Innovative Interaction in Mixed Market:
An Effect of Agency Problem on State-Owned Firm *

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Abstract

I investigate an innovative interaction followed by a quantity-setting competition in a mixed duopoly where a state-owned firm and a private firm compete with each other. I find that an agency problem in the former firm, when not extremely serious, can improve the expected social welfare, implying that contracting-outs and privatizations can damage the expected social welfare. Further, a bureaucratic system with a zero minimum wage level and a guaranteed utility level equal to the reservation utility level is almost always suboptimal from the viewpoint of expected social welfare. This implies that bureaucrats’ higher utility level and lesser responsibility for bad performance is desirable.

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1 Introduction

The agency problem due to a bureaucratic management is one of the most important problems in Japan. People often criticize bureaucratic managements for their inefficiency. Moreover, some of them require contracting-outs or privatizations in order to improve the efficiency of managements. In fact, the Liberal Democratic Party, which attached considerable emphasis to the privatization of the national postal mail, savings, and insurance services, achieved a significant victory in the 2005 elections to the House of Representatives in Japan.

Although contracting-outs and privatizations could improve the efficiency of management in state-owned firms as likely as not, they affect not only the management of the firm but also that of private competitors in mixed duopolistic markets. In fact, some critics have pointed out that the privatization of the national postal mail, savings, and insurance services could damage the performance of private competitors in the delivery service, banking, and insurance industries in Japan. However, most of them only consider the ex-post allocation between the ex-state-owned firms and other firms, i.e., the outcomes of market competition. I consider that the more important viewpoints are how the policies (contracting-outs or privatizations) change ex-ante incentive to innovate technology in private firms and whether or not it improves the expected social welfare. I investigate an innovative interaction followed by a quantity-setting competition in a mixed duopoly where a state-owned firm and a private firm compete with each other. I find that although it reduces the investment level of the state-owned firm, an agency problem can increase the investment level of the private firm and consequently improve the expected social welfare, if it is not extremely serious. In other words, a small degree of inefficiency in state-owned firms can improve welfare in mixed markets. This result implies that contracting-outs and privatizations can damage the expected social welfare.

I also investigate bureaucratic systems with minimum wage and guaranteed utility levels. Setting the minimum wage level higher heightens the equilibrium investment level of the state-owned firm, while setting the guaranteed utility level higher lowers the equilibrium investment level. I show that a bureaucratic system where the minimum wage level is zero and the guaranteed utility level is equal to the reservation utility level is suboptimal for almost every case. It is often considered that while managers in certain state-owned firms enjoy higher utility levels, those in other state-owned firms are less responsible for bad performance. My results imply, however, that these aspects can be desirable
from the viewpoint of expected social welfare.

Several studies, for example, De Fraja (1993), Schmidt (1996), Hart, Shleifer and Vishny (1997), and Corneo and Rob (2003), investigate the agency problem in state-owned firms. Although these studies examine various effects on both ex-ante investment and ex-post allocation, they do not consider the effects on strategic interactions between public and private sectors.

Analyses on the strategic interaction in mixed markets have been a popular subject in recent years. The most remarkable aspect in a mixed market is that when the behaviors of state-owned and private firms are strategic substitutes, welfare-maximizing state-owned firms behave over aggressively. In other words, the equilibrium behaviors of state-owned firms are so aggressive, such as choosing a higher quantity level in quantity-setting competitions, that making the behaviors less aggressive improves social welfare. It follows that providing a commitment device to behave less aggressively could improve social welfare. In my study, an efficiency loss derived from the agency problem in a state-owned firm works as a device.

The rest of this article is organized as follows. In Section 2, I formulate the basic model. Next, I present the benchmark outcomes in the absence of the agency problem (under a direct management) in Section 3, and I investigate the effects of the agency problem in a state-owned firm and in an optimal

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1De Fraja (1993) studies the problem using a complete contracting approach. He shows that in a good state of the world, state-owned ownership always leads to a higher degree of productive efficiency. This is because the benefit of enhancing productive efficiency is higher for the public owner than for the private owner. Schmidt (1996), on the other hand, analyzes the problem by employing an incomplete contracting approach. In his study, he supposes that the government cannot offer any long-term contingent contracts. Given this assumption, although public ownership always leads to allocative efficiency in any technological environment, it cannot offer any incentive to enhance productive efficiency. Hart et al. (1997) and Corneo and Rob (2003) analyze the problem by using multi-task models. Hart et al. (1997) show that incentive contracts increase the cost-reducing investments but lower the quality of services. Corneo and Rob (2003), on the other hand, show that public owners offer less intensive incentive contracts when the agents derive a private benefit from one task (cooperative task).

2The studies on mixed markets originated with the works of Merrill and Schneider (1966) and Harris and Wiens (1980). In recent years, the strategic interactions in the mixed market have been analyzed in various contexts. For example, Matsushima and Matsumura (2003) analyze the location-setting pattern in a mixed oligopoly market. Fjell and Heywood (2004) analyze optimal subsidy levels and effects of privatization of the state-owned firm in a mixed oligopoly market, and Chang (2005) analyzes optimal trade and privatization policies in an international duopoly where a state-owned firm competes with a more efficient foreign firm. Matsushima and Matsumura (2004) and Ishibashi and Matsumura (2006) analyze cost-reduction and R&D competition in a mixed duopoly market, although they do not consider an agency problem in a state-owned firm. For a survey on these studies, see Nett (1993) for example.

3Several studies use the aspect of mixed market in their investigation. De Fraja and Delbono (1989) and Matsumura (1998), for example, use it in the quantity-setting competition and show that privatization in mixed oligopoly and partial privatization in a mixed duopoly, respectively, can improve social welfare. Ishibashi and Matsumura (2006) use the effect in R&D competition and show that committing less investment by imposing a budget constraint improves social welfare.

4When it can commit contracts, the government uses a contract to control the equilibrium outcome. Barros (1995) considers the contract as a strategic commitment device and shows that privatization in mixed oligopoly and partial privatization in a mixed duopoly, respectively, can improve social welfare. Ishibashi and Matsumura (2006) use the effect in R&D competition and show that committing less investment by imposing a budget constraint improves social welfare.
bureaucratic system in Section 4. Finally, in Section 5, I provide the concluding remarks. Every proof is presented in Appendix A.

2 Model

Consider an industry with two firms – 0 and 1 – that produce a homogeneous commodity. For this commodity, the market demand function is given by \( D(p) = a - p \). Let \( p(Q) \equiv D^{-1}(Q) = a - Q \) be the inverse demand function.

Let \( c(q; \theta) = \frac{1}{2} bq^2 + \theta (b > 0) \) denote the cost function common for both firms. The cost of production depends not only on the quantity \( q \) but also on the technological level \( \theta \in \{ \theta^g, \theta^b \} \), where \( \theta^g < \theta^b < a \). Assume \( b(a - \theta^b) \geq \theta^b - \theta^g \) for analytical simplicity.\(^5\)

The technological level of one firm is attained in a stochastic manner independent of the other’s level. Let \( v(e) \) denote the probability that a firm attains a good technological level \( \theta^g \). It is common for both firms and depends on the investment (or effort) level \( e \in [0, \overline{e}] \) the manager of the firm has undertaken.\(^6\) This level is measured in terms of the units of disutility the manager has incurred. Assume the following: (i) \( v' \geq 0 \) and \( v'' < 0 \) for all \( e \in [0, \overline{e}] \), (ii) \( v(0) = 0 \) and \( v(\overline{e}) = 1 \), (iii) \( v'(0) = +\infty \) and \( v'(\overline{e}) = 0 \), (iv) \( v'' \) is continuous, and (v) \( \frac{v''}{v'} \) is non-increasing.\(^7\) The technological environments in the market competition are divided into four combinations: \( gg, gb, bg, \) and \( bb \). For example, \( gb \) indicates that firm 0 possesses good technology and firm 1 has bad technology. I use a superscript \( s \in \{ gg, gb, bg, bb \} \) when describing some outcomes depending on the environment.

Firm 0 is owned by a government G, and firm 1 is owned and managed by a private entrepreneur P. G is unable or unwilling to manage the state-owned firm by itself and must employ a bureaucrat B as the manager of the firm. Although P has sufficient wealth, B has no wealth and is protected by limited liability. Every player is risk neutral.

The timing and information structure is as follows. At the beginning of the initial stage, G offers a contract to B. Assume that P cannot observe the contract for the time being.\(^8\) Next, B and P choose

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\(^5\)This condition guarantees that the market demand is so huge that even a firm facing a cost disadvantage can provide a positive amount.

\(^6\)For simplicity, I assume that the innovation technology is common for both firms.

\(^7\)Assumptions (i)-(iii) focus on an interior solution. (iv) guarantees the existence of equilibrium and (v) is imposed for analytical simplicity. Note \( v' \) is not as restrictive. When (i)-(iii) hold, \( \frac{v''}{v'} \) could be increasing for an interval of \( e \) only when \( v'' \) is rapidly increasing. \( \sqrt{\overline{e}^2 - (e - \overline{e})^2} \) is one example of \( v(e) \) satisfying these assumptions.

\(^8\)Later, I will consider a case where G can disclose the contract.
the investment levels for the firms $e_0$ and $e_1$ respectively. These levels are private information for each of them. At the end of this stage, a technological environment is stochastically realized and publicly observed. The contract between G and B specifies the wage level contingent on the technological levels of firm 0, i.e., $(w^g, w^b)$. In the second stage, G and P simultaneously decide the output level of their own firm $q_i \in \mathbb{R}_+$. 

Since they are risk neutral, G, P, and B maximize the expected values of their respective payoffs. Here, P’s payoff is given by $\pi_1 - e_1$, where $\pi_1$ represents firm 1’s profit and is equal to $p(q_0 + q_1)q_1 - c(q_1; \theta_1)$. Note that firm 1’s profit is dependent on not only the actions of both G and P but also its own technological level $\theta_1$. Further, B’s payoff is given by $w - e_1$, where $w$ is either $w^g$ or $w^b$. Finally, G’s payoff is given by

$$C_S + \pi_0 - w + \pi_1 - e_1 + \zeta (w - e_0),$$

where $C_S$ represents the consumer surplus and is given by

$$\int_0^{q_0 + q_1} (p(Q) - p(q_0 + q_1))dQ,$$

$\pi_0$ represents firm 0’s profit and is equal to $p(q_0 + q_1)q_0 - c(q_0; \theta_0)$, and $\zeta$ is a parameter in $[0, 1]$. The government G is rather benevolent in the sense that it cares for the consumers as well as for agents P and B. However, G discounts the others’ payoffs following from an evaluation loss. In order to focus on the agency problem between G and B, I ignore the loss concerning consumer surplus and P’s payoff. Note that the social welfare is given by $C_S + \pi_0 + \pi_1 - e_0 - e_1 \equiv Z - e_0 - e_1$, where $Z$ represents the gross social welfare. Therefore, G’s payoff is rewritten as $Z - e_0 - e_1 - (1 - \zeta)(w - e_0)$, where the last term represents the value of evaluation loss.

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9 This assumption is plausible when I consider the investment as an effort for cost reduction.

10 G could offer a contract contingent on the environments. However, since the investment outcome of firm 0 is independent of that of firm 1, the contract $(w^g, w^b)$ is sufficient for controlling B’s incentive.

11 This type of formalization of the government’s objective is similar to that in De Fraja (1993). Another possible formalization is that in Caillaud, Guesnerie, Rey and Tirole (1988). In Caillaud et al. (1988), transferring one dollar from the public sector to the private sector costs society more than one dollar due to a certain kind of taxation cost. Even under this type of formalization, although analyses on comparative statics were more complicated, I can present similar results.
3 Outcomes under a Direct Management

3.1 Market Competition

First, I investigate the market competition in the second stage.\textsuperscript{12} Let $CS^s*$, $\pi^s_0^*$, $\pi^s_1^*$, and $Z^s* = CS^s* + \pi^s_0^* + \pi^s_1^*$ denote the equilibrium outcomes in an environment $s \in \{gg, gb, bg, bb\}$. Then, I have the following.

Lemma 1. Among the equilibrium outcomes, the following relationships hold: (i) $Z^{gb*} - Z^{bb*} > Z^{gg*} - Z^{bg*} \geq 0$ and $Z^{bg*} - Z^{bb*} > Z^{gg*} - Z^{gb*} \geq 0$, (ii) $\pi^{bg*}_1 - \pi^{bb*}_1 > \pi^{gg*}_1 - \pi^{gb*}_1 \geq 0$, and (iii) $CS^{gg*} + \pi^{gg*}_0 \geq CS^{gb*} + \pi^{gb*}_0$ and $CS^{bg*} + \pi^{bg*}_0 > CS^{bb*} + \pi^{bb*}_0$.

(i) implies that although innovation in one firm unfailingly increases the gross social welfare, the increment is larger when the other firm does not innovate. This is intuitive since innovation in one firm leads to not only an increase in the total supply but also product substitution, when the other firm does not innovate. (ii) implies that although innovation in firm 1 always increases the firm’s profit, the degree of increase is larger when firm 0 does not innovate. This is intuitive since innovation in firm 1 relaxes the competition and allows firm 1 to enjoy a cost advantage when firm 0 does not innovate. (iii) describes a feature of the external effect derived from innovation in firm 1. On one hand, it increases the total supply and subsequently the consumer surplus, while on the other hand, it decreases firm 0’s profit. It implies that the former effect outweighs the latter effect.

At the end of this subsection, I note my specification of market competition as follows: the market demand function is linear, the cost function is quadratic, and the agents are involved in simultaneous quantity-setting competition. I am afraid that my specification is not general, although it is one of the ordinary specifications. However, the results later in this study stand upon the relationships in Lemma 1 only. Therefore, my analyses are sustainable if the relationships hold under certain types of market competition. For example, under G-leader price-setting competition with two levels of constant marginal cost, the relationships hold.\textsuperscript{13}

\textsuperscript{12} Since the environment specifying each firm’s cost function is observed by both G and P, a competition in this stage is a sub-game of the whole game.

\textsuperscript{13} In contrast, under a simultaneous quantity-setting competition with two levels of constant marginal cost, relationship (iii) does not hold.
3.2 Innovative Interaction

In this subsection, I investigate the innovative interaction between G and P. As a benchmark, I suppose that G directly controls firm 0’s investment level $e_0$. Since G and P simultaneously decide their respective firm’s investment level, the equilibrium investment level $e^d = (e^d_0, e^d_1)$ satisfies

\[ v'(e^d_0) \left[ v(e^d_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e^d_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] = 1 \]

\[ v'(e^d_1) \left[ v(e^d_0) \left( \pi^{gg*}_1 - \pi^{gb*}_1 \right) + (1 - v(e^d_0)) \left( \pi^{bg*}_1 - \pi^{bb*}_1 \right) \right] = 1, \]

following from first-order conditions.\(^{14}\) For explanatory simplicity, assume the equilibrium as unique.\(^{15}\)

Note that $e^d$ is not the maximizer of the expected social welfare; this is because P does not maximize the expected social welfare. Let $EW(e_0, e_1)$ be the expected social welfare, given a set of investment levels. Moreover, let $R^d_1(e)$ be P’s response function, i.e.,

\[ R^d_1(e) = v'^{-1} \left( \frac{1}{v(e) \left( \pi^{gg*}_1 - \pi^{gb*}_1 \right) + (1 - v(e)) \left( \pi^{bg*}_1 - \pi^{bb*}_1 \right)} \right). \]

Then I have the following.

**Lemma 2.** There exists a non-empty set $A_0 \subset [0, e^d_0)$ such that

$e_0 \in A_0 \iff EW(e_0, R^d_1(e_0)) > EW(e^d_0, e^d_1) \equiv EW^d$.

This lemma implies that the equilibrium investment level of G under a direct management, denoted by $e^d_0$, is over aggressive in the sense that making G’s decision less aggressive can result in an improvement in the expected social welfare. In addition, it also implies that G’s investment level that maximizes the expected social welfare given the response of P is less than the equilibrium level under a direct management. This result is attributed to the strategic substitutability and P’s under-investment.\(^{16}\)

A marginal decrease in $e_0$ from $e^d_0$ does not directly change the expected social welfare, since $e^d_0$ is the

\(^{14}\)Second-order conditions are satisfied following from the assumption on \(v\) and (i) in Lemma 1.

\(^{15}\)I can easily confirm the existence of equilibrium.

\(^{16}\)I have $\frac{dR^d_1(e)}{de} < 0$. See the proof of Lemma 2.
maximizer when \( e_1 = e_1^d \) is given. However, such a decrease increases \( e_1 \), which indirectly increases the expected social welfare since \( e_1^d \) is less than the welfare-maximizing investment level. As a result, it increases the expected social welfare.\(^{17}\)

## 4 Agency Problem in the State-Owned Firm

### 4.1 Agency Problem and Substitution of Investment

Now, I analyze the effect of agency problem in the state-owned firm. Note that the competition outcome in the second stage is not affected by the agency problem. Then, I can focus on the investigation on innovative interaction. G’s maximization problem responding to \( e_1 \) is given by

\[
\begin{align*}
\max_{e_0, (w^g, w^b)} & \quad EW(e_0, e_1) - (1 - \zeta) \left[ v(e_0)w^g + (1 - v(e_0))w^b - e_0 \right] \\
\text{s.t.} & \quad v(e_0)w^g + (1 - v(e_0))w^b - e_0 \geq w \\
& \quad v'(e_0)(w^g - w^b) = 1 \\
& \quad w^g \geq 0 \text{ and } w^b \geq 0,
\end{align*}
\]

where the constraints are the individual rationality (IR), incentive compatibility (IC), and limited liability (LL) constraints.

Let \( e^b \) denote the set of levels of the equilibrium investment. First, I consider the case of \( \zeta = 1 \), wherein G completely evaluates B’s payoffs. Then, I have the following.

**Proposition 1.** \( e^b = e^d \), if \( \zeta = 1 \).

Note first that the distortion derived from the agency problem is generated only when the LL constraint \( w^b \geq 0 \) is binding, since both G and B are risk neutral. When \( \zeta = 1 \), the wage levels do not affect G’s objective since the wage is just a transfer from the firm to B. Therefore, G has no incentive to restrain the wage levels, and consequently the LL constraint \( w^b \geq 0 \) is never binding. Proposition 1 implies that the investment level of the state-owned firm is over aggressive, when the government is completely benevolent following from Lemma 2.

\(^{17}\) Several studies consider similar aspects in different settings: De Fraja and Delbono (1989) and Matsumura (1998), in the quantity-setting competition and Ishibashi and Matsumura (2006), in R&D competition, for example.
Next, I consider the case of $\zeta \in [0,1)$. Herein, the LL constraint $w^b \geq 0$ is binding depending on the parameters and P’s action $e_1$. Let $\hat{w}$ be the increment in expected social welfare that G can generate by investment under a direct management when P’s investment level is given by $e^d_1$, i.e.,

$$\hat{w} \equiv EW^d - EW(0, e^d_1) = v(e^d_0) \left[ v(e^d_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e^d_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] - e^d_0.$$ 

Note that $\hat{w} > 0$ following from the assumptions on $v(e)$. Then, I have the following.

**Proposition 2.** Suppose that $\zeta \in [0,1)$. Then, $e^b_0 < e^d_0$ and $e^b_1 > e^d_1$, if and only if $w < \hat{w}$.

When $\zeta \in [0,1)$. G is willing to restrain the expected payoff of B to the level of reservation utility. Given $e_1 = e^d_1$, $e^d_0$ should be such that

$$w^g \geq v(e^d_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e^d_1)) \left( Z^{gb*} - Z^{bb*} \right)$$

due to the LL constraint $w^b \geq 0$. However, when $w < \hat{w}$, this inequality implies that

$$v(e^d_0)w^g + (1 - v(e^d_0))w^b - e^d_0 \geq \hat{w} + (1 - v(e^d_0))w^b > w.$$ 

Therefore, G reduces the target investment level from $e^d_0$. Responding to this reduction, P increases his/her investment level since the reduction in $e_0$ reduces the probability of innovation in firm 0 and then heightens the marginal benefit of investment for P, which is provided by the chance of enjoying a cost advantage.

### 4.2 Effect on Expected Social Welfare

From here on, I suppose that $\zeta \in [0,1)$. Following from Proposition 2, the agency problem in firm 0 reduces $e_0$ and increases $e_1$ instead. The reduction in $e_0$ indubitably damages the expected social welfare. However, in contrast, a certain degree of increase in $e_1$ improves the expected social welfare. This is because $e^d_1$ is lower than the expected social welfare maximizing level, as firm 1 does not take the others’ payoff, particularly the consumer surplus, into account. Whether the agency problem damages or improves the expected social welfare balances these two effects. Let $EW^b = EW(e^b_0, e^b_1)$ be the expected social welfare in equilibrium. Then, $EW^b > EW^d$, if and only if $e^b_0 \in A_0$ following from
Lemma 2.

Let \( \bar{e} = (\bar{e}_0, \bar{e}_1) \) be a set of investment levels such that

\[
v'(\bar{e}_0) \left[ v(\bar{e}_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(\bar{e}_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] = 1 - \frac{(1 - \zeta)v(\bar{e}_0)v''(\bar{e}_0)}{[v'(\bar{e}_0)]^2} \]

\[
v'(\bar{e}_1) \left[ v(\bar{e}_0) \left( \pi^{gg*}_1 - \pi^{gb*}_1 \right) + (1 - v(\bar{e}_0)) \left( \pi^{bg*}_1 - \pi^{bb*}_1 \right) \right] = 1.
\]

For simplicity, assume it is unique. Then, I have the following.

**Proposition 3.** There exists a non-empty set \( W \subset [0, \hat{w}) \) such that \( EW^b > EW^d \) for \( w \in W \). Moreover, \( W = [0, \hat{w}) \), i.e., \( EW^b > EW^d \), for all \( w < \hat{w} \) if \((\bar{e}_0, e^d_0) \subset A_0 \).

For \( w < \hat{w} \), a decrease in \( w \) increases the degree of distortion, i.e., reduces \( e^b_0 \). However, \( e^b_0 \) has a lower bound, i.e., \( e^b_0 \geq \bar{e}_0 \). When \( w \) is extremely low, G gives up to keep the expected payoff of B at \( w \), since the cost to do so is too high.\(^{18}\)

Proposition 3 implies that the agency problem in the state-owned firm can improve the expected social welfare depending on the reservation utility level of the bureaucrat. Furthermore, it implies that when the maximum cost of agency problem is low, which is the case when \((\bar{e}_0, e^d_0) \subset A_0 \), the agency problem in the state-owned firm improves the expected social welfare if the level of reservation utility is not too high, and does not damage the expected social welfare.

When \( w < \hat{w} \), firm 0’s investment level \( e^b_0 \) is not efficient, given firm 1’s investment level \( e^b_1 \). However, if G controls the investment level to an efficient level, the set of investment levels results in \( e^d \), and consequently, the expected social welfare can be damaged. The inefficiency of bureaucratic managements is often criticized. However, when the agency problem in state-owned firms is considered, the response of the private sector and the expected social welfare in equilibrium also need to be evaluated.

The driving force of the result is that making the decision of the government less aggressive can improve the expected social welfare when the decisions of the government and the private sector are strategic substitutes. However, in such a situation, committing to be less aggressive is not self-enforcing and requires a certain kind of commitment device. In my analysis, the employment of B that produces the agency problem in the state-owned firm is the commitment device.\(^{19}\) I consider that G cannot use...

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\(^{18}\)Technically, \( \bar{e} \) represents the set of investment levels when the IC constraint and one of the LL constraints, i.e., \( w^b \geq 0 \), are binding, but the IR constraint is not binding.

\(^{19}\)I suppose that it is too costly for G to fire and alter B during the initial stage.
the contract as the device. Even if \( G \) discloses the contract before \( P \)'s decision, it does not necessarily imply that \( G \) can commit to the contract, or equivalently the investment level it draws from \( B \).

At the end of this subsection, I comment on the effect of contracting-outs and privatizations, which are considered to improve the efficiency of management. Suppose that there are a sufficient number of potential entrants with sufficient wealth. \( G \) can contract with one of them to commission firm 0’s management. When \( G \) can observe the realized technological environment, it can directly induce the agent to choose the output level similar to that under a direct management in each environment. Moreover, \( G \) can draw an investment level similar to that under a direct management \( e_0^d \) by an appropriate incentive contract, since the agent has sufficient wealth. Note that a contract that targets a lower investment level is not self-enforcing. Furthermore, contracting with an agent without sufficient wealth is not self-enforcing, if \( G \) can easily re-contract with another agent.\(^{20}\) Therefore, a contracting-out can damage the expected social welfare when the government can use a sophisticated incentive scheme. Even in a case wherein \( G \) sells out the state-owned firm to an entrant, i.e., privatizes the firm, similar outcomes, pertaining to both the output and investment levels, can be achieved in equilibrium when \( G \) can use a sophisticated selling-out contract embedding appropriate contingent terms.\(^{21}\) Therefore, privatization can also damage the expected social welfare in such a case as mentioned above.\(^{22}\)

### 4.3 Optimal Bureaucratic System

I have investigated the equilibrium outcomes given an exogenous reservation utility level \( w \) and the non-negative wage limitation. I now consider bureaucratic systems setting a minimum wage level \( w_{\text{min}} \geq 0 \) and a guaranteed utility level \( w' \geq w \). Suppose that the government can commit any bureaucratic

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\(^{20}\) I consider that altering the commissioned entrepreneurs is easier than altering the employees.

\(^{21}\) More precisely, \( G \) can accomplish it with ex-post transfers that internalize \( G \)'s objective into the agent’s objective.

\(^{22}\) These results pertaining to contracting-outs and privatizations are attributed to the consideration that the government controls the management completely by means of a sophisticated system. If the government cannot control the management completely or it can commit to control it incompletely, the equilibrium outcomes will differ from those in this study. Analyses on these cases are beyond the scope of this study.
Then, the IR constraint and LL constraints in G’s problem are altered by

\[ v(e_0)w^g + (1 - v(e_0))w^b - e_0 \geq w' \]
\[ u^g \geq w_{\text{min}} \text{ and } w^b \geq w_{\text{min}}, \]

respectively.

Let \( e^b \) denote the equilibrium set of investment levels in the original problem. Then, I have the following.

**Proposition 4.** Setting \((w_{\text{min}}, w') = (0, w)\) is suboptimal from the viewpoint of expected social welfare, if \( e^b_0 \notin \arg \max_{e_0 \in \tilde{e}_0, e_0} EW(e_0, R_1^I(e_0)) \).

Both the minimum wage and guaranteed utility systems are considered as devices for guaranteeing a certain utility level of bureaucrats. However, the two systems also control the incentive of management and consequently affect the expected social welfare. The minimum wage system makes incentive contracts less intensive and leads to a less aggressive behavior of bureaucrats. On the other hand, the guaranteed utility system makes the contracts more intensive and leads to more aggressive bureaucratic behavior. An optimal bureaucratic system, which maximizes the expected social welfare, is a system \((w_{\text{min}}, w')\) that induces an optimal investment level \( e_0 \in \arg \max_{e_0 \in \tilde{e}_0, e_0} EW(e_0, R_1^I(e_0)) \) in equilibrium.

Suppose that the optimal investment level is unique \( e^{**}_0 = \arg \max_{e_0 \in \tilde{e}_0, e_0} EW(e_0, R_1^I(e_0)) \) for simplicity. There is continually infinite number of optima since the wage is merely a transfer from the state-owned firm to the bureaucrat. However, \( w_{\text{min}} = 0 \) is never optimal when \( e^b_0 > e^{**}_0 \). G cannot induce \( e^{**}_0 \) under any guaranteed utility system given \( w_{\text{min}} = 0 \), and increase in \( w_{\text{min}} \) reduces the target investment level and helps G attain the optimal investment level. Similarly, \( w' \) is never optimal when \( e^b_0 < e^{**}_0 \). Proposition 4 implies that the bureaucratic system \((w_{\text{min}}, w') = (0, w)\) is suboptimal for almost every case.

People often consider that managers in some state-owned firms enjoy higher utility levels and that those in other state-owned firms are less responsible for bad performance. My results imply, however,

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\(^{23}\)It seems to be expedient to assume that committing to systems is sustainable, although committing to contracts is not. However, I consider that designing a system requires very complex political procedures and that the government cannot change the system in the short run.
that these aspects can be desirable from the viewpoint of expected social welfare. In other words, setting the utility level of state-owned firms’ managers higher than the required level is possibly necessary to make more intensive contracts feasible when the agency problem in state-owned firms is considerably serious. Adversely, setting the minimum wage level positive, which implies lessening the responsibility of bureaucrats, can be required to restrain the intensity of contracts when the agency problem in state-owned firms is not extremely serious.

5 Concluding Remarks

I investigate an innovative interaction followed by a quantity-setting competition in a mixed duopoly where a state-owned firm and a private firm compete with each other. The agency problem derived from the limited liabilities in a state-owned firm reduces the investment level of the state-owned firm and increases that of a competing private firm. This substitution of the investment levels can improve the expected social welfare. In other words, a small degree of inefficiency in state-owned firms does not damage but improve welfare in mixed markets. This result implies that contracting-outs and privatizations can damage expected social welfare.

Even when the equilibrium outcome under a bureaucratic management is preferable to that in the absence of the agency problem (under a direct management), the inefficiency of the bureaucratic management is criticized. This is because the external effects derived from the bureaucratic management are neglected. It is true that the investment level under a bureaucratic management is less than the optimal level, given the investment level of the private competitor. However, inducing a higher investment level reduces the private competitor’s investment level and consequently can damage the expected social welfare in equilibrium. It is often considered that the government should contract out or privatize state-owned firms in order to improve the efficiency of managements. However, if the government can completely control the agent’s behavior by a sophisticated contract, the equilibrium outcome under contracting-outs or privatization is similar to that under direct managements. Therefore, these policies damage the expected social welfare, if the inefficiency of bureaucratic managements improves the expected social welfare.

I apply the model to investigate bureaucratic systems. The government can either use a minimum wage system as a commitment device to behave less aggressively or a guaranteed utility system, to
behave more aggressively. Thus, the government can maximize the expected social welfare, given the response of private competitors, by means of these systems. I find that a bureaucratic system with the minimum wage level equal to zero and the guaranteed utility level equal to the reservation utility level is suboptimal for almost every case. It is often pointed out that managers in some state-owned firms enjoy higher utility levels and that those in other state-owned firms are less responsible for bad performance; further, these aspects are cited as evidence that bureaucratic managements are inefficient. However, these aspects can be desirable from the viewpoint of expected social welfare.

I want to emphasize that I do not conclude that inefficient bureaucratic management is always preferable to efficient contracting-out and privatization. I merely want to state that when we evaluate bureaucratic managements in a mixed market, we should consider the external effects on the competitors’ behaviors and the expected social welfare.

Finally, I must concede that my analyses are restrictive. For example, when the information structure with respect to technological environments and output levels is asymmetric, the agency problem affects not only the incentive of investment but also that of production. Therefore, the effects are considerably more complex. Investigations on these effects will be a topic of future research.

Appendix A

A.1 Lemma 1

Let \((q_0, q_1)\) and \((\theta_0, \theta_1)\) be sets of quantities and technological levels of firms 0 and 1 respectively. Then, the objectives of G and P at the beginning of stage 2 are written as

\[
\int_{q_0}^{q_0 + q_1} p(Q)dQ - c(q_0; \theta_0) - c(q_1; \theta_1)
\]

and

\[
p(q_0 + q_1)q_1 - c(q_1; \theta_1),
\]

respectively. Following from first-order conditions, the best responses of G and P are written as

\[
BR_0(q_1) = \max \left( \frac{a - \theta_0 - q_1}{b + 1}, 0 \right)
\]
and 
\[
BR_1(q_0) = \max \left( \frac{a - \theta_1 - q_0}{b + 2}, 0 \right),
\]
respectively. Let \((q_0^b, q_1^b)\) be the equilibrium quantity set depending on the technological environment.

Then, I have
\[
\begin{align*}
q_0^* &= \frac{1}{2(b^2 + 5b + 1)} \{ (1 + b)a - (2 + b)\theta^k + \theta^l \}, \\
q_1^* &= \frac{1}{2(b^2 + 5b + 1)} \{ ba + \theta^k - (1 + b)\theta^l \},
\end{align*}
\]

Therefore, I have the following:
\[
\begin{align*}
Z^{gg^*} - Z^{gb^*} &= \frac{(b^2 + 4b + 2)(\theta^b - \theta^g) \{ b(a - \theta^b) + b(a - \theta^g) - (\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} > 0, \\
Z^{gb^*} - Z^{bb^*} &= \frac{(b^2 + 4b + 2)(\theta^b - \theta^g) \{ b(a - \theta^b) + b(a - \theta^g) + (\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} > 0, \\
\left( Z^{gb^*} - Z^{bb^*} \right) - \left( Z^{gg^*} - Z^{gb^*} \right) &= \frac{2(b^2 + 4b + 2)(\theta^b - \theta^g)^2}{2(b^2 + 3b + 1)^2} > 0, \\
Z^{gb^*} - Z^{bb^*} &= \frac{(\theta^b - \theta^g) \{ 2(b^2 + 4b^2 + 3b + 1)(a - \theta^b) + (b^2 + 3b^2 - b - 1)(\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} \\
& \geq \frac{(\theta^b - \theta^g) \{ 2(a - \theta^b) + (b^2 + 5b^2 + 7b + 5)(\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} > 0, \\
\left( Z^{gb^*} - Z^{bb^*} \right) - \left( Z^{gg^*} - Z^{gb^*} \right) &= \frac{2(b^2 + 4b + 2)(\theta^b - \theta^g)^2}{2(b^2 + 3b + 1)^2} > 0, \\
\pi_1^{gg^*} - \pi_1^{gb^*} &= \frac{(b^2 + 3b + 2)(\theta^b - \theta^g) \{ b(a - \theta^b) + b(a - \theta^g) - (\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} > 0, \\
\pi_1^{gb^*} - \pi_1^{bb^*} &= \frac{(b^2 + 3b + 2)(\theta^b - \theta^g) \{ b(a - \theta^b) + b(a - \theta^g) + (\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} > 0, \\
\left( \pi_1^{bb^*} - \pi_1^{gb^*} \right) - \left( \pi_1^{gg^*} - \pi_1^{gb^*} \right) &= \frac{2(b^2 + 3b + 2)(\theta^b - \theta^g)^2}{2(b^2 + 3b + 1)^2} > 0, \\
(CS_{gb^*}^* + \pi_0^{gb^*}) - (CS_{gb^*}^* + \pi_0^{gb^*}) &= \frac{b(\theta^b - \theta^g) \{ b(a - \theta^b) + b(a - \theta^g) - (\theta^b - \theta^g) \}}{2(b^2 + 3b + 1)^2} > 0,
\end{align*}
\]
and

\[ (CS^{gb^*} + \pi_0^{bg^*}) - (CS^{bb^*} + \pi_0^{bb^*}) = \frac{b(\theta_b - \theta_g)\{b(a - \theta_b) + b(a - \theta_g) + (\theta_b - \theta_g)\}}{2(b^2 + 3b + 1)^2} > 0 \]

A.2 Lemma 2

Since

\[ EW(e_0, R^d_1(e_0)) = v(e_0)[v(R^d_1(e_0))Z^{gg^*} + (1 - v(R^d_1(e_0)))Z^{gb^*}] + (1 - v(e_0))[v(R^d_1(e_0))Z^{gb^*} + (1 - v(R^d_1(e_0)))Z^{bb^*}] - e_0 - R^d_1(e_0), \]

the derivative of this is denoted by

\[ v'(e_0) \left[ v(R^d_1(e_0)) \left( Z^{gg^*} - Z^{gb^*} \right) + (1 - v(R^d_1(e_0))) \left( Z^{gb^*} - Z^{bb^*} \right) \right] - 1 + \frac{dR^d_1(e_0)}{de_0} \left[ v'(e_1^d) \left[ v(e_0^d) \left( Z^{gg^*} - Z^{gb^*} \right) + (1 - v(e_0^d)) \left( Z^{gb^*} - Z^{bb^*} \right) \right] - 1 \right]. \]

Then, I have

\[ \frac{dEW(e_0^d, R^d_1(e_0^d))}{de_0} = \frac{dR^d_1(e_0^d)}{de_0} \left[ v'(e_1^d) \left[ v(e_0^d) \left( Z^{gg^*} - Z^{gb^*} \right) + (1 - v(e_0^d)) \left( Z^{gb^*} - Z^{bb^*} \right) \right] - 1 \right] \]

following from (1). Here,

\[ \frac{dR^d_1(e_0^d)}{de_0} = \frac{v'(e_0^d)v'(e_1^d)\left\{ \pi_1^{bg^*} - \pi_1^{bb^*} - \left( \pi_1^{gg^*} - \pi_1^{gb^*} \right) \right\}}{v'(e_1^d)\left[ v(e_0^d) \left( \pi_1^{gg^*} - \pi_1^{gb^*} \right) + (1 - v(e_0^d)) \left( \pi_1^{bb^*} - \pi_1^{gb^*} \right) \right]} < 0 \]

following from (3) and (i) and (ii) in Lemma 1, and

\[ v'(e_1^d) \left[ v(e_0^d) \left( Z^{gg^*} - Z^{gb^*} \right) + (1 - v(e_0^d)) \left( Z^{gb^*} - Z^{bb^*} \right) \right] > v'(e_1^d) \left[ v(e_0^d) \left( \pi_1^{gg^*} - \pi_1^{gb^*} \right) + (1 - v(e_0^d)) \left( \pi_1^{bb^*} - \pi_1^{gb^*} \right) \right] = 1, \]

following from (2) and (iii) in Lemma 1. Therefore, I have \( \frac{dEW(e_0^d, R^d_1(e_0^d))}{de_0} < 0 \) and this lemma.
A.3 Proposition 1

The Lagrangian of G’s problem is given by

\[
L = EW(e_0, e_1) - (1 - \zeta) \left[ v(e_0) w^g + (1 - v(e_0)) w^b - e_0 \right] \\
+ \lambda \left[ v(e_0) w^g + (1 - v(e_0)) w^b - e_0 - w \right] + \gamma \left[ v'(e_0) \left( w^{g'} - w^{b'} \right) - 1 \right],
\]

where \( \lambda \geq 0 \) and \( \gamma \geq 0 \) denote the Lagrange multipliers on the IR and IC constraints respectively. Let \((e_0', w^{g'}, w^{b'}, \lambda', \gamma')\) be the solution, given \(e_1\). Then, the first-order conditions can be reduced to the following:

\[
v'(e_0') \left[v(e_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] = 1 - \frac{\gamma' v''(e_0')}{v'(e_0')}, \quad (A1) \\
v(e_0') w^{g'} + (1 - v(e_0')) w^{b'} - e_0' \geq w, \quad (A2) \\
v'(e_0') \left( w^{g'} - w^{b'} \right) = 1, \quad (A3) \\
(1 - \zeta - \lambda') v(e_0') - \gamma' v'(e_0') = 0, \quad (A4) \\
(1 - \zeta - \lambda')(1 - v(e_0')) + \gamma' v'(e_0') \geq 0, \quad (A5) \\
\lambda' \left[v(e_0') w^{g'} + (1 - v(e_0')) w^{b'} - e_0' - w \right] = 0, \quad (A6)
\]

\[w^{g'} > w^{b'} \geq 0, \quad \lambda' \geq 0, \text{ and } \gamma' \geq 0.\]

Note that the LL constraint \( w^{g'} \geq 0 \) is never binding and that (A4) holds since I have \( w^{g'} > w^{b'} \) following from (A3) and \( e_0' \in (0, \bar{e}) \) for all \( e_1 \) due to the assumption on \( v(e) \).

Substituting \( \zeta = 1 \) into (A4) and (A5), I have

\[-\lambda' v(e_0') - \gamma' v'(e_0') = 0, \]

\[-\lambda'(1 - v(e_0')) + \gamma' v'(e_0') \geq 0.\]

Substituting the former into the latter, I have \(-\lambda' \geq 0\), which implies that \( \lambda' = 0 \). Since \( e_0' > 0 \), I also have \( \gamma' = 0 \). It follows that (A1) is equivalent to (1). Therefore, the set of equilibrium investment levels are given by \((e_0^d, e_1^d)\).
A.4 Proposition 2

Let \( R_{d0}^d(e_1) \) be G’s response function under a direct management, that is,

\[
R_{d0}^d(e_1) = v^{-1}\left(\frac{1}{v(e) (Z_{g^{g*}} - Z_{b^{g*}}) + (1 - v(e)) (Z_{g^{b*}} - Z_{b^{b*}})}\right).
\]  

(A7)

Then I have the following.

Lemma 3. \( e_0' < R_{d0}^d(e_1) \) if and only if the following inequality holds.

\[
v(R_{d0}^d(e_1)) > R_{d0}^d(e_1) + w
\]  

(A8)

Proof. Since \( e_0' \in (0, \bar{e}) \), \( \bar{e}, \gamma' = 0 \) if and only if \( \lambda' = 1 - \zeta \) and (A5) with equality hold. This implies that the IR constraint is binding, while the LL constraint \( w^b \geq 0 \) is not binding, following from (A4) and (A5). Therefore, I have

\[
\gamma' > 0 \iff v(R_{d0}^d(e_1))w^g > R_{d0}^d(e_1) + w \text{ for } w^g \text{ s.t } v(R_{d0}^d(e_1))w^g = 1
\]

\[
\iff \frac{v(R_{d0}^d(e_1))}{v'(R_{d0}^d(e_1))} > R_{d0}^d(e_1) + w
\]

\[
\iff (A8).
\]  

(A9)

Note that \( e_0' \leq R_{d0}^d(e_1) \) following from (1) and (A1). Then, I have this lemma. \( \square \)

Suppose that (A8) holds. Then, I have

\[
\lambda' \left(\frac{v(e_0')}{v'(e_0')} - e_0' - w\right) = 0
\]

following from (A3) and (A6). When \( \lambda' > 0 \), which implies that the IR constraint is binding, \( e_0' \) is a constant \( e_0^{LL} \) such that

\[
\frac{v(e_0^{LL})}{v'(e_0^{LL})} - e_0^{LL} - w = 0
\]  

(A10)

independent of \( e_1 \). \( e_0^{LL} \) is unique following from (A10) and the assumption on \( v(e) \).
On the other hand, when \( \lambda' = 0 \), which implies that the IR constraint is no longer binding, I have

\[
\gamma' = \frac{(1 - \zeta)v(e_0')}{v'(e_0')}
\]

following from (A4). Substituting this into (A1), I have

\[
v'(e_0') \left[ v(e_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] = 1 - \frac{(1 - \zeta)v(e_0')v''(e_0')}{[v'(e_0')]^2}. \tag{A11}
\]

Let \( R_0^d(e_1) \) be the solution of equation (A11) with respect to \( e_0' \). Following from the assumption on \( v(e) \), \( R_0^d(e_1) \) is unique, continuous with respect to \( e_1 \), and decreasing in \( e_1 \).

**Lemma 4.** \( e_0' \) is continuous with respect to \( e_1 \).

**Proof.** Depending on \( e_1, e_0' \) can be \( R_0^d(e_1), e_0^{LL} \), or \( R_0^d(e_1) \). Since the three candidates are continuous with respect to \( e_1 \), it should suffice to show that \( e_0' \) is continuous at possible thresholds. Suppose there exists a value \( e_1^{LL} \in (0, \bar{e}) \) such that

\[
v(R_0^d(e_1^{LL})) \left[ v(e_1^{LL}) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e_1^{LL})) \left( Z^{gb*} - Z^{bb*} \right) \right] - R_0^d(e_1^{LL}) = \bar{w}. \tag{A12}
\]

Then, \( e_0' = R_0^d(e_1) \) for \( e_1 \geq e_1^{LL} \) and \( e_0' < R_0^d(e_1) \) for \( e_1 < e_1^{LL} \) following from

\[
\frac{d}{de_1} \left[ v(R_0^d(e_1)) \left[ v(e_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] - R_0^d(e_1) \right] = \frac{dR_0^d(e_1)}{de_1} \left[ v'(R_0^d(e_1)) \left[ v(e_1) \left( Z^{gg*} - Z^{bg*} \right) + (1 - v(e_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] - 1 \right] + v'(R_0^d(e_1))v(e_1) \left[ \left( Z^{gg*} - Z^{bg*} \right) - \left( Z^{gb*} - Z^{bb*} \right) \right] < 0
\]

and Lemma 3. Since \( e_0' = R_0^d(e_1^{LL}) \) when \( e_1 = e_1^{LL} \), I have

\[
\frac{v(R_0^d(e_1^{LL}))}{v'(R_0^d(e_1^{LL}))} - R_0^d(e_1^{LL}) - \bar{w} = 0
\]

and \( R_0^d(e_1^{LL}) = e_0^{LL} \) following from (A7) and (A12). This result implies that \( e_0' \) is continuous at \( e_1 = e_1^{LL} \).
Next, consider a system of \((\hat{\lambda}, \hat{\gamma})\) as follows.

\[
\begin{cases}
v'(e_0) \left[ v(e_1) \left( Z^{gg*} - Z^{gb*} \right) + (1 - v(e_1)) \left( Z^{gb*} - Z^{bb*} \right) \right] = 1 - \hat{\gamma} \frac{v''(e_0)}{v'(e_0)} \\
(1 - \zeta - \hat{\lambda})v(e_0) - \hat{\gamma}v'(e_0) = 0
\end{cases}
\]

I can see that \(\hat{\lambda}\) is continuous and increasing with respect to \(e_0\) and \(e_1\). Suppose that there exists a value \(e_1^{IR} \in (0, \bar{e})\) such that \(\hat{\lambda} = 0\) for \((e_0, e_1) = (e_0^{LL}, e_1^{IR})\). Then, I have

\[
e_0' = \begin{cases} R_0^d(e_1) & \text{for } e_1 \leq e_1^{IR} \\
e_0^{LL} & \text{for } e_1 \geq e_1^{IR}
\end{cases}
\]

at the neighborhood of \(e_1^{IR}\). This result implies that \(e_0'\) is continuous at \(e_1 = e_1^{IR}\).

Following from Lemma 4 and \(e_0' \leq R_0^d(e_1)\) for all \(e_1\), \(e_0^b < e_0^d\) if and only if \(e_0' < R_0^d(e_1)\) at \(e_1 = e_1^{d}\). Moreover, \(e_0' < R_0^d(e_1)\) at \(e_1 = e_1^{d}\) is equivalent to \(w < \hat{w}\) following from Lemma 3. Since \(e_0^b < e_0^d\) is equivalent to \(e_1^b > e_1^d\) following from \(e_1^b = R_0^d(e_0^b)\) and \(\frac{dR_0^d(e_0)}{de_0} < 0\), I have this proposition.

### A.5 Proposition 3

I want to show the relationship between the level of reservation utility \(w\) and the equilibrium investment levels \((e_0^b, e_1^b)\). First, I show the following.

**Lemma 5.** \(e_0^{LL}\) is increasing in \(w\).

**Proof.** Note that \(e_0^{LL}\) satisfies (A10). Therefore, I have this lemma since

\[
\frac{d}{de} \left( \frac{v(e)}{v'(e)} - e \right) = -\frac{v(e)v''(e)}{[v'(e)]^2} > 0.
\]

Next, I show the following.

**Lemma 6.** For any \(e_1\) such that \(e_0' < R_0^d(e_1)\), \(e_0' = R_0^d(e_1)\) if \(e_0^{LL} < R_0^d(e_1)\).
Proof. I will show the contraposition. Note that \( e_0' \) is \( e_0^{LL} \) or \( R_0^d(e_1) \) for any \( e_1 \) such that \( e_0' < R_0^d(e_1) \).

I have this lemma since

\[
e_0' = e_0^{LL} \neq R_0^d(e_1) \Rightarrow \lambda' > 0
\]

\[
\Rightarrow \gamma' < (1 - \zeta) \frac{v(e_0^{LL})}{v'(e_0^{LL})}
\]

\[
\Rightarrow e_0^{LL} > R_0^d(e_1)
\]

following from (A6), (A4), and (A1).

Combining Lemmas 5 and 6, it can be seen that \( e_0^d \) decreases and stops at \( \hat{e}_0 \), as \( \hat{w} \) reduces from \( \hat{w} \) to 0. Then, I have this proposition.

A.6 Proposition 4

The change of setting alters (A10) to

\[
\frac{v(e_0^{LL})}{v'(e_0^{LL})} - e_0^{LL} + w_{\text{min}} - w' = 0,
\]

which implies that an increase in \( w_{\text{min}} \) and a decrease in \( w' \) reduce the constant \( e_0^{LL} \) and keep \( R_0^d(e_1) \) unchanged. Then, any \( e_0 \in [\hat{e}_0, e_0^d] \) can be induced in equilibrium by controlling \( w_{\text{min}} \) and/or \( w' \). Therefore, I have this proposition.

References


*Australian Economic Papers* 44(3), 275–289