Oligopoly with a large number of competitors: Asymmetric limit result*

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Abstract

We address how profitable innovation is in a competitive market by investigating the asymmetric oligopoly model in which 1 firm (innovator) has a cost advantage that is not drastic enough for her to become a monopolist, and by inducing asymmetric limit outcomes when the number of the other firms (laggard firms) goes to infinity. If the innovator is the Stackelberg leader, two cases can arise: (i) the innovator behaves as in the competitive market or (ii) she occupies the entire market but cannot make the price. While the innovator earns the ordinary price-taking profit in the case (i), the case (ii) brings a larger profit than the price-taking profit. If we consider Cournot competition, the innovator becomes the partial monopolist. In this case, the innovator's profit can be smaller than the price-taking profit.

Keywords and Phrases: Stackelberg game; Cournot game; Limit result; Marginal-cost advantage; Arrow effect.

JEL Classification Numbers: L11; L12; L13.

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1 Introduction

> Entrepreneurship is rewarded with extraordinary profit. It is the profit for being first (Kamien and Schwartz [4], pp.8). There is a positive relationship between innovation and monopoly power with the concomitant above normal profits (Kamien and Schwartz, pp.22). Realizing extraordinary profits is of course the motive ... for developing an innovation (Kamien and Schwartz, pp.27).

This is a simple explanation of the Schumpeterian hypothesis. We often encounter the situation where one firm succeeds in acquiring a new technology in a competitive market. In such a market, before the new technology spreads, only one firm has the technical advance. Following Schumpeter, many economists believe that the innovation will bring some sort of monopoly power and extraordinary profit. Is this belief true? If so, what kind of market structure arises and how much will the realized profit be? Our aim is to provide rigorous game theoretic answers to these questions.

We address these questions by investigating the quantity-setting oligopoly model in which there is 1 firm that has a technical advantage (innovator) and n firms that have the same prevailing technology (laggard firms).[1] Corresponding to the competitiveness of the pre-innovation market, n should be a large number. Therefore, we induce limit outcomes when the number of laggard firms goes to infinity. We investigate both types of games: those in which the innovator is the leader (Stackelberg competition) and those in which all the firms move simultaneously (Cournot competition). Compatible with the Schumpeterian hypotheses, this model results in a monopoly market when innovation is drastic in both Stackelberg and Cournot games.

When the innovation is non-drastic, the following results are supported in the limit. When the innovator is the Stackelberg leader, two cases arise: if the innovation is minor, the innovator still produces the competitive outcome; if not so minor, the result in Arrow's classic analysis of invention is supported. Therefore, the innovator with the minor innovation earns the ordinary price-taking profit. On the other hand, Arrow's outcome brings a larger profit than the price-taking profit. When we consider Cournot competition, only one case arises for non-drastic innovation: partial monopoly à la Forchheimer. In this, the innovator's profit can be smaller than the price-taking profit.

[1] Kamien and Schwartz [4] involves an earlier work addressing the Schumpeterian hypothesis in a Cournot-type oligopoly model with linear costs and demand.

As mentioned above, two classic analyses are provided with game theoretic bases in our model. Arrow [1] is an earlier work that addresses our main question. According to his analysis, even if the cost advantage is non-drastic, the inventor gets the power to supply the entire market but cannot enjoy the pure monopoly price and takes the competitive price of the pre-invention market. Meanwhile, partial monopoly, which was historically presented by Forchheimer, is another kind of classic discussion about a market with one advanced firm.[2] The partial monopolist is considered as a price maker among perfectly competitive firms.[3]

Two fields relate to our limit results. One field is the literature of the Cournot limit theorem (e.g., Ruffin [11], Novshek [7]). This field aims to show that the price converges to the competitive price when all the firms' technologies shrink relatively to the scale of the market size. By contrast, in our analysis, the innovator's scale need not shrink since the innovative technology is given. Therefore, it would be appropriate to call our results *asymmetric* limit results. The other field is that of patent licenses (e.g., Kamien and Tauman [5], Kamien [3]). In these papers, essentially the same situation as that of the Stackelberg limit case is investigated; however, this is done under a simple, specific setting with linear costs and demand. In our paper, we generalize the setting. In particular, if we consider strictly convex costs, the competitive outcome of minor innovation and the adverse effect of the "Arrow effect" are observed.

The rest of the paper is organized as follows. Section 2 presents our model. In Section 3, we explain our main theorems, which describe the limit world. Section 4 provides the formal analysis to induce these results. In Section 5, a specific example of our model is examined and some simulations are offered to describe the path to the limit. By doing so, we can delve deeper into the reasons underlying our results. Section 6 considers some interesting applications. The adverse effect of the Arrow effect is discussed here. Finally, Section 7 concludes the paper and provides future topics for discussion.

[2] Reid [9] interprets and summarizes Forchheimer's original discussion in English.
[3] Reid [8] offers a formal model according to this definition.

2 The model

We focus our attention on one homogeneous good's market in the economy. In this market, 1 firm with a new (innovative) technology and $n \in \mathbb{N}$ firms with an old (prevailing) technology are in quantity-setting oligopoly competition. We call the firm with the new technology the *innovator*; those with the old technology *laggard firms*. Label the innovator 0 and the laggard firms $1, \dots, n$. Let L^n be the set of laggard firms, i.e., $L^n \equiv \{1, \dots, n\}$.

$C : \mathbb{R}^2_+ \to \mathbb{R}_+$ is the cost function of a firm. $C(q, \delta)$ denotes the cost for a firm with marginal-cost disadvantage δ to produce q-unit goods. Hereafter, we use a simpler notation $C_\delta(q) \equiv C(q, \delta)$, to treat the cost function as if it were an ordinary one-variable function. Let C_δ be a differentiable, increasing, and strictly convex function. Moreover, let $C_\delta(0) = 0$ for all $\delta \in \mathbb{R}_+$.[4] For any q, let $C_\delta(q)$ be continuous in δ. We assume that $C_\delta(q) = C(q, \delta)$ has strictly increasing differences in (q, δ), i.e., $C'_{\bar{\delta}}(q) > C'_{\underline{\delta}}(q)$ for all $\bar{\delta}, \underline{\delta} \in \mathbb{R}_+$ such that $\bar{\delta} > \underline{\delta}$ and all $q \in \mathbb{R}_+$.[5] We assume that $\delta = 0$ for the new technology's cost disadvantage and $\delta = d \in \mathbb{R}_{++}$ for the laggard technology's.

$P : \mathbb{R}_{++} \to \mathbb{R}_+$ is the inverse demand function. We assume that P is continuous. Let P be a differentiable and strictly decreasing function in $P^{-1}(\mathbb{R}_{++})$. Further, we posit some restrictions on the shape of the demand function.

Assumption 1 (Second order and stability condition)**.**
$P'(Q) + P''(Q)q < 0$ for all $Q \in P^{-1}(\mathbb{R}_{++})$ and all $q \in [0, Q]$.

Assumption 2 (Interior solution)**.**
Incentive to supply a nearby zero is positive,[6] i.e., $\lim_{Q \downarrow 0} \left(P(Q) + P'(Q)Q - C'_\delta(Q)\right) > 0$.

Assumption 3 (No positive lower bound)**.**
Lower bound of price is zero[7] so that $\lim_{Q \to \infty} P(Q) = 0$.

Each firm $i \in \{0\} \cup L^n$ is supposed to maximize its profit by choosing its output level q_i. As a matter of course, if $q_i = 0$, the profit of firm i is equal to $-C_\delta(0) = 0$. From Assumption 3 and the strict

[4] This assumption means that there is no entry cost since C_δ is continuous at 0. This assumption is reasonable when we focus our attention on the most competitive market under the old technology, i.e., $n \to \infty$. We discusses how to introduce fixed costs in our model in Section 7.

[5] If we assume that $C(q, \delta)$ is differentiable in (q, δ), we can say the same thing by the expression $\partial C'_\delta(q)/\partial\delta \equiv \partial^2 C(q, \delta)/\partial q \partial \delta > 0$.

[6] This assumption implies that demand is so adequate that $\lim_{Q \downarrow 0} P(Q) > C'_d(0)$.

[7] In our assumption, it does not matter whether or not there exists $\bar{Q} \in \mathbb{R}_{++}$ such that for all $Q \geq \bar{Q}$, $P(Q) = 0$.

convexity of C''_δ, a firm's profit must be strictly negative if its output exceeds some level. Therefore, we can regard each firm's action space of q_i as a compact interval between zero and a sufficiently large number.

We consider two types of games: one is the Stackelberg game; the other, the Cournot game. In the Stackelberg game, firm 0 is the leader and the remaining n firms are followers. In the Cournot game, all firms move simultaneously. Let $\mathcal{S}^{d,n}$ ($\mathcal{C}^{d,n}$) denote the Stackelberg (Cournot) game such that each laggard firm's cost disadvantage is d and the number of laggard firms is n.

In our setting, we can show that game $\mathcal{C}^{d,n}$ has the unique Nash equilibrium.[8] Let $q_0^{\mathcal{C}^{d,n}}$ denote the equilibrium strategy of the innovator in $\mathcal{C}^{d,n}$. We can also show that there exists a subgame perfect equilibrium for the game $\mathcal{S}^{d,n}$. The game $\mathcal{S}^{d,n}$, however, can have multiple equilibria.[9] Take arbitrarily one of the equilibrium strategies of the innovator in $\mathcal{S}^{d,n}$ and let $q_0^{\mathcal{S}^{d,n}}$ denote this. We will research where the sequences of the innovator's equilibrium outputs, $\left(q_0^{\mathcal{S}^{d,n}}\right)_{n\in\mathbb{N}}$ and $\left(q_0^{\mathcal{C}^{d,n}}\right)_{n\in\mathbb{N}}$, converge when n goes to infinity.

Observe that if the innovation is drastic, in all the games $\mathcal{S}^{d,n}$ and $\mathcal{C}^{d,n}$, the innovator's equilibrium output is its monopoly quantity q^{M},[10] which is given by $P\left(q^{\mathrm{M}}\right)+P'\left(q^{\mathrm{M}}\right)q^{\mathrm{M}} = C'_0\left(q^{\mathrm{M}}\right)$. More precisely, assume that there exists $\bar{d} \in \mathbb{R}_{++}$ such that $P\left(q^{\mathrm{M}}\right) = C'_{\bar{d}}(0)$. Then, we can show that $P\left(q^{\mathrm{M}}\right) \leq C'_d(0)$ if and only if $d \geq \bar{d}$. In other words, if $d \geq \bar{d}$, each laggard firm's marginal cost is larger than even the monopoly price for the innovator. Therefore, the innovator can deter the laggard firms from entering the market by supplying its monopoly quantity and can enjoy the monopoly profit regardless of the number of laggard firms. Thus, it is clear that the innovator's equilibrium output converges to its monopoly quantity in both Stackelberg and Cournot games. Hereafter, we will focus on the case where $d < \bar{d}$, i.e., the innovation is not drastic, which is the case where we cannot readily jump to the conclusion of the innovator's monopoly.

[8] Rosen [10] proved that our Cournot game has the unique Nash equilibrium in a more general setting.

[9] It is well known that the multiple equilibria sometimes arise in the Stackelberg model. This is because we cannot see whether the second order condition of a leader is globally met since the leader takes the followers' reactions into account. Our analysis works well even in such a situation.

[10] Another equilibrium does not exist when the innovation is drastic. As stated above, the uniqueness of the Cournot-Nash equilibrium is preserved by our setting. With regard to Stackelberg games, the innovator's profit never exceeds its monopoly profit since the leader has less residual demand when its rivals' outputs are positive than when they are zero. Thus, if the innovator can earn a monopoly profit, the innovator's monopoly must be the unique equilibrium.

3 Main results

In this section, we will first present our main results. The formal analysis to induce these results will be presented in the next section.

3.1 Benchmark: Perfect competition

As a benchmark, we consider perfect competition, where all the firms including firm 0 are price takers. Let $\mathcal{P}^{d,n}$ denote this model and denote the innovator's equilibrium output by $q_0^{\mathcal{P}^{d,n}}$.

The following lemma tells us which level of quantity the innovator produces if there are a large number of laggard firms.

Lemma 1.

$$\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = \begin{cases} q_d^{\mathrm{P}} & \text{if } C_d'(0) \leq P\left(q^{\mathrm{P}}\right), \\ q^{\mathrm{P}} & \text{if } C_d'(0) > P\left(q^{\mathrm{P}}\right), \end{cases}$$

where q^{P} is defined by $P\left(q^{\mathrm{P}}\right) = C_0'\left(q^{\mathrm{P}}\right)$ and q_d^{P} by $C_d'(0) = C_0'\left(q_d^{\mathrm{P}}\right)$.

Proof. See Appendix B. **Q.E.D.**

This is a well-known result in the limit of perfect competition. The innovator prices at the marginal cost and produces the competitive fringe of the demand.

3.2 The innovator's limit output in the Stackelberg game

In our limit result of the Stackelberg game, the competitive quantity q_d^{P} and the following quantity can be supported.

Definition 1. We call q_d^{QM} the *quasi-monopoly quantity* of the innovator, which is given by $P\left(q_d^{\mathrm{QM}}\right) = C_d'(0)$.

q_d^{QM} defined here is the output level just to undercut the pre-innovation competitive price $C_d'(0)$.[11] When $q_0 = q_d^{\mathrm{QM}}$, the innovator occupies an entire market under the pre-innovation competitive price. Although at a glance, such an innovator seems to be a monopolist, she does not have the power to

[11] When all the firms use the old technology, the price goes down close to $C_d'(0)$ as n increases. Thus, $P = C_d'(0)$ is the competitive price in our pre-innovation market with a large number of firms.

make the price.[12] Thus, we call this situation quasi monopoly. Arrow [1] is an earlier work that suggests that this quantity is the output level produced by the innovator.[13]

Two quantities, q_d^{P} and q_d^{QM}, have the relation expressed in the following lemma:

Lemma 2. *There exists $\underline{d} \in (0, \bar{d})$ such that $d \leq \underline{d} \Leftrightarrow q_d^{\mathrm{P}} \leq q_d^{\mathrm{QM}}$.*

Formal proof of this lemma is presented in Appendix A.1. It is more plausible if the reason for this lemma is presented graphically. Line 1 in Figure 1 depicts $C_d'(0)$ in the situation where $d < \underline{d}$. In this case, we have $q_d^{\mathrm{QM}} > q_d^{\mathrm{P}}$ as in Figure 1. If $d \geq \underline{d}$, as in Line 2, the solid line that represents $C_d'(0)$ lies over the lower dotted line. Then, we have $q_d^{\mathrm{QM}} \leq q_d^{\mathrm{P}}$ (equality holds when $d = \underline{d}$).

Theorem 1 (Stackelberg asymmetric limit output)**.**
(S-i) $\lim_{n\to\infty} q_0^{\mathcal{S}^{d,n}} = q_d^{\mathrm{P}}$ *if* $d \in \left(0, \underline{d}\right]$.
(S-ii) $\lim_{n\to\infty} q_0^{\mathcal{S}^{d,n}} = q_d^{\mathrm{QM}}$ *if* $d \in \left(\underline{d}, \bar{d}\right)$.

This theorem describes the limit world of the Stackelberg game.[14] (S-i) is the case where the cost advantage is so minor that the innovator cannot cover the entire demand under the competitive price. At such a time, the innovator provides competitive quantity just as in the classical price theory.[15] The case of (S-ii) indicates moderate innovation, which is not minor or drastic. In this case, since the innovator has the ability to supply the entire demand under the competitive price but since competitive price is lower than the new monopoly price, the innovator supplies a quasi-monopoly quantity to deter the laggard firms from entering the market.[16]

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[12] Note that $q_d^{\mathrm{QM}} > q^{\mathrm{M}}$ for non-drastic innovations that we are now focusing on. This is because $P\left(q^{\mathrm{M}}\right) > C_d'(0) = P\left(q_d^{\mathrm{QM}}\right)$ when $d < \bar{d}$.

[13] We provide a simple graphical explanation of Arrow's discussion in Appendix D.

[14] This result completely coincides with a Nash equilibrium of the Bertrand game with the efficient rationing rule. Let $\mathcal{B}^{d,n}$ denote the Bertrand game such that each laggard firm's cost disadvantage is d and the number of laggard firms is n. Let $s^{\mathcal{B}^{d,n}}$ be a strategy profile of $\mathcal{B}^{d,n}$ such that each player choose price $p^{\mathcal{B}^{d,n}}$ which satisfies $p^{\mathcal{B}^{d,n}} = C_0'(q_0) = C_d'(q_1) = P(q_0 + nq_1)$ for some q_0 and q_1 if $d \in \left(0, \underline{d}\right]$, and $p^{\mathcal{B}^{d,n}} = C_d'(0)$ if $d \in \left(\underline{d}, \bar{d}\right)$. $s^{\mathcal{B}^{d,n}}$ is a Nash equilibrium of $\mathcal{B}^{d,n}$. By the efficient rationing rule, the innovator's output and each laggard firm's in $s^{\mathcal{B}^{d,n}}$ are respectively $q_0^{\mathcal{B}^{d,n}}$ and $q_1^{\mathcal{B}^{d,n}}$ such that $p^{\mathcal{B}^{d,n}} = C_0'\left(q_0^{\mathcal{B}^{d,n}}\right) = C_d'\left(q_1^{\mathcal{B}^{d,n}}\right) = P\left(q_0^{\mathcal{B}^{d,n}} + nq_1^{\mathcal{B}^{d,n}}\right)$ if $d \in \left(0, \underline{d}\right]$, and $p^{\mathcal{B}^{d,n}} = C_d'(0) = P\left(q_0^{\mathcal{B}^{d,n}}\right)$ and $q_1^{\mathcal{B}^{d,n}} = 0$ if $d \in \left(\underline{d}, \bar{d}\right)$. Thus, we have

$$\lim_{n\to\infty} q_0^{\mathcal{B}^{d,n}} = \begin{cases} q_d^{\mathrm{P}} & \text{if } d \in \left(0, \underline{d}\right] \\ q_d^{\mathrm{QM}} & \text{if } d \in \left(\underline{d}, \bar{d}\right). \end{cases}$$

[15] Observe that when the marginal cost of innovative technology is sufficiently close to a horizontal line, $\underline{d}$ will be arbitrarily small and the case (S-i) in Theorem 1 disappears. Since Arrow [1] considered the simple constant marginal costs, in his discussion, the innovator always produce q_d^{QM} when $d \leq \bar{d}$. In the case of the increasing marginal cost, it is possible that innovation is so minor that the leader settles for behaving as in the competitive market.

[16] The possibility of other firms entering the market prevents a firm from supplying at the monopoly price even if this

3.3 The innovator's limit output in the Cournot game

To investigate what happens if the innovator cannot commit her outputs, we provide the limit result of the Cournot case. In our limit result of the Cournot game, the following quantity can be supported.

Definition 2. We call q_d^{PM} the *partial-monopoly quantity* of the innovator, which is given by $P\left(q_d^{\mathrm{QM}}\right) + P'\left(q_d^{\mathrm{QM}}\right) q_d^{\mathrm{PM}} = C_0'\left(q_d^{\mathrm{PM}}\right)$.

From the definition above,[17] notice that $(q_0, Q_{-0}) = (q_d^{\mathrm{PM}}, q_d^{\mathrm{QM}} - q_d^{\mathrm{PM}})$ solves $P(q_0 + Q_{-0}) + P'(q_0 + Q_{-0})q_0 = C_0'(q_0)$ and $P(q_0 + Q_{-0}) = C_d'(0)$ simultaneously. Figure 2 represents this relation graphically. As seen here, the partial-monopoly quantity is the monopoly outcome for its residual demand under the pre-innovation competitive price $C_d'(0)$. Thus, the partial monopolist is considered as the price maker among perfectly competitive firms. This concept was historically presented by Forchheimer and is often referred to in old industrial organization textbooks.

The following theorem reveals where the innovator's equilibrium output goes in the limit world of the Cournot game.

Theorem 2 (Cournot asymmetric limit output)**.**
$\lim_{n\to\infty} q_0^{C^{d,n}} = q_d^{\mathrm{PM}}$ *if* $d \in (0, \bar{d})$.

In contrast to the Stackelberg game, the laggard firms always enter the market in equilibrium. The limit output of the innovator corresponds to that of the partial monopolist. It is worth noting that the innovator's output never converges to a competitive one in our asymmetric limit results of the Cournot model.[18]

3.4 The innovator's limit profit

How profitable is innovation in a competitive market? Since the firms' profit in the pre-innovation market is zero in the limit,[19] the innovator's limit post-innovation profit is the fruit of the innovation.

firm's market share is 1. The idea of contestable markets have already pointed out such a phenomenon in a symmetric case. For details regarding the characteristics of contestable markets, see Baumol [2].

[17] q_d^{PM} are well defined. The existence and uniqueness of q_d^{PM} are held by P decreasing, the strict convexity of C, Definition 1, and $C_d'(0) - C_0'(0) > 0$.

[18] We must have $q_d^{\mathrm{PM}} < q_d^{\mathrm{P}}$ since $C_0'(q_d^{\mathrm{PM}}) < P(q_d^{\mathrm{QM}}) = C_d'(0) = C_0'(q_d^{\mathrm{P}})$.

[19] Suppose that all the firms are symmetric. Then, regardless of the Stackelberg game, Cournot game, or perfect competition, a firm's profit goes to zero when n goes to infinity.

Denote the equilibrium profit of the innovator in $\mathcal{S}^{d,n}$ ($\mathcal{C}^{d,n}$, $\mathcal{P}^{d,n}$) by $\pi_0^{\mathcal{S}^{d,n}}$ ($\pi_0^{\mathcal{C}^{d,n}}$, $\pi_0^{\mathcal{P}^{d,n}}$). The following theorem reveals how much the innovation earns.

Theorem 3 (Asymmetric limit profit)**.**
(S-i) $\lim_{n\to\infty} \pi_0^{\mathcal{S}^{d,n}} = \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ *if* $d \in \left(0, \underline{d}\right]$.
(S-ii) $\lim_{n\to\infty} \pi_0^{\mathcal{S}^{d,n}} > \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ *if* $d \in \left(\underline{d}, \bar{d}\right)$.
(C) There exists $d^* \in \left(\underline{d}, \bar{d}\right)$ *such that (C-i)* $\lim_{n\to\infty} \pi_0^{\mathcal{C}^{d,n}} < \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ *if* $d \in (0, d^*)$,
(C-ii) $\lim_{n\to\infty} \pi_0^{\mathcal{C}^{d,n}} \geq \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ *if* $d \in [d^*, \bar{d})$.

Essentially, these results follow Theorems 1 and 2. In the Stackelberg game, though the moderate innovation in (S-ii) still earns a larger profit than the price-taking profit, the profit of minor innovation in (S-i) finally approximates the ordinary price-taking profit. In the Cournot game, when the innovation is close to a drastic innovation, like that in (C-ii), the innovator earns a larger profit than the price taker, similar to the Stackelberg case. However, for a sufficiently small cost advantage as in (C-i), the innovator's profit will be smaller than the price-taking profit.

4 Induction of limit results

4.1 Laggard firms' reactions

We consider the reactions of firms $1, \ldots, n$ to firm 0's output $q_0 \in \mathbb{R}_+$. Given an action profile $\mathbf{q} = (q_k)_{k\in L^n} \in \mathbb{R}_+^n$, the profit of firm $i \in L^n$ is described as

$$\pi_i^{d,n}(\mathbf{q}; q_0) = P\left(\sum_{k=0}^{n} q_k\right) q_i - C_d(q_i). \tag{1}$$

Since all the laggard firms are identical, their solution is symmetric.[20] Thus, we denote each firm's equilibrium output as $q_1^{d,n}(q_0)$. Let $Q^{d,n}(q_0) \equiv q_0 + nq_1^{d,n}(q_0)$. From the first order condition of laggard firms,

$$P(Q^{d,n}(q_0)) + P'(Q^{d,n}(q_0))q_1^{d,n}(q_0) \leq C_d'(q_1^{d,n}(q_0)). \tag{2}$$

[20] When the innovator's output q_0 is given, laggard firms are engaged in Cournot-type competition. Rosen [10] proved that this Cournot game has the unique Nash equilibrium in more general setting. Therefore, there are no solutions except for the symmetric equilibrium we focus on.

with equality if $q_1^{d,n}(q_0) > 0$.

When do the laggard firms quit entering the market? To answer this question, we can use the quasi-monopoly quantity q_d^{QM} as a threshold.

Lemma 3. *Take any $n \in \mathbb{N}$. Then, $q_1^{d,n}(q_0) = 0$ if and only if $q_0 \geq q_d^{\mathrm{QM}}$.*

Proof. Suppose $q_1^{d,n}(q_0) = 0$. Then, (2) turns out to be $P(q_0) \leq C_d'(0)$. If $q_0 < q_d^{\mathrm{QM}}$, $P(q_0) > C_d'(0)$ from Definition 1, which is a contradiction. Suppose $q_0 \geq q_d^{\mathrm{QM}}$. If $q_1^{d,n}(q_0) > 0$, (2) holds with equality. However, $P(Q^{d,n}(q_0)) + P'(Q^{d,n}(q_0))q_1^{d,n}(q_0) < P\left(q_d^{\mathrm{QM}}\right) + P'(Q^{d,n}(q_0))q_1^{d,n}(q_0) < P\left(q_d^{\mathrm{QM}}\right) = C_d'(0) < C_d'(q_1^{d,n}(q_0))$, which is a contradiction. **Q.E.D.**

Using the lemma above, we can show that $Q^{d,n}$ is continuous at $q_0 = q_d^{\mathrm{QM}}$ and thus a continuous function.

Lemma 4. *For any $n \in \mathbb{N}$, $Q^{d,n}$ is continuous on $\mathbb{R}_+$.*

Proof. See Appendix A.2. **Q.E.D.**

Now, we consider the impact of the innovator's output on the laggard firms' outputs when the innovator's output q_0 is in $\left[0, q_d^{\mathrm{QM}}\right)$. By Lemma 3, $q_1^{d,n}(q_0) > 0$. Thus, (2) holds with equality. Differentiate both sides of (2) by q_0, and then we get

$$\frac{dq_1^{d,n}(q_0)}{dq_0} = \frac{P'(Q^{d,n}(q_0)) + P''(Q^{d,n}(q_0))q_1^{d,n}(q_0)}{C_d''(q_1^{d,n}(q_0)) - P'(Q^{d,n}(q_0)) - n\left(P'(Q^{d,n}(q_0)) + P''(Q^{d,n}(q_0))q_1^{d,n}(q_0)\right)} < 0. \tag{3}$$

Therefore, each laggard firm's supply falls as the innovator increases her supply. Then, what is the impact on total supply? Using (3), we immediately obtain

$$\frac{dQ^{d,n}(q_0)}{dq_0} = \frac{C_d''(q_1^{d,n}(q_0)) - P'(Q^{d,n}(q_0))}{C_d''(q_1^{d,n}(q_0)) - P'(Q^{d,n}(q_0)) - n\left(P'(Q^{d,n}(q_0)) + P''(Q^{d,n}(q_0))q_1^{d,n}(q_0)\right)} > 0. \tag{4}$$

Therefore, the total market scale expands as the innovator increases her supply. Hence, each laggard firm's market share falls as the innovator increases her supply.

4.2 Limit price

From the reactions of the laggard firms, we can induce the limit price, given q_0.

Theorem 4 (Limit price in a symmetric case)**.**
$\left(P \circ Q^{d,n}\right)_{n\in\mathbb{N}}$ *converges pointwise to* $C'_d(0)$ *as* $n \to \infty$ *on* $\left[0, q_d^{\mathrm{QM}}\right)$.

Proof. See Appendix A.3. **Q.E.D.**

If there is no innovator and all the firms have the prevailing symmetric technology, i.e., $q_0 = 0$, this lemma is the so-called Cournot limit theorem,[21] which states that the price converges to a competitive one as the number of firms goes to infinity. However, if one firm has the innovative asymmetric technology, Theorem 4 is not sufficient to indicate where the equilibrium price converges both in the Stackelberg and Cournot games. This is because Theorem 4 just ensures pointwise convergence for fixed q_0. In the sequence of equilibria, firm 0's output is generally not constant when n changes.

We find out that Theorem 4 can be extended to stronger one.

Theorem 5 (Limit price in an asymmetric case)**.**
$\left(P \circ Q^{d,n}\right)_{n\in\mathbb{N}}$ *converges uniformly to* $C'_d(0)$ *as* $n \to \infty$ *on* $\left[0, q_d^{\mathrm{QM}}\right)$.

Proof. Fix $n \in \mathbb{N}$. Since P is decreasing, (4) holds by $q_1^{d,n}(q_0) > 0$ for any $q_0 \in \left[0, q_d^{\mathrm{QM}}\right)$, and $Q^{d,n}$ is continuous by Lemma 4, we have $P(Q^{d,n}(q_0)) \geq P\left(Q^{d,n}\left(q_d^{\mathrm{QM}}\right)\right)$ for any $q_0 \in \left[0, q_d^{\mathrm{QM}}\right)$. For any $q_0 \in \left[0, q_d^{\mathrm{QM}}\right)$, $P(Q^{d,n}(q_0)) \geq P\left(q_d^{\mathrm{QM}}\right)$ by Lemma 3, and $P(Q^{d,n}(q_0)) \geq C'_d(0)$ by the definition of q_d^{QM}. Thus,

$$\sup_{q_0 \in \left[0, q_d^{\mathrm{QM}}\right)} \left|P(Q^{d,n}(q_0)) - C'_d(0)\right| = \sup_{q_0 \in \left[0, q_d^{\mathrm{QM}}\right)} \left(P(Q^{d,n}(q_0)) - C'_d(0)\right).$$

Since P is decreasing and (4) holds for any $q_0 \in \left[0, q_d^{\mathrm{QM}}\right)$,

$$\sup_{q_0 \in \left[0, q_d^{\mathrm{P}}\right)} \left|P(Q^{d,n}(q_0)) - C'_d(0)\right| = P(Q^{d,n}(0)) - C'_d(0).$$

Taking n to infinity, we obtain $\lim_{n\to\infty} \left|P(Q^{d,n}(0)) - C'_d(0)\right| = 0$ by Theorem 4. Hence, $\left(P \circ Q^{d,n}\right)_{n\in\mathbb{N}}$ converges uniformly to $C'_d(0)$. **Q.E.D.**

Uniform convergence is the key concept to make the limit theorem applicable to our asymmetric

[21] See Ruffin [11] for details regarding the Cournot limit theorem. However, Ruffin's proof has a jump when Lemma 7 in our proof is induced. Thus, we provide a formal proof of Theorem 4 in Appendix A.3.

case.[22] If a function uniformly converges to a function with constant value, there exists the most slowly converging point in the domain. Since the price converges to $C'_d(0)$ even at this slowest point, the price also converges whenever q_0 staggers. In other words, for any sequence $(q_0^n)_{n\in\mathbb{N}}$ such that $q_1^{d,n}(q_0^n) > 0$, we have $\lim_{n\to\infty} P\left(Q^{d,n}\left(q_0^n\right)\right) = C'_d(0)$. Therefore, Theorem 5 implies that in both the Stackelberg and Cournot games involving an innovator, the equilibrium price in the limit is $C'_d(0)$.

4.3 Proof of Theorem 1

In game $\mathcal{S}^{d,n}$, given $Q^{d,n}(q_0)$ induced in Subsection 4.1, the innovator maximizes its profit $\pi_0^{d,n}(q_0)$ with respect to q_0, where the innovator's profit is described as

$$\pi_0^{d,n}(q_0) = P(Q^{d,n}(q_0))q_0 - C_0(q_0). \tag{5}$$

Note that since $\pi_0^{d,n}$ is continuous in q_0 by Lemma 4, by the Weierstrass' theorem, there exists an equilibrium in game $\mathcal{S}^{d,n}$ for all $n \in \mathbb{N}$. We have denoted one of the equilibrium strategies of the innovator in $\mathcal{S}^{d,n}$ by $q_0^{\mathcal{S}^{d,n}}$, i.e.,

$$q_0^{\mathcal{S}^{d,n}} \in \arg\max_{q_0\in\mathbb{R}_+} \pi_0^{d,n}(q_0). \tag{6}$$

Henceforth, we will use one mathematical fact provided in Appendix C to investigate the Stackelberg limit outcomes. Roughly speaking, this fact tells us that if the "limit payoff function," which $\left(\pi_0^{d,n}\right)_{n\in\mathbb{N}}$ uniformly converges to, is continuous and has the unique maximizer on the compact area, the sequence $(q_0^{\mathcal{S}^{d,n}})_{n\in\mathbb{N}}$ cannot escape from that maximizer as n goes to infinity. Thus, once this "limit payoff function" is identified, we can find the convergent point of the innovator's outputs without characterizing each finite game.

Indeed, the innovator's profit uniformly converges as follows.

[22] In the proofs below, we will make full use of the structure of uniform convergence, which is the mathematical nature of our limit results. Some readers may think that using such a concept is redundant. Indeed, we can describe the Cournot limit result without uniform convergence in a way that is almost similar to that of our proof. As for the Stackelberg limit result, however, the proof is remarkably simplified by using uniform convergence. Moreover, Theorem 5 says more than the limit price of the Cournot and Stackelberg models, as seen in footnote 25 or in concluding remarks. Therefore, we believe our proof strategy is a noteworthy one.

Lemma 5. $(\pi_0^{d,n})_{n\in\mathbb{N}}$ *uniformly converges to* π_0^d *as* $n \to \infty$*, which is given by*

$$\pi_0^d(q_0) \equiv \begin{cases} \pi^{\mathrm{M}}(q_0) \equiv P(q_0)q_0 - C_0(q_0) & \text{if } q_0 \geq q_d^{\mathrm{QM}} \\ \pi_d^{\mathrm{P}}(q_0) \equiv C_d'(0)q_0 - C_0(q_0) & \text{if } q_0 < q_d^{\mathrm{QM}}. \end{cases} \tag{7}$$

Proof. (i) By Lemma 3, when $q_0 \geq q_d^{\mathrm{QM}}$, for any $n \in \mathbb{N}$, $\pi_0^{d,n}(q_0) = \pi^{\mathrm{M}}(q_0)$. (ii) By

$$\sup_{q_0 \in \left[0, q_d^{\mathrm{QM}}\right)} \left|\pi_0^{d,n}(q_0) - \pi_d^{\mathrm{P}}(q_0)\right| = \sup_{q_0 \in \left[0, q_d^{\mathrm{QM}}\right)} \left|P(Q^{d,n}(q_0)) - C_d'(0)\right| q_0 \leq \sup_{q_0 \in \left[0, q_d^{\mathrm{QM}}\right)} \left|P(Q^{d,n}(q_0)) - C_d'(0)\right| q_d^{\mathrm{QM}}$$

and Theorem 5, $\sup_{q_0 \in \left[0, q_d^{\mathrm{QM}}\right)} \left|\pi_0^{d,n}(q_0) - \pi_d^{\mathrm{P}}(q_0)\right|$ converges to 0 as n goes to infinity. By (i) and (ii), we reach the conclusion. **Q.E.D.**

Now, we are ready to prove Theorem 1.

Proof. Immediately, Lemma 5 implies that $\left(\pi_0^{d,n}\right)_{n\in\mathbb{N}}$ uniformly converges to π_0^d on any compact interval in $\mathbb{R}_+$. Note that by Definition 1, π_0^d is continuous at q_d^{QM}. Therefore, what remains is finding the unique maximizer of π_0^d.

(S-i) Suppose that $d \in (0, \underline{d}]$. Then, $q_d^{\mathrm{P}} \leq q_d^{\mathrm{QM}}$ by Lemma 2 and $q^{\mathrm{M}} < q_d^{\mathrm{QM}}$ by $\underline{d} < \bar{d}$ (See footnote 12). Thus, $\arg\max_{q_0 \in \mathbb{R}_+} \pi_0^d(q_0) = \left\{q_d^{\mathrm{P}}\right\}$ from the definition of q_d^{P} and the fact that π^{M} is strictly concave. By Lemma 8 in Appendix C, we have $\lim_{n\to\infty} q_0^{S^{d,n}} = q_d^{\mathrm{P}}$.

(S-ii) Suppose that $d \in (\underline{d}, \bar{d})$. Then, $q^{\mathrm{M}} < q_d^{\mathrm{QM}} < q_d^{\mathrm{P}}$ (See Lemma 2 and footnote 12). Thus, from the concavity of π_d^{P} and π^{M}, $\arg\max_{q_0 \in \mathbb{R}_+} \pi_0^d(q_0) = \left\{q_d^{\mathrm{QM}}\right\}$. By Lemma 8 in Appendix C, we have $\lim_{n\to\infty} q_0^{S^{d,n}} = q_d^{\mathrm{QM}}$. **Q.E.D.**

4.4 Proof of Theorem 2

Even in game $C^{d,n}$, each laggard firm's reaction is the same as that described in Subsection 4.1. However, the innovator does not take their reactions into account. Hence, we can induce firm 0's equilibrium output $q_0^{C^{d,n}}$ in game $C^{d,n}$ by substituting the reaction function obtained in the previous section into the first order condition of firm 0 as follows:

$$P\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) + P'\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) q_0^{C^{d,n}} - C_0'\left(q_0^{C^{d,n}}\right) = 0. \tag{8}$$

Suppose that $d \in (0, \bar{d})$. Hereafter, using condition (8), we will show that the innovator's output is guaranteed to converge as $\lim_{n\to\infty} q_0^{C^{d,n}} = q_d^{\mathrm{PM}}$.

Proof. Since $d < \bar{d}$, we have $q^{\mathrm{M}} < q_d^{\mathrm{QM}}$ (footnote 12). Fix $n \in \mathbb{N}$. Suppose that $q_0^{C^{d,n}} \geq q_d^{\mathrm{QM}}$. By Lemma 3, $q_1^{d,n}\left(q_0^{C^{d,n}}\right) = 0$. Since firm 0's best response is q^{M}, $q_0^{C^{d,n}} = q^{\mathrm{M}}$ must hold. Thus, by $q^{\mathrm{M}} < q_d^{\mathrm{QM}}$, $q_0^{C^{d,n}} < q_d^{\mathrm{QM}}$, which is a contradiction. Therefore, $q_0^{C^{d,n}} < q_d^{\mathrm{QM}}$ for any $n \in \mathbb{N}$. Thus, by Theorem 5 and by Definition 1, $\lim_{n\to\infty} P\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) = C_d'(0) = P\left(q_d^{\mathrm{QM}}\right)$. This, with the continuity of P, implies that $\lim_{n\to\infty} Q^{d,n}\left(q_0^{C^{d,n}}\right) = q_d^{\mathrm{QM}}$, and thus, $\lim_{n\to\infty} P'\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) = P'\left(q_d^{\mathrm{QM}}\right)$. Hence, $\forall \delta > 0, \exists n' \in \mathbb{N}, \forall n \geq n'$,

$$\left|\left(P\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) + P'\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) q_0^{C^{d,n}} - C_0'\left(q_0^{C^{d,n}}\right)\right) - \left(P\left(q_d^{\mathrm{QM}}\right) + P'\left(q_d^{\mathrm{QM}}\right) q_0^{C^{d,n}} - C_0'\left(q_0^{C^{d,n}}\right)\right)\right| < \delta.$$

Now suppose the contrary, $\exists \epsilon > 0, \forall \bar{n} \in \mathbb{N}, \exists n \geq \bar{n}, \left|q_0^{C^{d,n}} - q_d^{\mathrm{PM}}\right| > \epsilon$. Then, since $P'\left(q_d^{\mathrm{QM}}\right) q_0^{C^{d,n}} - C_0'\left(q_0^{C^{d,n}}\right)$ is decreasing in $q_0^{C^{d,n}}$, $\exists \delta' > 0, \forall \bar{n} \in \mathbb{N}, \exists n \geq \bar{n}$,

$$\left|\left(P\left(q_d^{\mathrm{QM}}\right) + P'\left(q_d^{\mathrm{QM}}\right) q_0^{C^{d,n}} - C_0'\left(q_0^{C^{d,n}}\right)\right) - \left(P\left(q_d^{\mathrm{QM}}\right) + P'\left(q_d^{\mathrm{QM}}\right) q_d^{\mathrm{PM}} - C_0'\left(q_d^{\mathrm{PM}}\right)\right)\right| > \delta'.$$

Therefore, when we take $\delta < \delta'$, there exists $n \in \mathbb{N}$ such that

$$P\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) + P'\left(Q^{d,n}\left(q_0^{C^{d,n}}\right)\right) q_0^{C^{d,n}} - C_0'\left(q_0^{C^{d,n}}\right) \neq P\left(q_d^{\mathrm{QM}}\right) + P'\left(q_d^{\mathrm{QM}}\right) q_d^{\mathrm{PM}} - C_0'\left(q_d^{\mathrm{PM}}\right) = 0$$

Therefore, (8) is not satisfied for such an n, which is a contradiction. **Q.E.D.**

4.5 Proof of Theorem 3

Finally, we show how profitable the innovation is in the limit of each Stackelberg and Cournot model.

Proof. (S-i) is obvious from Theorem 1. Therefore, we will prove (S-ii) and (C).

(S-ii) Since $d > \underline{d}$, by Lemma 2, we have $q_d^{\mathrm{P}} > q_d^{\mathrm{QM}}$. Hence, by Lemma 1, $\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = q^{\mathrm{P}}$. Observe that $q^{\mathrm{P}} > q_d^{\mathrm{QM}}$. Thus, since $\lim_{n\to\infty} q_0^{\mathcal{S}^{d,n}} = q_d^{\mathrm{QM}}$ by (S-ii) of Theorem 1, $q_0^{\mathcal{P}^{d,n}} > q_0^{\mathcal{S}^{d,n}}$ for a sufficiently large n. The Stackelberg leader can produce $q_0^{\mathcal{P}^{d,n}}$, but it chooses $q_0^{\mathcal{S}^{d,n}}$ in the equilibrium. Therefore, the profit must be higher when $q_0^{\mathcal{S}^{d,n}}$ is chosen than when $q_0^{\mathcal{P}^{d,n}}$ is. This implies that $\lim_{n\to\infty} \pi_0^{\mathcal{S}^{d,n}} > \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$.

(C) First, suppose that $d \leq \underline{d}$. Then, since $\lim_{n\to\infty} q_0^{\mathcal{C}^{d,n}} = q_d^{\mathrm{PM}} < q_d^{\mathrm{P}} = \lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}}$ and the limit price is the same both in games $\mathcal{C}^{d,n}$ and $\mathcal{P}^{d,n}$, $\lim_{n\to\infty} \pi_0^{\mathcal{C}^{d,n}} < \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ holds. Second, suppose that $d = \bar{d}$. Then, we must have $\lim_{n\to\infty} \pi_0^{\mathcal{C}^{d,n}} > \lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ since $\pi_0^{\mathcal{C}^{d,n}}$ is firm 0's monopoly profit and $\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = q^{\mathrm{P}} \neq q^{\mathrm{M}}$. Finally, consider $\underline{d} \leq d \leq \bar{d}$. In this case, while $\lim_{n\to\infty} \pi_0^{\mathcal{P}^{d,n}}$ does not depend on d since $\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = q^{\mathrm{P}}$, $\lim_{n\to\infty} \pi_0^{\mathcal{C}^{d,n}}$ is increasing in d since both the limit outcome q_d^{PM} and the limit price $C_d'(0)$ is increasing in d. Note that q_d^{PM} is increasing in d because for all $\bar{\delta}$ and $\underline{\delta}$ such that $\bar{\delta} > \underline{\delta}$,

$$P\left(q_{\underline{\delta}}^{\mathrm{QM}}\right) + P'\left(q_{\underline{\delta}}^{\mathrm{QM}}\right) q_{\underline{\delta}}^{\mathrm{PM}} - C_0'\left(q_{\underline{\delta}}^{\mathrm{PM}}\right) < P\left(q_{\bar{\delta}}^{\mathrm{QM}}\right) + P'\left(q_{\bar{\delta}}^{\mathrm{QM}}\right) q_{\underline{\delta}}^{\mathrm{PM}} - C_0'\left(q_{\underline{\delta}}^{\mathrm{PM}}\right)$$

by Assumption 1 and $q_{\underline{\delta}}^{\mathrm{QM}} > q_{\bar{\delta}}^{\mathrm{QM}}$; thus, $q_{\underline{\delta}}^{\mathrm{PM}} < q_{\bar{\delta}}^{\mathrm{PM}}$ by $P'(q_{\bar{\delta}}^{\mathrm{QM}}) - C_0''(q_d^{\mathrm{PM}}) < 0$. Combining these three facts, we can find the desired d^* in $(\underline{d}, \bar{d})$ (See Figure 3). **Q.E.D.**

5 Path to the limit

To describe the path to the limit, we specify the functions as follows: $P(Q) \equiv \max\{1 - Q, 0\}$ and $C_\delta(q_i) \equiv e(q_i)^2 + \delta q_i$. In this section, we assume that d may be 0.

5.1 Results of computation

From the results of the computation below, we can find that $\underline{d} = \frac{2e}{2e+1}$ and $\bar{d} = \frac{2e+1}{2e+2}$ in this specification.

Perfect competition As a benchmark, we consider the perfect competition, where all the firms including firm 0 are price takers. The equilibrium output profile $\left(q_k^{\mathcal{P}^{d,n}}\right)_{k\in\{0\}\cup L^n}$ in the perfect competition is as follows:

$$q_0^{\mathcal{P}^{d,n}} = \begin{cases} \frac{dn+2e}{2e(n+2e+1)} & \text{if } d < \frac{2e}{2e+1} \\ \frac{1}{2e+1} & \text{if } \frac{2e}{2e+1} \leq d \end{cases}$$

and

$$q_i^{\mathcal{P}^{d,n}} = \begin{cases} \frac{2e-(2e+1)d}{2e(n+2e+1)} & \text{if } d < \frac{2e}{2e+1} \\ 0 & \text{if } \frac{2e}{2e+1} \leq d \end{cases}$$

for $i \in L^n$.

Let $Q_{-0}^{\mathcal{P}^{d,n}}$ be the sum of the laggard firms' equilibrium outputs in perfect competition. We can compute the equilibrium outputs of $\mathcal{P}^{d,n}$:

$$\left(q_0^{\mathcal{P}^{d,n}}, Q_{-0}^{\mathcal{P}^{d,n}}\right) = \begin{cases} \left(\frac{dn+2e}{2e(n+2e+1)}, \frac{n\{2e-(2e+1)d\}}{2e(n+2e+1)}\right) & \text{if } d < \frac{2e}{2e+1} \\ \left(\frac{1}{2e+1}, 0\right) & \text{if } d \geq \frac{2e}{2e+1}. \end{cases} \tag{9}$$

From (9), we have the limit quantities. This result corresponds to Lemma 1.

$$\lim_{n\to\infty} \left(q_0^{\mathcal{P}^{d,n}}, Q_{-0}^{\mathcal{P}^{d,n}}\right) = \begin{cases} \left(\frac{d}{2e}, \frac{2e-(2e+1)d}{2e}\right) & \text{if } d < \frac{2e}{2e+1} \\ \left(\frac{1}{2e+1}, 0\right) & \text{if } d \geq \frac{2e}{2e+1}. \end{cases} \tag{10}$$

Stackelberg competition We compute the equilibrium of $\mathcal{S}^{d,n}$ by the backward induction. Each laggard firm's equilibrium strategy is as follows:

$$q_1^{d,n}\left(q^0\right) = \begin{cases} \frac{1-d-q^0}{n+2e+1} & \text{if } q^0 < 1-d \\ 0 & \text{if } q^0 \geq 1-d. \end{cases}$$

Thus, the profit which the innovator obtain with her output q^0 is induced as

$$\pi_0^{d,n}(q^0) = \begin{cases} -\frac{ne+(2e+1)(e+1)}{n+2e+1}\left(q^0 - \frac{2e+1+nd}{2\{ne+(2e+1)(e+1)\}}\right)^2 + \frac{(2e+1+nd)^2}{4\{ne+(2e+1)(e+1)\}(n+2e+1)} & \text{if } q^0 < 1-d \\ -(1+e)\left(q^0 - \frac{1}{2(1+e)}\right) + \frac{1}{4(1+e)} & \text{if } q^0 \geq 1-d. \end{cases}$$

Hence, the innovator's equilibrium strategy is as follows:

$$q_0^{\mathcal{S}^{d,n}} = \begin{cases} \frac{dn+2e+1}{2(en+(2e+1)(e+1))} & \text{if } d \leq \frac{2e}{2e+1} \\ \begin{cases} \frac{dn+2e+1}{2(en+(2e+1)(e+1))} & \text{if } n < \bar{n}\,(d,e) \\ 1-d & \text{if } n \geq \bar{n}\,(d,e) \end{cases} & \text{if } \frac{2e}{2e+1} < d < \frac{2e+1}{2e+2} \\ \frac{1}{2e+2} & \text{if } d \geq \frac{2e+1}{2e+2}, \end{cases}$$

where $\bar{n}\,(d,e) = \frac{(2e+1)(2(e+1)d-2e-1)}{2e-(2e+1)d}$.

Let $Q_{-0}^{\mathcal{S}^{d,n}}$ be the sum of the laggard firms' equilibrium outputs of $\mathcal{S}^{d,n}$ ($Q_{-0}^{\mathcal{S}^{d,n}} \equiv nq_1^{d,n}\left(q_0^{\mathcal{S}^{d,n}}\right)$). We can compute the equilibrium outputs of $\mathcal{S}^{d,n}$:

$$\left(q_0^{\mathcal{S}^{d,n}}, Q_{-0}^{\mathcal{S}^{d,n}}\right) =$$

$$\begin{cases} \left(\frac{dn+2e+1}{2(en+(2e+1)(e+1))}, \frac{n[2(1-d)\{en+(2e+1)(e+1)\}-dn-2e-1]}{2(n+2e+1)\{en+(2e+1)(e+1)\}}\right) & \text{if (i) } d \leq \frac{2e}{2e+1} \\ \begin{cases} \left(\frac{dn+2e+1}{2(en+(2e+1)(e+1))}, \frac{n[2(1-d)\{en+(2e+1)(e+1)\}-dn-2e-1]}{2(n+2e+1)\{en+(2e+1)(e+1)\}}\right) & \text{if (ii-a) } n < \bar{n}\,(d,e) \\ (1-d, 0) & \text{if (ii-b) } n \geq \bar{n}\,(d,e) \end{cases} & \text{if (ii) } \frac{2e}{2e+1} < d < \frac{2e+1}{2e+2} \\ \left(\frac{1}{2e+2}, 0\right) & \text{if (iii) } d \geq \frac{2e+1}{2e+2}. \end{cases} \tag{11}$$

From (11), we have the limit quantities. This result corresponds to Theorem 1.

$$\lim_{n\to\infty}\left(q_0^{\mathcal{S}^{d,n}}, Q_{-0}^{\mathcal{S}^{d,n}}\right) = \begin{cases} \left(\frac{d}{2e}, \frac{2e-(2e+1)d}{2e}\right) & \text{if (i) } d \leq \frac{2e}{2e+1} \\ (1-d, 0) & \text{if (ii) } \frac{2e}{2e+1} < d < \frac{2e+1}{2e+2} \\ \left(\frac{1}{2e+2}, 0\right) & \text{if (iii) } d \geq \frac{2e+1}{2e+2}. \end{cases} \tag{12}$$

Cournot competition The Nash equilibrium $\left(q_k^{C^{d,n}}\right)_{k\in\{0\}\cup L^n}$ of $C^{d,n}$ is as follows:

$$q_0^{C^{d,n}} = \begin{cases} \frac{dn+2e+1}{(1+2e)(n+2e+2)} & \text{if } d < \frac{2e+1}{2e+2} \\ \frac{1}{2(e+1)} & \text{if } d \geq \frac{2e+1}{2e+2} \end{cases}$$

and

$$q_i^{C^{d,n}} = \begin{cases} \frac{1+2e-2(1+e)d}{(1+2e)(n+2e+2)} & \text{if } d < \frac{2e+1}{2e+2} \\ 0 & \text{if } d \geq \frac{2e+1}{2e+2} \end{cases}$$

for $i \in L^n$.

Let $Q_{-0}^{C^{d,n}}$ be the sum of the laggard firms' equilibrium outputs of $C^{d,n}$. We can compute the equilibrium outputs of $C^{d,n}$:

$$\left(q_0^{C^{d,n}}, Q_{-0}^{C^{d,n}}\right) = \begin{cases} \left(\frac{dn+2e+1}{(1+2e)(n+2e+2)}, \frac{n\{1+2e-2(1+e)d\}}{(1+2e)(n+2e+2)}\right) & \text{if (i) } d < \frac{2e+1}{2e+2} \\ \left(\frac{1}{2(e+1)}, 0\right) & \text{if (ii) } d \geq \frac{2e+1}{2e+2}. \end{cases} \tag{13}$$

From (13), we have the limit quantities. This result corresponds to Theorem 2.

$$\lim_{n\to\infty}\left(q_0^{C^{d,n}}, Q_{-0}^{C^{d,n}}\right) = \begin{cases} \left(\frac{d}{1+2e}, \frac{1+2e-2(1+e)d}{1+2e}\right) & \text{if (i) } d < \frac{2e+1}{2e+2} \\ \left(\frac{1}{2(e+1)}, 0\right) & \text{if (ii) } d \geq \frac{2e+1}{2e+2}. \end{cases} \tag{14}$$

5.2 Path of market structure in equilibrium

(11) and (13) describe the equilibrium paths of Stackelberg and Cournot competitions, respectively. From (11), we can depict Figure 4, which reveals what kind of market structure arises as the number of firms gets larger in Stackelberg competition. (i) If the innovator's cost advantage is minor (i.e., in interval A in Figure 4), the laggard firms enter the market regardless of n, and (ii) if the innovator's cost advantage is moderate (i.e., in interval B in Figure 4), (ii-a) the laggard firms enter the market when n is small, whereas (ii-b) the laggard firms stay out when n is large enough. In the meantime, the paths of the Cournot equilibrium (13) are in contrast to (11). For any non-drastic cost advantage

(i.e., in interval A and B in Figure 4), the laggard firms always enter the market.[23]

To explain this contrast between the Stackelberg and Cournot models, it is plausible if we consider the effect of the number of laggard firms on the innovator's equilibrium output in the case where the innovator's cost advantage is sufficiently small and the case where it is sufficiently close to the drastic one. In Stackelberg competition, the increase of the number of laggard firms has two effects on the innovator's output. With a larger n, (i) since the innovator herself is in a more severe competition,[24] she has an incentive to produce a smaller quantity of goods; (ii) since the innovator can more easily steal the laggard firms' business,[25] it has an incentive to produce a greater quantity of goods. If the innovator's cost advantage is larger, effect (i) is smaller and effect (ii) is greater. If the cost advantage is sufficiently small, effect (i) dominates effect (ii), and thus, the innovator's output is decreasing in the number of laggard firms,[26] which is illustrated in the left panel of Figure 5. Hence, the laggard firms are not deterred. If the cost advantage is sufficiently close to the drastic one, effect (ii) dominates effect (i), and thus, the innovator's output is increasing in the number of laggard firms,[27] which is illustrated in the right panel of Figure 5.[28] In such a case, when $n \geq \bar{n}(d, n)$, the innovator's equilibrium output finally reaches q_d^{QM} and the laggard firms quit the market. On the other hand, in Cournot competition, the innovator cannot reduce the laggard firms' output by producing a large quantity before the laggard firms' decision. Thus, effect (ii) does not exist. Therefore, the innovator's quantity is always decreasing in the number of laggard firms[29] and the laggard firms are not deterred even for a large number of n.

The slope of the marginal cost plays an important role in determining the market structure. For $e \in \mathbb{R}_{++}$ and $n \in \mathbb{N}$, let $l_n^{\mathrm{C}}(e)$, $l_n^{\mathrm{QM}}(e)$, and $l_n^{\mathrm{M}}(e)$ be the lengths of intervals $\left[0, \frac{2en+(2e+1)^2}{(2e+1)(2e+2+n)}\right]$,

[23] As stated in Section 2, if the innovator's cost advantage is drastic (i.e., in interval C in Figure 4), in both the Stackelberg and Cournot models, the innovator can make the laggard firms stay out by supplying the monopoly quantity, regardless of the number of laggard firms.

[24] This comes from the fact each laggard firm's output is smaller. Laggard firms care less about the price reduction since their influence upon the market price is reduced. In addition, they produce the good more efficiently because of the convexity of cost function.

[25] This is observed in Theorem 5. Since the limit price is given for all q_0, an increase of the innovator's output almost perfectly crowds the the laggard firms' production out if n is sufficiently large.

[26] If $d < \frac{e}{e+1}$, $\frac{\partial q_0^{S^{d,n}}}{\partial n} = \frac{(e+1)(2e+1)}{8\{en+(2e+1)(e+1)\}^2}\left(d - \frac{e}{e+1}\right) < 0$.

[27] If $\frac{e}{e+1} \leq d \leq \frac{2e}{2e+1}$ or $\frac{2e}{2e+1} < d < \frac{2e+1}{2e+2} \wedge n < \bar{n}(d, e)$, $\frac{\partial q_0^{S^{d,n}}}{\partial n} = \frac{(e+1)(2e+1)}{8\{en+(2e+1)(e+1)\}^2}\left(d - \frac{e}{e+1}\right) \geq 0$.

[28] This panel depicts the case where $\frac{2e}{2e+1} < d < \frac{2e+1}{2e+2}$.

[29] If $d < \frac{2e+1}{2e+2}$, $\frac{\partial q_0^{C^{d,n}}}{\partial n} = \frac{2e+2}{(2e+1)(n+2e+2)^2}\left(d - \frac{2e+1}{2e+2}\right) < 0$.

$\left(\frac{2en+(2e+1)^2}{(2e+1)(2e+2+n)}, \frac{2e+1}{2e+2}\right)$, and $\left[\frac{2e+1}{2e+2}, 1\right]$, respectively. For $e \in \mathbb{R}_{++}$, let $l_{\infty}^{\mathrm{C}}(e)$, $l_{\infty}^{\mathrm{QM}}(e)$, and $l_{\infty}^{\mathrm{M}}(e)$ be the lengths of intervals $\left[0, \frac{2e}{2e+1}\right]$, $\left(\frac{2e}{2e+1}, \frac{2e+1}{2e+2}\right)$, and $\left[\frac{2e+1}{2e+2}, 1\right]$, respectively. This can be depicted in Table 1. Intuitively, these expressions imply the following. When e is large, the laggard firms can keep their marginal cost as low as the innovator's marginal cost by producing a slightly smaller quantity than that produced by the innovator. Thus, with a large e, the marginal cost disadvantage scarcely matters. Hence, as e gets larger, the laggard firms tend to enter the market, i.e., $l_n^{\mathrm{C}}(e)$ or $l_{\infty}^{\mathrm{C}}(e)$ is increasing in e. In contrast, when e is as small, the difference between the innovator's marginal cost and the laggard firms' marginal cost exists for almost all the combinations of outputs. Thus, the innovator can deter the laggard firms from entering the market for almost all d.

5.3 Path of innovator's equilibrium profits

Finally, we compare the innovator's equilibrium profits in the three abovementioned competitions when the cost disadvantage is sufficiently small. From the arguments above, when $d \leq \frac{2e}{2e+1}$, the innovator's equilibrium profit $\pi_0^{\mathcal{P}^{d,n}}$ in perfect competition is

$$\pi_0^{\mathcal{P}^{d,n}} = \frac{(dn+2e)^2}{4e\,(n+2e+1)^2};$$

when $d < \frac{2e+1}{2e+2}$, the innovator's equilibrium profit $\pi_0^{\mathcal{C}^{d,n}}$ in Cournot competition is

$$\pi_0^{\mathcal{C}^{d,n}} = (e+1)\left\{\frac{dn+2e+1}{(2e+1)\,(n+2e+2)}\right\}^2;$$

and when $d \leq \frac{2en+(2e+1)^2}{(2e+1)(2e+2+n)}$, the innovator's equilibrium profit $\pi_0^{\mathcal{S}^{d,n}}$ in Stackelberg competition is

$$\pi_0^{\mathcal{S}^{d,n}} = \frac{(dn+2e+1)^2}{4\,(n+2e+1)\,(en+(2e+1)(e+1))}.$$

Thus, when the cost disadvantage is sufficiently small, i.e., $d \leq \frac{2e}{2e+1}$, $\pi_0^{\mathcal{P}^{d,n}}$, $\pi_0^{\mathcal{C}^{d,n}}$ and $\pi_0^{\mathcal{S}^{d,n}}$ are calculated as above. It is noteworthy that on the one hand, the innovator's profit $\pi_0^{\mathcal{P}^{d,n}}$ in perfect competition is less than her profit $\pi_0^{\mathcal{C}^{d,n}}$ in Cournot competition if the number n of the laggard firms is sufficiently small, i.e., $n < \frac{2(2e+1)\left(e+\sqrt{e(e+1)}\right)}{d}$; on the other hand, $\pi_0^{\mathcal{P}^{d,n}}$ is greater than $\pi_0^{\mathcal{C}^{d,n}}$ if n is sufficiently large, i.e., $n > \frac{2(2e+1)\left(e+\sqrt{e(e+1)}\right)}{d}$. Figure 6 illustrates how these profits change according to the number of laggard firms.

6 Applications

6.1 Counter example of Arrow effect

When we consider the case of strictly convex costs and are convinced of Theorem 1, the Arrow effect, which asserts that the lack of competitors lessens the incentive to innovate,[30] turns out not to be robust.

Let $\pi^{\mathrm{M},\delta}(q) \equiv P(q)q - C_\delta(q)$. $\pi^{\mathrm{M},\delta}(q)$ is the profit that a monopolist with cost disadvantage δ obtains by producing a quantity of q. $\pi^{\mathrm{M}}(q) \equiv \pi^{\mathrm{M},0}(q)$. Let $q^{\mathrm{M},\delta}$ be the monopoly quantity with cost disadvantage δ. Then, $q^{\mathrm{M}} = q^{\mathrm{M},0}$. Consider the case that the innovation is moderate. By the innovation, a monopolist's profit increases from $\pi^{\mathrm{M},d}\left(q^{\mathrm{M},d}\right)$ to $\pi^{\mathrm{M}}\left(q^{\mathrm{M}}\right)$. Thus, $\pi^{\mathrm{M},d}\left(q^{\mathrm{M},d}\right) - \pi^{\mathrm{M}}\left(q^{\mathrm{M}}\right)$ expresses the incentive for a monopolist to innovate. Needless to say, a firm in perfect competition obtains profit 0. If she gets a new technology by the innovation and becomes the Stackelberg leader, she obtains profit $\pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right)$. Thus, $\pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right)$ expresses the incentive for a firm in perfect competition to innovate. The Arrow effect reveals that $\pi^{\mathrm{M}}(q^{\mathrm{M}}) - \pi^{\mathrm{M},d}(q^{\mathrm{M},d}) < \pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right)$ in the case of constant marginal costs. The following proposition states that the Arrow effect does not necessarily hold in the case that the marginal costs are not constant.

Proposition 1. *We can have* $\pi^{\mathrm{M}}(q^{\mathrm{M}}) - \pi^{\mathrm{M},d}(q^{\mathrm{M},d}) > \pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right)$.

To prove this proposition, the following example suffices.

Example 1. *Consider the case that* $P(Q) = \max\{1-Q, 0\}$ *and* $C_\delta(q) \equiv (a\delta + e)\,q^2 + \delta q$ *for some* $a \in \mathbb{R}_+$ *and some* $e \in \mathbb{R}_{++}$.[31] $q^{\mathrm{M},\delta}$, q_d^{QM}, $\bar{d}$, *and* $\underline{d}$ *are calculated as follows:* $q^{\mathrm{M},\delta} = \frac{1}{2(e+1)}$, $q_d^{\mathrm{QM}} = 1 - d$, $\bar{d} = \frac{2e+1}{2(e+1)}$, *and* $\underline{d} = \frac{2e}{2e+1}$. *Suppose that* $d \in \left[\underline{d}, \bar{d}\right]$. *Then, by Theorem 1, the innovator's equilibrium profit converges to*

[30] See Arrow [1]. We provide a simple graphical explanation of the Arrow Effect in Appendix D. Originally, Arrow considered the situation where the innovator can license her new technology to the other firms and make a take-it-or-leave-it offer of unit royalty and licence quantity before the laggard firms' decisions. If we put the means of license contractions in a black box, we can regard Arrow's story as essentially analyzing the reduced game where the innovator has the commitment power.

[31] If $a = 0$, this example is the same as that in Section 5.

$\pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right) = P\left(q_d^{\mathrm{QM}}\right) q_d^{\mathrm{QM}} - C_0\left(q_d^{\mathrm{QM}}\right) = \{(e+1)d - e\}(1-d)$ as $n \to \infty$. Since $\pi^{\mathrm{M},\delta}(q^{\mathrm{M},\delta}) = \frac{(1-\delta)^2}{4(a\delta+e+1)}$ in the setting of this example, $\pi^{\mathrm{M}}(q^{\mathrm{M}}) - \pi^{\mathrm{M},d}(q^{\mathrm{M,d}}) = \frac{1}{4(e+1)} - \frac{(1-d)^2}{4(ad+e+1)}$. From the above, $\left(\pi^{\mathrm{M}}(q^{\mathrm{M}}) - \pi^{\mathrm{M},d}(q^{\mathrm{M,d}})\right) - \pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right) = \frac{1}{4(e+1)} - \frac{(1-d)^2}{4(ad+e+1)} - \{(e+1)d - e\}(1-d) \equiv f(d,e,a)$. Note that by $d \in \left[\underline{d}, \bar{d}\right]$, $f(d,e,0) = \frac{1}{4(e+1)} - \frac{(1-d)^2}{4(e+1)} - \{(e+1)d - e\}(1-d) \leq 0$ and $\lim_{a\to\infty} f(d,e,a) = \frac{1}{4(e+1)} - \{(e+1)d - e\}(1-d) \geq 0$. Also, note that $f(d,e,0) < 0$ if $d > \underline{d}$, and $\lim_{a\to\infty} f(d,e,a) > 0$ if $d < \bar{d}$. Therefore, if $d > \underline{d}$ and a is sufficiently small, the incentive to innovate is stronger in a competitive market than it is in a monopoly market. On the other hand, if $d < \bar{d}$ and a is sufficiently large, it is stronger in a monopoly market than it is in a competitive market. Figure 7 illustrates a situation where $d < \bar{d}$ and a is sufficiently large.

Recall that in the Arrow effect, the lack of competitors results in a monopoly profit in the pre-innovation market and reduces the incentive to innovate. In our analysis, another adverse effect arises. If the marginal cost is increasing, the more a technology spreads, the greater the efficiency of the production of the entire industry. Therefore, the innovation becomes less profitable as the number of firms sharing the old technology increases. The monopolization of technology in a pre-innovation market prevents such rivalry by spreading worn-out technology, and increases the incentive to innovate.

6.2 Asymmetric tax exemption or subsidy

We regard cost advantage as asymmetric tax exemption or subsidy for a particular firm.[32] If the subsidy per unit for firm 0 is d and she is the only firm to receive the subsidy, cost functions for firm $i \in L^n$ are $C_d(q_i) = C_0(q_i) + dq_i$. Thus, we can evaluate the effect of asymmetric subsidy by using the framework of this paper. For example, if the rate of subsidy is sufficiently small and the subsidized firm is the Stackelberg leader, in the limit $n \to \infty$, all firms including the subsidized one are in perfect competition, wherein the equilibrium price is the one before the subsidy was introduced. If the rate of the subsidy is not drastic such that the subsidized firm does not become a monopolist, in both Stackelberg and Cournot games, in the limit $n \to \infty$, the subsidy does not vary the total supply of the market. Some other institutional advantages for a particular firm may be interpreted as cost advantage, similar to tax exemption or subsidy.

[32] For example, consider an open market where 1 home firm and n foreign firms compete and only the home firm can supply without the import tariff.

7 Concluding remarks

We addressed the following question: "In an initially competitive market, what kind of markets arise when one firm succeeds in inventing a new technology?" We have induced, focusing on non-drastic innovation, an asymmetric limit theorem of n-follower Stackelberg competition to give a game theoretic basis to the answer of this question. As a result, competitive markets and quasi-monopoly markets arise depending on the extent of cost advantage accomplished by the invention. Further, we also induced an asymmetric limit theorem of Cournot competition and proved that partial-monopoly markets arise for non-drastic innovation.

Although we have been interpreting our discussion as a story of cost-reducing innovation, it is interesting if we view it more theoretically. Since Theorem 5 indicates the limit price not only for Stackelberg or Cournot equilibria but also for any sequence of $q_0 \in [0, q_d^{\mathrm{QM}})$, this theorem is applicable to other principles of firm 0's behavior. Thus, we believe our analysis is useful for a wider class of games.

We conclude this paper with two remarks, which open up the possibility of further research.

Arrow [1] considered that the leadership of the innovator comes from patent licenses. We neglect this aspect instead of considering general cost reductions. Our model treats the world where there is no patent protection[33] and the innovator is supposed to keep her new technology to herself and produce it by herself. Meanwhile, following Arrow, there are literatures that highlight the aspect of patent license (e.g., Kamien and Tauman [5] [6], Kamien [3], Wang [12]). In these literatures, they treat the world with perfect patent protection and the technologies with constant marginal cost. In reality, patents are protected to some extent but not completely and the technologies are more general. Therefore, it is important to fill the gaps between these literatures and our analysis.

In the body of paper, we avoid discussing the set-up cost of the technology and endogenizing the number of firms. In other words, we assume the increasing average variable cost and make n diverge exogenously. Since we consider the period in which the new technology does not spread, the set-up cost might have sunk. In this case, our analysis works well as an approximation of the market with a sufficiently large n. However, it is also possible that some firms exit the market within

[33]Recently, the importance of considering such a primitive world is increasing, especially keeping in mind digital technologies.

this period. In such a case, we can offer a simple modification to apply the limit outcome to our analysis. Suppose that there are non-sunk set-up costs, $F_0 > 0$ for new technology and $F_d > 0$ for laggard technology. Consider that n is determined endogenously by a zero-profit condition. Then, the original pre-invention market grows into a competitive one as F_d goes to zero. Also, consider the limit outcome with $F_d \to 0$ in the post-invention market. Since $n \to \infty$ when $F_d \to 0$, this limit outcome is the same as that in the body of the paper.[34] Of course, converging F_d to zero is not the only way to make a pre-invention market competitive.[35] Further discussion is required to refine the discussion of this paragraph.

Appendix

A Proofs

A.1 Proof of Lemma 2

Proof. Recall that $\bar{d}$ is defined by $C'_{\bar{d}}(0) = P(q^{\mathrm{M}})$ and that q^{P} is defined by $C'_0(q^{\mathrm{P}}) = P(q^{\mathrm{P}})$. Define $\underline{d}$ as $C'_{\underline{d}}(0) = P(q^{\mathrm{P}})$. Notice that $q^{\mathrm{P}} > q^{\mathrm{M}}$, since if we supposed the contrary, $q^{\mathrm{P}} \leq q^{\mathrm{M}}$, the relation

$$C'_0\left(q^{\mathrm{P}}\right) \leq C'_0\left(q^{\mathrm{M}}\right) = P\left(q^{\mathrm{M}}\right) + P'\left(q^{\mathrm{M}}\right)q^{\mathrm{M}} < P\left(q^{\mathrm{M}}\right) \leq P\left(q^{\mathrm{P}}\right)$$

contradicts the definition of q^{P}. Hence, we obtain $C'_{\underline{d}}(0) = P(q^{\mathrm{P}}) < P(q^{\mathrm{M}}) = C'_{\bar{d}}(0)$ and this implies that $\underline{d} < \bar{d}$. By the definition of $\underline{d}$ and Definition 1, $d \leq \underline{d} \Leftrightarrow C'_d(0) \leq C'_{\underline{d}}(0) \Leftrightarrow P\left(q_d^{\mathrm{QM}}\right) \leq P(q^{\mathrm{P}}) \Leftrightarrow q_d^{\mathrm{QM}} \geq q^{\mathrm{P}}$ holds. Since $C'_0(q) - P(q)$ is strictly increasing, we have $d \leq \underline{d} \Leftrightarrow C'_0(q_d^{\mathrm{QM}}) - P(q_d^{\mathrm{QM}}) \geq C'_0(q^{\mathrm{P}}) - P(q^{\mathrm{P}})$. By the definition of q^{P}, we have

$$d \leq \underline{d} \Leftrightarrow C'_0(q_d^{\mathrm{QM}}) - P(q_d^{\mathrm{QM}}) \geq 0. \tag{15}$$

[34] Firm 0 will carry out its innovation project if F_0 is less than its post-invention profit induced in our paper.

[35] In the literature of the Cournot limit theorem, Ruffin [11] pointed out that when the average cost is U-shaped, the limit outcome with the indefinite number of firms does not converge to a competitive one. Novshek [7] resolved this difficulty by shrinking the scale of U-shaped average cost instead of diverging the number of firms.

Recall that q_d^{P} is defined by $C_d'(0) = C_0'\left(q_d^{\mathrm{P}}\right)$. Substituting this definition and Definition 1 into (15), we obtain

$$d \leq \underline{d} \Leftrightarrow C_0'(q_d^{\mathrm{QM}}) \geq C_d'(0) \Leftrightarrow C_0'(q_d^{\mathrm{QM}}) \geq C_0'(q_d^{\mathrm{P}}) \Leftrightarrow q_d^{\mathrm{QM}} \geq q_d^{\mathrm{P}}$$

Q.E.D.

A.2 Proof of Lemma 4

Proof. (i) Take any $q_0 < q_d^{\mathrm{QM}}$. By Lemma 3, $q_1^{d,n}(q_0) > 0$ and thus (2) holds with equality, which implies that $P\left(Q^{d,n}(q_0)\right) \geq C_d'\left(q_1^{d,n}(q_0)\right)$. Therefore, by $C_d'\left(q_1^{d,n}(q_0)\right) \geq C_d'(0) = P\left(q_d^{\mathrm{QM}}\right)$, $P\left(Q^{d,n}(q_0)\right) \geq P\left(q_d^{\mathrm{QM}}\right)$. Hence, $Q^{d,n}(q_0) \leq q_d^{\mathrm{QM}}$. By definition, $Q^{d,n}(q_0) \geq q_0$. Therefore, $q_0 \leq Q^{d,n}(q_0) \leq q_d^{\mathrm{QM}}$ for any $q_0 < q_d^{\mathrm{QM}}$. Hence, by $q_0 \uparrow q_d^{\mathrm{QM}}$, we have $q_d^{\mathrm{QM}} \leq \lim_{q_0 \uparrow q_d^{\mathrm{QM}}} Q^{d,n}(q_0) \leq q_d^{\mathrm{QM}}$, which implies that $\lim_{q_0 \uparrow q_d^{\mathrm{QM}}} Q^{d,n}(q_0) = q_d^{\mathrm{QM}}$. By Lemma 3, for any $q_0 \geq q_d^{\mathrm{QM}}$, $Q^{d,n}(q_0) = q_0$. Thus, $Q^{d,n}$ is continuous at q_d^{QM}. (ii) For any $q_0 < q_d^{\mathrm{QM}}$, by Lemma 3, $q_1^{d,n}(q_0) > 0$, and thus, (2) holds with equality. Thus, the implicit function theorem implies that $q_1^{d,n}$ is continuous on $\left[0, q_d^{\mathrm{QM}}\right)$. Thus, $Q^{d,n}$ is continuous on $\left[0, q_d^{\mathrm{QM}}\right)$. (iii) For any $q_0 > q_d^{\mathrm{QM}}$, by Lemma 3, $q_1^{d,n}(q_0) = 0$, and thus, $Q^{d,n}(q_0) = q_0$. Therefore, $Q^{d,n}$ is continuous on $\left(q_d^{\mathrm{QM}}, \infty\right)$. By (i)-(iii), $Q^{d,n}$ is a continuous function. **Q.E.D.**

A.3 Proof of Theorem 4

Proof. Take any $q_0 \in \left[0, q_d^{\mathrm{QM}}\right)$. By Lemma 3, $q_1^{d,n}(q_0) > 0$ for all $n \in \mathbb{N}$. Thus, (2) is reduced to equality. First, we prove the following two lemmas:

Lemma 6. $\lim_{n \to \infty} q_1^{d,n}(q_0) = 0$.

Proof. Suppose the contrary, $\exists \epsilon > 0, \forall \bar{n} \in \mathbb{N}, \exists n \geq \bar{n} : q_1^{d,n}(q_0) > \epsilon$. Then, we can make up a subsequence $\left(q_1^{d,m(n)}(q_0)\right)$ of $\left(q_1^{d,n}(q_0)\right)$ such that $q_1^{d,m(n)}(q_0) > \epsilon$. Since $m(n) q_1^{d,m(n)}(q_0) > m(n)\epsilon$ is boundless upward,

$$\lim_{n \to \infty} P\left(q_0 + m(n) q_1^{d,m(n)}(q_0)\right) = 0$$

holds by Assumption 3. Notice that $P'\left(q_0 + m(n) q_1^{d,m(n)}(q_0)\right) q_1^{d,m(n)}(q_0) \leq 0$. Then, if we take a sufficiently large $n' \in \mathbb{N}$, we have $\forall \delta > 0, \exists n' \in \mathbb{N}, \forall n \geq n'$,

$$\delta > P\left(q_0 + m(n)q_1^{d,m(n)}(q_0)\right) + P'\left(q_0 + m(n)q_1^{d,m(n)}\right)q_1^{d,m(n)}(q_0).$$

However, we also have $C_d'\left(q_1^{d,m(n)}(q_0)\right) > C_d'(\epsilon) > 0$ by $C_d'' > 0$. Taking a sufficiently large n, the first order condition is never satisfied with equality for such an n, which is a contradiction. **Q.E.D.**

Lemma 7. $\lim_{n\to\infty} P'(q_0 + nq_1^{d,n}(q_0))q_1^{d,n}(q_0) = 0$.

Proof. From the lemma above, we can conclude this proof if $\lim_{n\to\infty} P'(q_0 + nq_1^{d,n}(q_0)) \neq -\infty$. Since P is differentiable in an open interval $P^{-1}(\mathbb{R}_{++})$, it is possible that $\lim_{n\to\infty} P'(q_0 + nq_1^{d,n}(q_0)) = -\infty$ only if $q_0 + nq_1^{d,n}(q_0)$ converges to a boundary of $P^{-1}(\mathbb{R}_{++})$ as $n \to \infty$. If $\bar{Q}$ is a boundary of $P^{-1}(\mathbb{R}_{++})$, we must have $\bar{Q} = 0$ or $\bar{Q} > 0$ with $P(\bar{Q}) = 0$. First, assume that $q_0 + \lim_{n\to\infty} nq_1^{d,n}(q_0) = \bar{Q} = 0$. Then, we have $q_0 = 0$ and $\lim_{n\to\infty} nq_1^{d,n}(q_0) = 0$. By taking n to infinity in (2), we obtain

$$\lim_{n\to\infty}\left(P(nq_1^{d,n}(0)) + P'(nq_1^{d,n}(0))q_1^{d,n}(0)\right) = \lim_{n\to\infty} C_d'(q_1^{d,n}(0)).$$

Note that the convergence of the left hand side is derived from the convergence of the right hand side. Rearranging this equation, we get $\lim_{n\to\infty}\left(P(nq_1^{d,n}(0)) + P'(nq_1^{d,n}(0))q_1^{d,n}(0) - C_d'(q_1^{d,n}(0))\right) = 0$. However, since $\forall q \in [0, Q], P(Q) + P'(Q)q - C_\delta'(q) \geq P(Q) + P'(Q)Q - C_\delta'(Q)$, we must have

$$\lim_{n\to\infty}\left(P(nq_1^{d,n}(0)) + P'(nq_1^{d,n}(0))q_1^{d,n}(0) - C_d'(q_1^{d,n}(0))\right) > 0,$$

by Assumption 2 and $\lim_{n\to\infty} nq_1^{d,n}(q_0) = 0$, which is a contradiction. Second, we assume that $q_0 + \lim_{n\to\infty} nq_1^{d,n}(q_0) = \bar{Q} > 0$, where $P(\bar{Q}) = 0$. Then, we have

$$\lim_{n\to\infty} P(q_0 + nq_1^{d,n}(q_0)) = P(\bar{Q}) = 0 \leq C_0'(0) < C_d'(0) = \lim_{n\to\infty} C_d'(q_1^{d,n}(q_0)).$$

Thus, for a sufficiently large n, (2) cannot be satisfied with equality, which is a contradiction. **Q.E.D.**

From the lemmas above and (2) with equality, we obtain Theorem 4. **Q.E.D.**

B Alternative Proof of Lemma 1

Lemma 1 is shown in a similar, rather simplified way to the proof of the limit result in the Cournot game provided in the body of our paper. However, here, we dare provide an alternative proof which does not make use of the structure of uniform convergence. This is because readers can notice that even if we describe the limit result without uniform convergence, almost similar steps are needed to rigorously prove the limit result.

Proof. Note that $q_0^{\mathcal{P}^{d,n}} > 0$ by Assumption 2 and $C_d'(0) - C_0'(0) > 0$. Hence, the equilibrium output profile $\left(q_k^{\mathcal{P}^{d,n}}\right)_{k\in\{0\}\cup L^n}$ of $\mathcal{P}^{d,n}$ is derived from the conditions that

$$P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right) = C_0'\left(q_0^{\mathcal{P}^{d,n}}\right), \tag{16}$$

$$P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right) \leq C_d'\left(q_1^{\mathcal{P}^{d,n}}\right) \tag{17}$$

with equality if $q_1^{\mathcal{P}^{d,n}} > 0$, and that $q_1^{\mathcal{P}^{d,n}} = q_i^{\mathcal{P}^{d,n}}$ for all $i \in L^n$ where we make use of the symmetry of laggard firms.

Step 1 Take any $n \in \mathbb{N}$. Then, $q_1^{\mathcal{P}^{d,n}} = 0$ if and only if $q_0^{\mathcal{P}^{d,n}} \geq q_d^{\mathrm{QM}}$.

The proof is similar to that of Lemma 3. Suppose $q_1^{\mathcal{P}^{d,n}} = 0$. Then, (17) turns out to be $P(q_0^{\mathcal{P}^{d,n}}) \leq C_d'(0)$. If $q_0^{\mathcal{P}^{d,n}} < q_d^{\mathrm{QM}}$, $P(q_0^{\mathcal{P}^{d,n}}) > C_d'(0)$ from Definition 1, which is a contradiction. Suppose $q_0^{\mathcal{P}^{d,n}} \geq q_d^{\mathrm{QM}}$. If $q_1^{\mathcal{P}^{d,n}} > 0$, (17) holds with equality. However, $P(Q^{\mathcal{P}^{d,n}}) < P\left(q_d^{\mathrm{QM}}\right) < P\left(q_d^{\mathrm{QM}}\right) = C_d'(0) < C_d'(q_1^{\mathcal{P}^{d,n}})$, which is a contradiction. ■

Step 2 $\lim_{n\to\infty} P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right) = C_d'(0)$ if $\forall n \in \mathbb{N}$, $q_0^{\mathcal{P}^{d,n}} < q_d^{\mathrm{QM}}$.

The proof is similar to that of Theorem 4. First, we show that $\lim_{n\to\infty} q_1^{\mathcal{P}^{d,n}} = 0$ if $\forall n \in \mathbb{N}$, $q_0^{\mathcal{P}^{d,n}} < q_d^{\mathrm{QM}}$. Suppose the contrary, $\exists\epsilon > 0, \forall\bar{n} \in \mathbb{N}, \exists n \geq \bar{n} : q_1^{\mathcal{P}^{d,n}} > \epsilon$. Then, we can make up a subsequence $\left(q_1^{\mathcal{P}^{d,m(n)}}\right)$ of $\left(q_1^{\mathcal{P}^{d,n}}\right)$ such that $q_1^{\mathcal{P}^{d,m(n)}} > \epsilon$. Since $m(n)q_1^{\mathcal{P}^{d,m(n)}} > m(n)\epsilon$ is boundless upward,

$$\lim_{n\to\infty} P\left(q_0 + m(n)q_1^{\mathcal{P}^{d,m(n)}}\right) = 0$$

holds by Assumption 3. Then, if we take a sufficiently large $n' \in \mathbb{N}$, we have $\forall \delta > 0, \exists n' \in \mathbb{N}, \forall n \geq n'$,

$$\delta > P\left(q_0 + m(n) q_1^{\mathcal{P}^{d,m(n)}}\right).$$

However, we also have $C_d'\left(q_1^{\mathcal{P}^{d,m(n)}}\right) > C_d'(\epsilon) > 0$ by $C_d'' > 0$. Since $q_0^{\mathcal{P}^{d,n}} < q_d^{\mathrm{QM}}$, the step 1 implies that $q_1^{\mathcal{P}^{d,n}} > 0$. Thus, (17) holds with equality. Taking a sufficiently large n, (17) is never satisfied for such an n, which is a contradiction.

Therefore, by taking n to infinity in (17), we obtain

$$\lim_{n\to\infty} P\left(q_0^{\mathcal{P}^{d,n}} + n q_1^{\mathcal{P}^{d,n}}\right) = \lim_{n\to\infty} C_d'(q_1^{\mathcal{P}^{d,n}}) = C_d'(0).$$

Note that the convergence of the left hand side is derived from the convergence of the right hand side. ■

Step 3 $\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = q_d^{\mathrm{P}}$ if $\forall n \in \mathbb{N}$, $q_0^{\mathcal{P}^{d,n}} < q_d^{\mathrm{QM}}$.

The proof is similar to that of Theorem 2. Suppose that $q_0^{C^{d,n}} < q_d^{\mathrm{QM}}$ for any $n \in \mathbb{N}$. Then, by the step 2 and by Definition 1, $\lim_{n\to\infty} P\left(q_0^{\mathcal{P}^{d,n}} + n q_1^{\mathcal{P}^{d,n}}\right) = C_d'(0) = P\left(q_d^{\mathrm{QM}}\right)$. Hence, $\forall \delta > 0, \exists n' \in \mathbb{N}, \forall n \geq n'$,

$$\left|\left(P\left(q_0^{\mathcal{P}^{d,n}} + n q_1^{\mathcal{P}^{d,n}}\right) - C_0'\left(q_0^{\mathcal{P}^{d,n}}\right)\right) - \left(P\left(q_d^{\mathrm{QM}}\right) - C_0'\left(q_0^{\mathcal{P}^{d,n}}\right)\right)\right| < \delta.$$

Now suppose the contrary, $\exists \epsilon > 0, \forall \bar{n} \in \mathbb{N}, \exists n \geq \bar{n}, \left|q_0^{\mathcal{P}^{d,n}} - q_d^{\mathrm{P}}\right| > \epsilon$. Then, since $C_0'' > 0$, $\exists \delta' > 0, \forall \bar{n} \in \mathbb{N}, \exists n \geq \bar{n}$,

$$\left|\left(P\left(q_d^{\mathrm{QM}}\right) - C_0'\left(q_0^{\mathcal{P}^{d,n}}\right)\right) - \left(P\left(q_d^{\mathrm{QM}}\right) - C_0'\left(q_d^{\mathrm{P}}\right)\right)\right| > \delta'.$$

Therefore, when we take $\delta < \delta'$, there exists $n \in \mathbb{N}$ such that

$$P\left(q_0^{\mathcal{P}^{d,n}} + n q_1^{\mathcal{P}^{d,n}}\right) - C_0'\left(q_0^{\mathcal{P}^{d,n}}\right) \neq P\left(q_d^{\mathrm{QM}}\right) - C_0'\left(q_d^{\mathrm{P}}\right) = C_d'(0) - C_0'\left(q_d^{\mathrm{P}}\right) = 0$$

Therefore, (16) is not satisfied for such an n, which is a contradiction. ■

Step 4 Take any $n \in \mathbb{N}$. Then, $q_0^{\mathcal{P}^{d,n}} < q_d^{\mathrm{QM}}$ if $C_d'(0) < P\left(q^{\mathrm{P}}\right)$ and $q_0^{\mathcal{P}^{d,n}} \geq q_d^{\mathrm{QM}}$ if $C_d'(0) \geq P\left(q^{\mathrm{P}}\right)$.

When $C_d'(0) < P\left(q^{\mathrm{P}}\right)$, we have $q^{\mathrm{P}} < q_d^{\mathrm{QM}}$ by Definition 1. Thus, if we suppose that $q_0^{\mathcal{P}^{d,n}} \geq q_d^{\mathrm{QM}}$,

$$P\left(q_0^{\mathcal{P}^{d,n}}\right) \leq P\left(q_d^{\mathrm{QM}}\right) < P\left(q^{\mathrm{P}}\right) = C_0'\left(q^{\mathrm{P}}\right) < C_0'\left(q_d^{\mathrm{QM}}\right) \leq C_0'\left(q_0^{\mathcal{P}^{d,n}}\right).$$

Since $P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right) \leq P\left(q_0^{\mathcal{P}^{d,n}}\right)$ by $q_1^{\mathcal{P}^{d,n}} \geq 0$, we must have $P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right) < C_0'\left(q_0^{\mathcal{P}^{d,n}}\right)$, which contradicts (16). Similarly, when $C_d'(0) \geq P\left(q^{\mathrm{P}}\right)$, we have $q^{\mathrm{P}} \geq q_d^{\mathrm{QM}}$, and under the supposition that $q_0^{\mathcal{P}^{d,n}} > q_d^{\mathrm{QM}}$, we can show that $P\left(q_0^{\mathcal{P}^{d,n}}\right) > C_0'\left(q_0^{\mathcal{P}^{d,n}}\right)$. Thus, $P\left(q_0^{\mathcal{P}^{d,n}}\right) > P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right)$ by (16). This implies that $q_1^{\mathcal{P}^{d,n}} > 0$ and (17) holds with equality. Therefore,

$$P\left(q_0^{\mathcal{P}^{d,n}} + nq_1^{\mathcal{P}^{d,n}}\right) = C_d'\left(q_1^{\mathcal{P}^{d,n}}\right) > C_d'(0) \geq P\left(q^{\mathrm{P}}\right) = C_0'\left(q^{\mathrm{P}}\right) \geq C_0'\left(q_d^{\mathrm{QM}}\right) > C_0'\left(q_0^{\mathcal{P}^{d,n}}\right),$$

which contradicts (16). ■

From the step 3 and the step 4, $\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = q_d^{\mathrm{P}}$ if $C_d'(0) < P\left(q^{\mathrm{P}}\right)$. From the step 1 and the step 4, $\forall n \in \mathbb{N}$, $q_1^{\mathcal{P}^{d,n}} = 0$ if $C_d'(0) \geq P\left(q^{\mathrm{P}}\right)$. Hence, (16) turns out to be $P\left(q_0^{\mathcal{P}^{d,n}}\right) = C_0'\left(q_0^{\mathcal{P}^{d,n}}\right)$. This implies that $\forall n \in \mathbb{N}$, $q_0^{\mathcal{P}^{d,n}} = q^{\mathrm{P}}$ and thus, $\lim_{n\to\infty} q_0^{\mathcal{P}^{d,n}} = q^{\mathrm{P}}$. Note that $q_d^{\mathrm{P}} = q^{\mathrm{P}}$ when $C_d'(0) = P\left(q^{\mathrm{P}}\right)$. **Q.E.D.**

C Mathematical Methods

We provide a mathematical result suitable for our analysis. Pointwise convergence is not sufficient to prove the lemma below. Uniform convergence is the key condition.

Lemma 8 (Solution of maximization in the limit)**.**
Let $D \subset \mathbb{R}$ be compact. Suppose that a sequence of functions $(f^n : D \to \mathbb{R})_{n\in\mathbf{N}}$ uniformly converges to a continuous function $f : D \to \mathbb{R}$. If f has the unique maximizer denoted by $x^* \in D$ and f^n has a maximizer denoted by $x^n \in D$ for each n, we have $x^* = \lim_{n\to\infty} x^n$.

Proof. Suppose the contrary, $\exists\epsilon > 0, \forall n, \exists n' > n, x^{n'} \notin B_\epsilon(x^*)$, where $B_\epsilon(x^*)$ represents the ϵ-open ball around x^*. Since f is continuous and $D' \equiv D \setminus B_\epsilon(x^*)$ is compact, $\max_{x\in D'} f(x)$ exists by the Weierstrass' theorem. Since x^* is the unique maximizer, $x^* > \max_{x\in D'} f(x)$. Thus, $\exists\delta > 0$, $\forall n'$ such that $x^{n'} \notin B_\epsilon(x^*)$, $f(x^*) > f(x^{n'}) + \delta$. However, we have the fact that $(f^n)_{n\in\mathbf{N}}$ uniformly converges to f. Therefore, $\exists n'', \forall x, \forall n > n'', |f(x) - f^n(x)| < \delta/2$. If we take a sufficiently large n' such that $x^{n'} \notin B_\epsilon(x^*)$, $n' > n''$ is held. Thus, $|f(x^*) - f^{n'}(x^*)| < \delta/2$, $|f(x^{n'}) - f^{n'}(x^{n'})| < \delta/2$, and $f(x^*) > f(x^{n'}) + \delta$ at the same time. From these,

$$f^{n'}(x^{n'}) - f^{n'}(x^*) \leq |f^{n'}(x^{n'}) - f(x^{n'})| + f(x^{n'}) - f(x^*) + |f(x^*) - f^{n'}(x^*)| < \frac{\delta}{2} - \delta + \frac{\delta}{2} = 0.$$

Hence, we obtain $f^{n'}(x^*) > f^{n'}(x^{n'})$, which contradicts to the fact that $x^{n'}$ is the maximizer of $f^{n'}(x)$.

Q.E.D.

Note that Lemma 8 allows the maximizers of f^n to be multiple. It is adequate if x^n is one of the maximizers of f^n, i.e.,

$$x^n \in \arg\max_{x \in D} f^n(x).$$

D Reprising Arrow's Analysis

In this section, we will offer a brief discussion about the classic analysis by Arrow [1]. For that, consider a special game of our model. Although Arrow did not mention the game theoretic nature of his discussion clearly, we can reprise his classic analyses by investigating this game.

In the game, we make following assumptions. First, the innovator is only the player who behaves strategically and all the laggard firms are price takers. Second, all the cost functions have constant marginal costs. Let $C'_d(q) = c + d$. Then, the marginal cost is reduced to $C'_0(q) = c$ after the innovation. Third, the innovator can commit the quantity of her outputs before the laggard firms' choice.

We can easily see this game's result graphically in Figure 8, which is perfectly coincides with Arrow's result. If $d < \bar{d}$ (i.e., non-drastic innovation), the monopoly price with the new technology p^{M} is higher than $c + d$ as seen in Figure 8. In this case, the innovator can not drives out the old technology if she intends to be the pure monopolist. However, by committing the output level $|OB|$ (the length of a segment OB), the innovator occupies an entire market under the competitive price of pre-invention market c and earns her maximized profit $d|OB|$.[36] Then, the entire realized benefit is appropriated to the innovator.

Moreover, Arrow considered the case of monopoly, where there are no other firms than the innovator, and compared the incentive of innovation in this case with that under competition.

[36] Suppling more than $|OB|$ does not raise the innovator's profit since $|OB| > |OA|$ where $|OA|$ is the monopoly output corresponding to p^{M}.

Under monopoly, even before the invention, the innovator earns the monopoly profit corresponding to the old marginal cost $c + d$. Thus, the difference between the innovator's pre-invention profit and her post-invention profit is represented by the heavily shaded areas in Figure 8. On the other hand, under competition, the innovator must operate with zero profit before the invention since all the other firms are price takers. Therefore, the incentive to innovate under monopoly is less than under competition by the lightly shaded areas in Figure 8. This finding is often referred as "Arrow effect".

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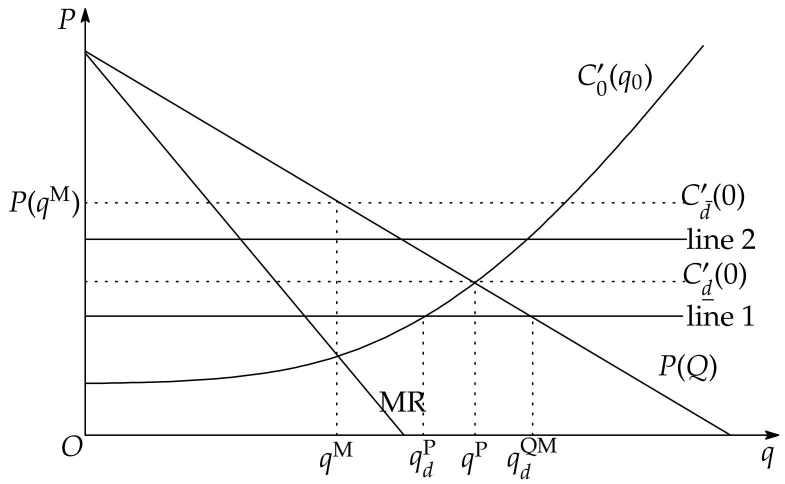

Figure 1: Graphical proof of Lemma 2

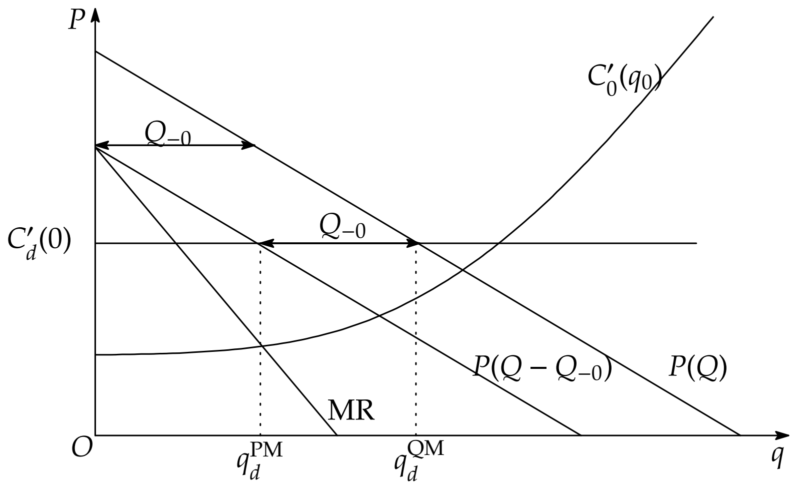

Figure 2: Quasi monopoly and partial monopoly

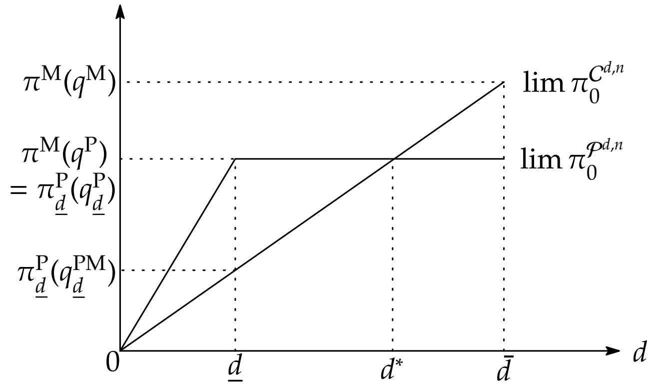

Figure 3: Limit profits in games $C^{d,n}$ and $\mathcal{P}^{d,n}$

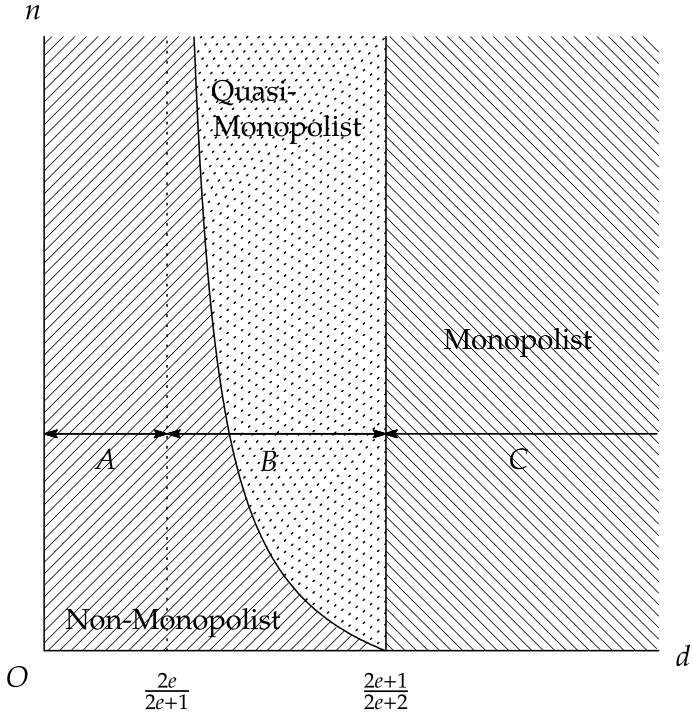

Figure 4: Effect of the laggard firms' number and cost disadvantage on equilibrium quantity in Stackelberg competition

	$l_n^{\mathrm{C}}(e)$	$l_n^{\mathrm{QM}}(e)$	$l_n^{\mathrm{M}}(e)$	$l_\infty^{\mathrm{C}}(e)$	$l_\infty^{\mathrm{QM}}(e)$	$l_\infty^{\mathrm{M}}(e)$
$\frac{d}{de}$	$+$	$-$	$-$	$+$	$-$	$-$
$\lim_{e\to 0}$	$\frac{1}{n+2}$	$\frac{n}{2(n+2)}$	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$\lim_{e\to\infty}$	1	0	0	1	0	0

Table 1: Effect of the slope of the marginal cost

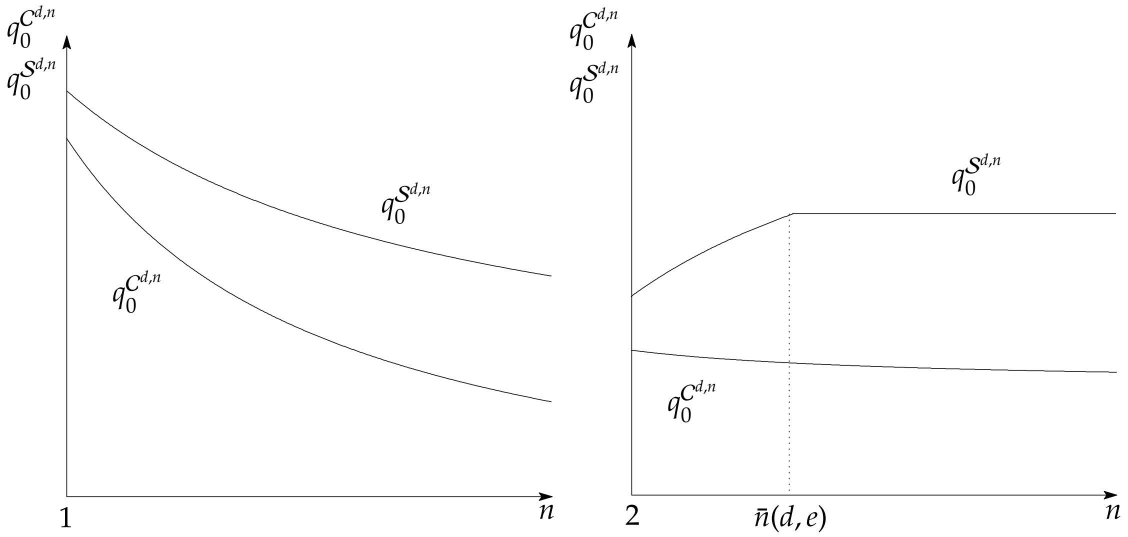

Figure 5: Effect of the number of laggard firms on the innovator's equilibrium output. The left panel illustrates the case where $e = 1$ and $d = \frac{1}{4}$. The right panel illustrates the case where $e = 1$ and $d = \frac{17}{24}$.

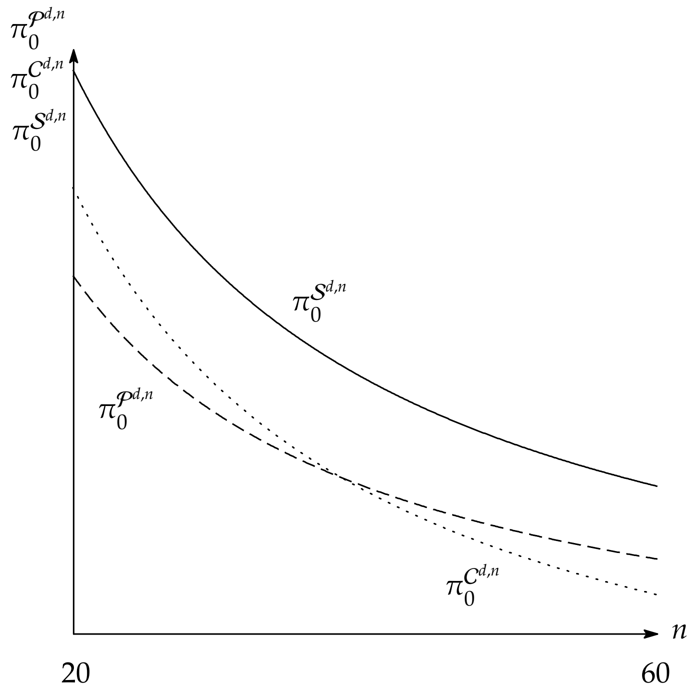

Figure 6: Innovator's profits $\pi_0^{\mathcal{P}^{d,n}}$, $\pi_0^{C^{d,n}}$, and $\pi_0^{S^{d,n}}$ change according to the number n of laggard firms, as depicted in this figure. The figure illustrates the case where $d = \frac{1}{4}$ and $e = 1$. When n is large, $\pi_0^{\mathcal{P}^{d,n}}$ is greater than $\pi_0^{C^{d,n}}$.

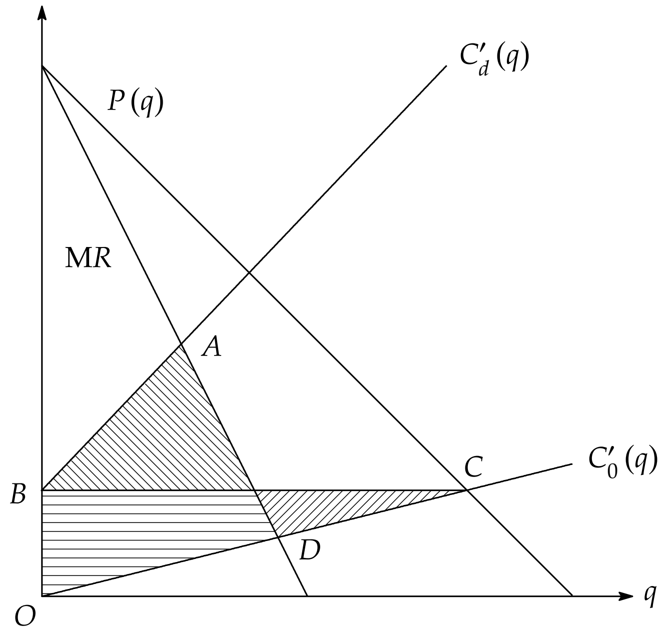

Figure 7: This figure illustrates the case where $d = \frac{1}{5}$, $e = \frac{1}{8}$ and $a = 2$. The areas of quadrilateral $ABOD$ and triangle BOC indicate $\pi^{\mathrm{M}}(q^{\mathrm{M}}) - \pi^{\mathrm{M},d}(q^{\mathrm{M,d}})$ and $\pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right)$, respectively. In this case, $\pi^{\mathrm{M}}(q^{\mathrm{M}}) - \pi^{\mathrm{M},d}(q^{\mathrm{M,d}}) > \pi^{\mathrm{M}}\left(q_d^{\mathrm{QM}}\right)$.

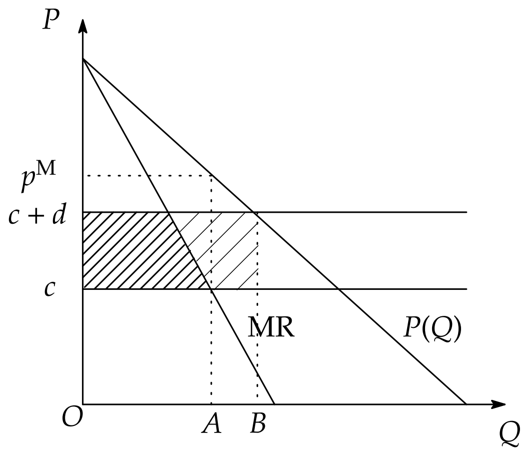

Figure 8: Reprising Arrow Effect