deviates to $s_{b^*} = 0$. ∎

The proposition depicts an essential difficulty in the security provision. If the tribes of bandits can confiscate from the economy as much as possible leaving residual goods in the economy, none of them wants to be a governor paying the manpower cost. The cost purely reduces its power of confiscation, without any kind of gain.

Therefore, for a governor to appear, there *has to be* some "encompassing interest" for the largest tribe guarding the market from the other tribes. The interest is not only important but also necessary. Especially when the tribe does not consider future events well, as in the model, such an interest must be realized quickly in order to attract the tribe.

3.4. Shopping equilibria

In the shopping equilibria, agents use signboards to advertise their profiles of demand and supply. In order to make the signboards conspicuous, they stay in a kind of (temporary) booth. Although such an advertisement system precludes friction from the market, a set of conspicuous signboards is also a convenient target for bandits. The author basically analyzes those shopping equilibria with unique media of exchange stated in 3.1.2.

3.4.1. Shopping equilibria with roving banditry

As entioned earlier, the peace rate in the shops is $p' \equiv 1 - Dc$. Unlike search equilibria, there is no guarantee that the rate is strictly positive. Therefore, the author has to assume $D < \frac{1}{c}$ to discuss shopping equilibria with roving banditry.

Assuming the fundamentally necessary condition, the author proves existence of shopping equilibria, one with commodity money and another with fiat money.

Proposition 4

There conditionally exist two kinds of shopping equilibria with roving banditry.

i) *If* $\frac{p'^2\beta}{1+p'\beta}u > w + \frac{p'\beta}{1+p'\beta}(w_+ - w)$, *there exists a commodity money shopping equilibrium, which satisfies:*

$l_3(1) = a(3,1), l_2(3) = (3,1),\ l_2(1) = a(2,1), l_1(2) = (2,1),$

$\tau_2(3,1) = 1,\ \tau_i(k,f) = 0, \tau_i(f,l) = 0\ \forall i \in \{1,2,3\}, \forall k \in \{1,f,i+1\}, i \neq l,$

$P_{31} = P_{12} = 1,\ P_{23} = \frac{1}{1+p'},\ P_{21} = \frac{p'}{1+p'}.$

ii) *If* $p'\beta u > w$ *there exists a fiat money shopping equilibrium, which satisfies:*

$l_3(1) = (1,f), l_1(f) = a(1-f),\ l_1(2) = (2,f), l_2(f) = a(2,f),\ l_2(3) = (3,f), l_3(f) = a(3,f),$

$\tau_2(3,1) = 0,\ \tau_i(k,f) = 1, \tau_i(f,l) = 0\ \forall i \in \{1,2,3\}, \forall k \in \{1,f,i+1\}, i \neq l,$

$\Pi = \frac{3p'}{1+p'},\ P_{12} = P_{23} = P_{31} = \frac{1}{1+p'},\ P_{1f} = P_{2f} = P_{3f} = \frac{p'}{1+p'}.$

Proof:

i) Similar to the proof of proposition 1, noting that a proportion $(1-p)$ of P_{21} flows into P_{23}, the reader can confirm the equilibrium state of the economy. On the equilibrium path, every type 2 agent remaining in the shops after the raiding succeeds in his/her trade.

Note that shops $(1,2)$ and $(1,3)$ are the only shopping zones where positive measure of the agents enter. Then a deviator from the equilibrium strategies necessarily falls into autarky at least for one period. If he/she deviates in his/her location policy, he/she comes to enter a place where there is none of good-holders. He/she has to hold his/her inventory at least for one period. If he/she deviates in his/her trading policy, again he/she has to keep his/her inventory at least for one period. And if he/she wants to trade successfully, he/she has to go back to the equilibrium strategy.

Therefore the strategies in i) of the proposition is a stationary equilibrium if all the values for the agents are positive. The reader can show that a type 2 agent's incentive compatibility condition is the strictest. The author explicitly checks the condition using value functions $V_2(3)$ and $V_2(1)$.

$$\begin{aligned} V_2(3) &= -w + p'\beta V_2(1) + (1-p')\beta V_2(3), \\ V_2(1) &= -w_+ + p'(u + \beta V_2(3)) + (1-p')\beta V_2(3). \end{aligned}$$

Note that in the shops there is no friction with respect to matching. The author solves for $V_2(3)$; $V_2(1) > V_2(3) = \frac{1}{1-\beta}\left\{-w - \frac{p'\beta}{1+p'\beta}(w_+ - w) + \frac{p'^2\beta}{1+p'\beta}u\right\} > 0$ if $\frac{p'^2\beta}{1+p'\beta}u > w + \frac{p'\beta}{1+p'\beta}(w_+ - w)$.

ii) $P_{i,i+1} = \frac{1}{1+p'}$ implies $p'P_{i,i+1} = P_{if}$. Then during every period, every fiat money holder successfully gains his/her consumption good and the state of the economy in the proposition becomes stationary. The logic of incentive compatibility is almost the same to that in i), and therefore the author omits the corresponding part of the proof. To check the agents' participation constraints, the author solves the following equations:

$$\begin{aligned} V_2(3) &= -w + p'\beta V_2(f) + (1-p')\beta V_2(3), \\ V_2(f) &= u + \beta V_2(3). \end{aligned}$$

Then the author derives $V_2(1) > V_2(3) = \frac{1}{1-\beta}\left(-\frac{1}{1+p'\beta}w + \frac{p'\beta}{1+p'\beta}u\right)$. $V_2(3)$ is positive if $p'\beta u > w$. Since the agents commit in their trading utilizing the shops, the author does not have to check the type 2 agents' mediation incentive. ∎

Naturally the existence condition for the fiat money shopping equilibrium is looser than that for the commodity money shopping equilibrium. Each equilibrium exists when the carrying costs are sufficiently moderate. Because fiat money incurs no carrying cost, the fiat money equilibrium more easily exists than its counterpart. Since there is no friction in trade through the shops, there is no microeconomic difference between the two equilibria except for the weights of the medium of exchange. The author reclaims this point later in welfare comparison.

Technically, if parameters satisfy some appropriate conditions, there are infinitely many fiat money shopping equilibria with $P_{12} = P_{23} = P_{31}$. However, the author focuses on a particular one described in the proposition, since it maximizes the agents' welfare among its infinite counterparts. Here the author shows that point without proving existence of other equilibria, since the author neglects the others in the rest of the analysis.

Lemma 3

Assume that $p'u > w$. Let $W_{sf\kappa}$ be the agents' welfare of a fiat money shopping equilibrium with roving banditry, which satisfies $P_{12} = P_{23} = P_{31} = \kappa$, $P_{1f} = P_{2f} = P_{3f} = 1 - \kappa$. Then

$$W_{sf,\frac{1}{1+p'}} = \max_{\kappa \in [0,1]} W_{sf\kappa}.$$

Proof: If $\kappa \geq \frac{1}{1+p'}$, since $p'P_{i,i+1} \geq P_{if}$, $W_{sf\kappa} = 3(1-\kappa)u - 3\kappa w = 3u - 3\kappa(u + w)$. Then the smallest κ maximizes $W_{sf\kappa}$. If $\kappa \leq \frac{1}{1+p'}$, on the other hand, since $p'P_{i,i+1} \leq P_{if}$, $W_{sf\kappa} = 3p'\kappa u - 3\kappa w = 3\kappa(p'u - w)$. As long as $p'u > w$, the largest κ maximizes $W_{sf\kappa}$. ∎

$p'u > w$ is a much weaker condition than that in proposition 3. Indeed, readers may notice that if it is not satisfied, there cannot be any kind of non-autarky shopping equilibrium with roving banditry.

Again as a preparation for subsection 3.5., the author evaluates the agents' welfare for the two equilibria in proposition 4.

Lemma 4

Let W_{sc} and W_{sf} be the agents' welfares of the commodity money shopping equilibrium and the fiat money shopping equilibrium, respectively. Then:

$$W_{sc} = \frac{p' + 2p'^2}{1 + p'}u - 3w - \frac{p'}{1 + p'}(w_+ - w), \quad W_{sf} = \frac{3p'}{1 + p'}u - \frac{3}{1 + p'}w.$$

Proof: The author routinely calculates the values using the states in proposition 4.
$W_{sc} = \left(p' \cdot \frac{1}{1+p'} + 2 \cdot p' \cdot \frac{p'}{1+p'}\right) u - 3w - \frac{p'}{1+p'}(w_+ - w) = \frac{p'+2p'^2}{1+p'}u - 3w - \frac{p'}{1+p'}(w_+ - w)$. $W_{sf} = 3 \cdot \frac{p'}{1+p'}u - 3 \cdot \frac{1}{1+p'}w = \frac{3p'}{1+p'}u - \frac{3}{1+p'}w$. ∎

3.4.2. Shopping equilibria with a governor

Based on exactly the same logic to that of proposition 3, the author states a non-existence condition of shopping equilibria with a governor.

Proposition 5

If $D < \frac{1}{c}$, there is no shopping equilibrium with a governor.

Proof: Omitted, since the logic is completely the same to that of proposition 3.

The proposition is logically ordinary but intuitively important. Whenever shopping equilibria with roving banditry exists, a shopping equilibria with a governor cannot exist. For a governor to emerge, it must emerge with the new efficiency-improving infrastructure, the shops.

Let d be the disposability rate under governor, $d \equiv 1 - D(c_{b^*} - c_{b'})$. In order to depict the governor as a developer of the shops, the author shows existence of shopping equilibria with a governor.

Proposition 6

If $\frac{1}{c} < D$, $2\beta u(1-\frac{c_{b^*}}{c}) > \beta w + \sqrt{\beta^2 w^2 + 4\beta u w}$, *and* $\frac{c_{b^*}}{c} \leq D(c_{b^*} - c_{b'}) < \frac{2\beta u - \beta w - \sqrt{\beta^2 w^2 + 4\beta u w}}{2\beta u}$, *there exist a commodity money shopping equilibrium with a governor and a fiat money shopping equilibrium with a governor.*

Proof: When $\frac{1}{c} < D$, the tribe b^* confiscates $\frac{c_{b^*}}{c}$ of the commodities in the economy. Therefore if it can confiscate at least the same proportion of the commodities by settling in, namely $\frac{c_{b^*}}{c} \leq D(c_{b^*} - c_{b'})$, it does not deviate from $s_{b^*} = 1$.

However, simultaneously the tax rate $1-d \equiv D(c_{b^*} - c_{b'})$ must satisfy the agents' participation constraints. Analogous with proposition 4, that is $\frac{d^2\beta}{1+d\beta}u > w + \frac{d\beta}{1+d\beta}(w_+ - w)$. The author solves for $1-d$ to derive $1-d < \frac{2\beta u - \beta w_+ - \sqrt{\beta^2 w_+^2 + 4\beta u w}}{2\beta u}$.

The right hand side must be positive and, furthermore, larger than $\frac{c_{b^*}}{c}$. It is straightforward to derive $2\beta u(1-\frac{c_{b^*}}{c}) > \beta w_+ + \sqrt{\beta^2 w_+^2 + 4\beta u w}$. ∎

The second condition is satisfied if u is sufficiently larger than w and w_+. However the security provision still faces substantial difficulty. On the one hand, in order to attract the largest tribe to rule the market, its tax rate has to be higher than the confiscation rate it retains when it continues to be a tribe of bandits. On the other hand, the same rate must be sufficiently low so that the agents do not give up trading. Therefore the third condition implies that, for the largest tribe b^* to be a governor, it must be sufficiently larger than the other tribes *and simultaneously* appropriately small as a confiscator of the agents.

3.5. Welfare comparison

As long as the author compares the agents' welfares, he deals the welfares just as functions of parameters. That is, the values of welfares exist whichever corresponding equilibria exist or not.

The first comparison is of two search equilibria.

Proposition 7

If $p > \left(\frac{-1+\sqrt{5}}{2}\right)^2$ *and* $\frac{2p(\sqrt{p}-1)+2p^2}{(3-p)(1+\sqrt{p})^2}u > 3w + \frac{p}{3-p}(w_+ - w)$, $W_{mc} > \bar{W}_{mf}$.

Proof: Let the author remind that $W_{mc} = \frac{p}{3-p}u - 3w - \frac{p}{3-p}(w_+ - w)$, $\bar{W}_{mf} = \frac{p}{(1+\sqrt{p})^2}u$. Then $W_{mc} - \bar{W}_{mf} = \frac{2p(\sqrt{p}-1)+2p^2}{(3-p)(1+\sqrt{p})^2}u - 3w - \frac{p}{3-p}(w_+ - w)$. The inequality $W_{mc} - W_{mf} > 0$ implies $\frac{2p(\sqrt{p}-1)+2p^2}{(3-p)(1+\sqrt{p})^2}u > 3w + \frac{p}{3-p}(w_+ - w)$. For $\frac{2p(\sqrt{p}-1)+2p^2}{(3-p)(1+\sqrt{p})^2}u$ to be positive, $p > \left(\frac{-1+\sqrt{5}}{2}\right)^2$ must stand. ∎

In spite of fiat money's apparent superiority to commodity money, namely its zero carrying cost and robustness against raiding, still the commodity money search equilibrium sometimes yields more welfare than its fiat money counterpart. This is due to search externality. In trade with commodity money, the economy needs one less step of trading than trade with fiat money for whole types of the agents acquire their consumption goods. In the economy with matching frictions, such easiness in trade enhances the agents' welfare performance.

Second the author compares of two shopping equilibria.

Proposition 8

If $p' \in (0,1)$ *or* $p' \cdot w > 0$, $W_{sf} > W_{sc}$. *If* $p' = 1$ *and* $w_+ = w = 0$, $W_{sf} = W_{sc}$.

Proof: $W_{sf} - W_{sc} = \frac{2p'(1-p')}{1+p'}u + \frac{3p'}{1+p'}w + \frac{p'}{1+p'}(w_+ - w)$. If $p' \in (0,1)$, $\frac{2p'(1-p')}{1+p'}u$ is strictly positive. If $p' \cdot w > 0$, $\frac{3p'}{1+p'}w$ is strictly positive. In either of the cases, $W_{sf} - W_{sc}$ is strictly positive. However if $p' = 1$ and $w_+ = w = 0$, all of $\frac{2p'(1-p')}{1+p'}u$, $\frac{3p'}{1+p'}w$, and $\frac{p'}{1+p'}(w_+ - w)$ are equal to 0. ∎

Although the proposition shows general superiority of the fiat money shopping equilibrium, the author notes that both the commodity money system and the fiat money system exhibit the same welfare performance when the economy is perfectly safe and the commodities have no weight. When the agents trade through the shops, arbitrary money's performance with respect to its exchange mediation is independent of its intrinsic property. Money is money, whether it is a commodity or an artificial token.

Third the author exercises system-wise comparison, namely comparison of W_{sf} and W_{mf}.

Lemma 5

Assume $(2p+6p\sqrt{p}+2p^2)u > 3(1+\sqrt{p})^2 w$. *There exists* $\bar{D}_0 > 1$ *such that* $W_{sf} > \bar{W}_{mf}$ *if* $\bar{D}_0 > D$.

Proof: $W_{sf} - \bar{W}_{mf} = \left(\frac{3p'}{1+p'} - \frac{p}{(1+\sqrt{p})^2}\right) u - \frac{3}{1+p'} w$, where $p' = 1-(1-p)D$ and $1 > (1-p)D$.

$$
\begin{aligned}
& W_{sf} - \bar{W}_{mf} > 0 \\
\Leftrightarrow\ & \left(\frac{3-3(1-p)D}{2-(1-p)D} - \frac{p}{(1+\sqrt{p})^2}\right) u - \frac{3}{2-(1-p)D} w > 0, \\
\Leftrightarrow\ & [(1+\sqrt{p})^2\{3-3(1-p)D\} - p\{2-(1-p)D\}]u - 3(1+\sqrt{p})^2 w > 0, \\
\Leftrightarrow\ & (3+6\sqrt{p}+p)u - 3(1+\sqrt{p})^2 w > (3+6\sqrt{p}+2p)(1-p)uD \\
\Leftrightarrow\ & \frac{(3+6\sqrt{p}+p)u - 3(1+\sqrt{p})^2 w}{(3+6\sqrt{p}+2p)(1-p)u} > D.
\end{aligned}
$$

If $(2p+6p\sqrt{p}+2p^2)u > 3(1+\sqrt{p})^2 w$, $(3+6\sqrt{p}+p)u - 3(1+\sqrt{p})^2 w > (3+6\sqrt{p}+2p)(1-p)u$. Let the left hand side be the level of $\bar{D}_0$. It satisfies the property in the statement. ∎

Trade through the shops has no friction. Trade through the random matching process cannot avoid friction. Naturally, therefore, there is a range of parameter wherein the former is better than the latter with respect to welfare.

Supplementarily, the author confirms that there is a case where trade through the shops (with fiat money) is better for the agents than trade through the random matching process.

Lemma 6

There exists $\bar{D}_1 > 1$ *such that* $W_{sf} > W_{mc}$ *if* $\bar{D}_1 > D$.

Proof: $W_{sf} - W_{mc} = \left(\frac{3p'}{1+p'} - \frac{p}{3-p}\right) u + \left(3 - \frac{3}{1+p'}\right) w + \frac{p}{3-p}(w_+ - w)$. Noting that $p' = 1-(1-p)D$ and $1 > (1-p)D$,

$$
\begin{aligned}
& W_{sf} - W_{mc} > 0 \\
\Leftrightarrow\ & \left(\frac{3-3(1-p)D}{2-(1-p)D} - \frac{p}{3-p}\right) u + \left(3 - \frac{3}{2-(1-p)D}\right) w + \frac{p}{3-p}(w_+ - w) > 0 \\
\Leftrightarrow\ & [(3-p)\{3-3(1-p)D\} - p\{2-(1-p)D\}]u \\
& \quad +[3(3-p)\{2-(1-p)D\} - 3(3-p)]w + p\{2-(1-p)D\}(w_+ - w) > 0 \\
\Leftrightarrow\ & (9-5p)u + (9-3p)w + 2p(w_+ - w) \\
& \quad > \{(9-2p)(1-p)u + 3(3-p)(1-p)w + p(1-p)(w_+ - w)\}D \\
\Leftrightarrow\ & \frac{(9-5p)u + (9-3p)w + 2p(w_+ - w)}{(9-11p+2p^2)u + (9-3p)(1-p)w + p(1-p)(w_+ - w)} > D.
\end{aligned}
$$

Let the left hand side be the level of $\bar{D}_1$. Since $9-5p > 9-11p+2p^2$, $9-3p > (9-3p)(1-p)$, and $2p > p(1-p)$, it is larger than 1. It satisfies the property in the statement. ∎

If the agents can hide effectively from the bandits even when they utilize the shops, they may improve their welfare by simply shifting to trade through the shops, without any aid by the governor. In that case, as shown in proposition 4, no governor appears.

Consider a situation in which the trade through the shops is impossible without the governor. As a corollary of proposition 5, proposition 6, lemma 5, and lemma 6, the author states a case where the fiat money shopping equilibrium with a governor exists and yields the best agents' welfare of all. Let the author define the welfare of fiat money shopping equilibrium with a governor. It is a function in which W_{sf} transforms by substituting d for p'. That is:

$$W_{sfg} = \frac{3d}{1+d}u - \frac{3}{1+d}w.$$

Corollary
Define $\bar{D}$ *as* $\bar{D} \equiv \min(\bar{D}_0, \bar{D}_1)$. *Assume* $(2p + 6p\sqrt{p} + 2p^2)u > 3(1+\sqrt{p})^2 w$. *If* $\frac{1}{c} < D$, $2\beta u(1 - \frac{c_{b^*}}{c}) > \beta w + \sqrt{\beta^2 w^2 + 4\beta uw}$, *and* $\frac{c_{b^*}}{c} \leq D(c_{b^*} - c_{b'}) < \min\left[\bar{D}c, \frac{2\beta u - \beta w - \sqrt{\beta^2 w^2 + 4\beta uw}}{2\beta u}\right]$, *there exists a fiat money shopping equilibrium with a governor and* $W_{sfg} > \max(W_{mc}, W_{mf})$ *stands.*

There is a range of parameters which satisfies the conditions in the corollary. Verification of its existence is in the appendix. A typical situation is as following. The detection coefficient is sufficiently large $(1 - Dc < 0)$. There are many tribes of bandits so that even c_{b^*} is a small fraction of c. c is large enough to make the interval $[\frac{c_{b^*}}{c}, \bar{D}c)$ substantially wide. With an appropriate level of $c_{b^*} - c_{b'}$, the conditions in the corollary are satisfied.

The author summaraizes the characteristics of the constraints into two points. First, the shopping system has to be vulnerable against the bandits' raiding so that it necessitates a governor. Second, the candidate tribe for governor must be strong enough to attain "encompassing interest" by settling in, and reasonably weak so that it leaves surplus from the efficiency improvement of exchange system for the agents.

Olson (1993) notes that the state-maker refrains from over-confiscation since there is some "encompassing interest" from nurturing the economy in the long run. However, it is not so easy to find a situation with such an interest from a simple and purely microeconomic setup. There should be some efficiency-enhancing infrastructure, mechanism of endogenous growth, or taxation-supporting system, which is vulnerable to intrusion of an armed force. And furthermore, there should be some factor which controlles the taxation power of governor into an appropriate range.

Historically, a king often transfered his/her decision right on tax rate to his/her nations.[12] By an appropriate allocation of property rights, a trading pattern utilizing efficient infrastructure remains. Namely, if the governor and the agents collaborate to adjust the taxation rate, they can drastically loosen the third condition in corollary. Furthermore, if they can collaborate ex ante, the transition is possible even if the first condition is not satisfied. In the author's opinion, the political adjustment is an essential procedure to sustain a state as a group.

4. Discussions

4.1. On Taxation Rights

The problem of a nations' property rights is worth considering. Along with the analysis, reallocation of decision rights on tax is an enhancing step for satisfying the conditions on incentives

[12] The reason of such re-allocation of right is various among regions and eras. In 17th century Guinea, for example, suzerain state Ardra left the administration of the port city Whydah to the city itself since living close to a seacoast was a religious taboo for the Ardra people (Polanyi (1966)).

of both the governor and the people. Especially in the long run, political and social changes in the economy often make the conditions substantial, and therefore make the political adjustment on tax essential.

Fairly long after the settlement, it is possible that many of the other bandit tribes immigrate somewhere else or surrender to the governor, since they cannot confiscate from the economy anymore. Then the governor does not have to use much of his/her manpower to exclude the other tribes. This loosening of manpower increases the governor's taxation ability. Besides, the nations may move into the governor's territory and start producing there. This will also increase the governor's ability, since he/she can more easily observe the people's activities. If the people do not regulate the taxation appropriately, the governor will start over-confiscation, which may destroy the sustainability of the equilibrium.

If the governor cannot use his/her surplus manpower to tax, it is likely that the governing officers will engage in some productive activities, in order to increase their gains from the economy. While they may produce some goods by themselves, it is more likely that they may exercise some economic policies that enhance economic activities. Such policies include, for example, developing infrastructure for trade and transportation, reclaiming lands in the territory, researching more efficient techniques of production, education, and so on.

Myopicity of the governor makes the security provision rather difficult. If the governor has a dynamic objective function with a positive discount factor, the problem of over-confiscation may be less serious. Especially, as an interesting extension, by introducing a dynamic behavior of the governor, a researcher may be able to describe a situation in which the governor voluntarily curbs its tax rate, in order to keep its nations in the territory.[13]

4.2. Interdisciplinary view

From the viewpoint of history or anthropology, an economy is always just a part of a society. An economic system is "embedded" in a social structure (Polanyi (1957)). Especially when you discuss economies in early societies, it is important to note that the economies are often under political administration. People do not necessarily trade in the laissez-faire manner. The governor, on the other hand, often administrates a market in order to acquire relative authority in the surrounding region, rather than direct revenue from tax.[14]

Economic equilibria in the model are stable or sustainable trading systems from such a viewpoint. When some union of merchants or a governor enforces one of the equilibrium trading patterns, the pattern is stable even when an individual or a governor consider behaving self-interestedly. With occasional interventions, an administrator can practically enforce the equilibrium behavior.

From an economic viewpoint, an authority-oriented governor supports the shopping economy. Since such a governor does not necessarily confiscate the largest amount of goods from the economy, it is more likely that the agents prefer trading under the guard of a governor. The value of authority increases mutual benefit between the governor and the agents.

5. Concluding remarks

The author introduced a microstructure of the shopping market into the economy in order to study the possible change in exchange system along with the security provision. Under security provision by a governor, if the level of the governor's power is in an appropriate range, there may appear a trading pattern that utilizes the shops with fiat money as the only medium of exchange.

[13] However the author should note that the tribes' dynamic inference may also result in survival of multiple tribes. A situation with tragedy of commons might be better for the largest tribe than its sole governing.

[14] As for Japan, Nakamura (2005) emphasizes that a local military lord in a region supplied security to acquire relative authority against other lords, not purely economic interest.

However, a political procedure which allocates decision rights on tax to the nations is desirable to sustain a state in the long run, because a strong state-maker always has an incentive to confiscate much from the economy.

The author focuses on the physical amount of goods as the source of welfare and has not considered aspects of quality. Also the author neglected sustainability of the agents' lives which naturally depend on their frequency of consumption. Such problems will be considered in future research.

Appendix : Verification for Corollary

The author shows that there exists a range of parameters which satisfy the following conditions, assuming $(2p+6p\sqrt{p}+2p^2)u > 3(1+\sqrt{p})^2 w$.

$\frac{1}{c} < D$, $2\beta u(1-\frac{c_{b^*}}{c}) > \beta w+\sqrt{\beta^2w^2+4\beta uw}$, and $\frac{c_{b^*}}{c} \leq D(c_{b^*}-c_{b'}) < \min\left[\bar{D}c, \frac{2\beta u-\beta w-\sqrt{\beta^2w^2+4\beta uw}}{2\beta u}\right]$ where $\bar{D} \equiv \min(\bar{D}_0, \bar{D}_1)$.

The author focuses on cases wherein $w > 0$. Explicit values of $\bar{D}_0$ and $\bar{D}_1$ are:

$$\bar{D}_0 = \frac{(3+6\sqrt{p}+p)u-3(1+\sqrt{p})^2w}{(3+6\sqrt{p}+2p)(1-p)u}, \; \bar{D}_1 = \frac{(9-5p)u+(9-3p)w+2p(w_+-w)}{(9-11p+2p^2)u+(9-3p)(1-p)w+p(1-p)(w_+-w)}.$$

Noting that $c = 1-p$, the author calculates $\bar{D}_0c$ and $\bar{D}_1c$.

$$\bar{D}_0c = \frac{(3+6\sqrt{p}+p)u-3(1+\sqrt{p})^2w}{(3+6\sqrt{p}+2p)u}, \; \bar{D}_1c = \frac{(9-5p)u+(9-3p)w+2p(w_+-w)}{(9-2p)u+(9-3p)w+p(w_+-w)}.$$

Next the author takes the limit of $\bar{D}_0c$ and $\bar{D}_1c$ when $\frac{u}{w_+} \to \infty$.

$$\lim_{\frac{u}{w_+}\to\infty} \bar{D}_0c = \frac{3+6\sqrt{p}+p}{3+6\sqrt{p}+2p} > \frac{10}{11}, \; \lim_{\frac{u}{w_+}\to\infty} \bar{D}_1c = \frac{9-5p}{9-2p} > \frac{4}{7}.$$

Additionally the author can verify that $\lim_{\frac{u}{w}\to\infty} \frac{2\beta u-\beta w-\sqrt{\beta^2w^2+4\beta uw}}{2\beta u} = 1$.

Then, if u is sufficiently larger than w and w_+, the author has a bundle of sufficient conditions as below:

$$\frac{1}{c} < D, \; \frac{c_{b^*}}{c} \leq D(c_{b^*}-c_{b'}) < \frac{4}{7}.$$

If $\frac{1}{c} < D$ and $c_{b^*} < \frac{4}{7}c$, with sufficiently large B, there clearly exists a range of the level of $c_{b^*}-c_{b'}$ which satisfies $\frac{c_{b^*}}{c} \leq D(c_{b^*}-c_{b'}) < \frac{4}{7}$. ∎

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