Construction of Preference and Information Flow: I. General Theory *

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Abstract

In this article, we propose a general framework of "representation," which enables us to see "preference relation," the fundamental structure of economics, as a structure derived by agent's own perception. In other words, we regard "preference relation" not as a given static order but as a flexible structure generated according to agent's own perception that, for example, a is preferable than b because a satisfies some desires but b doesn't.

We usually deem some particulars as satisfying some desires from limited observations. This is what we call "representation." Connections between particulars in front of us and the images in our mind are considered to be made by works of "representation." We will propose a "representation system" that formulates these functions of perception. Preference relation between particulars is defined according to the extent these particulars can satisfy desires.

Taking preference relation as a flexible structure, our approach enables us within a unified formal language to formalize various criteria of decision regardless of that of standard modern economics or that of behavioral economics. Such formalizations have not been adequately accomplished by neither psychology nor behavioral economics.

Keywords: Construction of Preference; Channel Theory; Representation System

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1 Construction of Preference, Agent's Interpretation, A New Formulation of Knowledge

In this article we present a new framework of decision making from a view point of construction of preference. Although our framework is rather cognitive and applicable for practical cases of decision making, but is logical and mathematical at the same time. Our framework is distinctive in two major ways compared to the traditional decision theory or economics. One is a standpoint that preference relations should be regarded as derivatives of more fundamental structure. The other is a new formulation of knowledge, describing the structure of knowledge of the agent who makes decisions, which is different from the one in traditional theory.

Construction of Preference

Whether it is regarded as the one which is merely formal or the one which captures the agent's actual thinking, preference relation is established axiomatically as a primitive in most of the decision theory. In contrast to this traditional view, we think preference relation should be derived

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from more primitive structure “desire” and “knowledge.” We also think that it is not a formal conceptual apparatus but an expression of actual thinking of the agent in decision making and a theory of derivation of preference relation models the process of thinking. Regarded as such, the structure of preference relation, which can be seen as an axiomatic foundation of utility-based decision theory, comes to be more realistic and practicable and, at the same time, becomes possible to analyze decision making more finely in descriptiveness and normativeness.

The necessity of a theory of construction of preference seems to be gaining significance. Behavioral and experimental economics has been illustrated choice behaviors such that the established idea of preference which presumes that agents have fixed preference relation in their mind and choose the best option according to it has difficulty explaining. Psychologists (for instance, Slovic (1995)) who promoted behavioral economics have been maintained the necessity of the theory of thoughtful process in constructing preference, which should be applied to the case such that there is indecisive trade-offs between options in the first situation (in many cases experiments would be designed as such) or agents do not realize what they want immediately because of the multiplicity of criterion for evaluation. On the other hand, they also state that axiomatic utility-based decision theory provides compelling argument for applicable case (Tversky and Kahneman (1986)). That is to say, it seems that psychologists consider the division of roles between orthodox decision theory and theory of construction of preference according to whether the decision maker has “rational preference order” in her mind or not. Here highlights the difference in theoretical position for preference relation between psychologists considering it actual being and economists considering it sometimes formal assumption. Our framework aims a unified formal description of individual decision making and so regard preference relation as a construct of judgement in concrete decision making rather than a formal assumption, and wherefore we introduce “desire” and “knowledge” as primitives.

Setting up preference relation on the basis of desire means, for example, that the demand for certain goods is a sequel of the desire for characteristics or attributes the consumer attributes to them. However, it is not a novel idea for economics. Characteristic approach (Lancaster (1971)) considers demand is a sequel of wants for physical characteristics of goods. It criticized traditional consumer theory for its difficulty treating of the effect of product change, model change, or new product launching. Although we appreciate and accede this criticism, we do not accede the methodology of evaluating for characteristics of goods which is by the function on the space of objective characteristics. This is because, firstly, we think that a relation which is expressed by such a function should be derived by “desire” and “knowledge,” structures deemed to be more primitive.

Secondly, characteristics or attributes are not what is embedded in goods objectively but what is necessarily to be read as such by the decision maker. Different characteristics would be found by different agents in the same goods and even by the same agent depending on the situations. Especially in the latter case, assuming that the utility function on the characteristic space is fixed for the same agent, the same criticism by characteristic approach to traditional consumer theory could be applied to characteristic approach itself.

Interpretive Momentum of Cognitive Agents

It seems necessary for decision theory to capture the interpretive momentum by decision maker. Findings in behavioral economics and experimental economics also show this. For example, the principle of description invariance (Arrow (1982)) says that if alternatives of the choice problem are defined as extensionally identical, the outcome of choice behavior is the same. However, existence of phenomena violating this principle is well-known in behavioral economics and its impact is sometimes referred to as “framing effect” (Tversky and Kahneman (1981)). The following “Asian disease” problem in their paper is a famous example.

> Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:
>
> If Program A is adopted, 200 people will be saved.
>
> If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved.
>
> Which of the two programs would you favor?

It is reported that 72% of 152 university students chose Program A. Contrastively, 78% of another group of 155 students chose Program D to the following question.

> If Program C is adopted 400 people will die.
>
> If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die.
>
> Which of the two programs would you favor?

One can easily confirm that Program A and C, Program B and D, and thus the two problems are substantially identical except the descriptions. Nevertheless, the choice pattern between two groups is reversed.

This result states that decision maker provides different evaluation of alternatives depending on their descriptions. Therefore, introducing interpretive momentum of decision maker is essential for explaining these phenomena. In Kahneman and Tversky (1979), in line with prospect theory, a psychological original point "reference point" from which divergence is evaluated as gain or loss plays a crucial role in explaining framing effect. However, prospect theory is generally less-obvious about how reference point is defined, in other words, how the problem is framed. Our framework could explain framing effect, which we will formalize in another article, by modeling interpretive momentum of decision maker.

Then how can we model interpretive momentum of decision maker? Our framework owes its paraphernalia to *channel theory* by two logicians Barwise and Seligman (Barwise and Seligman (1997)). Therefore, reviewing what kind of problem they tried to solve and how they solve them in the context of the theory coming in would outline our standpoint. Broadly speaking, the context of channel theory would be twofold. Formal semantics of natural language and theory of the flow of information (but they are closely related). Let us take a brief look at them.

Formal Semantics of Natural Language

Semantics explains systematically that a certain linguistic expression bears a certain meaning. Since the late 19th century, when modern logic established, various formal semantics of formal artificial language of which vocabulary and grammar are rigorously specified have been defined. Semantics based on logic is called "model-theoretic semantics." In the late 20th century, so-called "possible world semantics" based on modal logic came on the scene and became mainstream.

Ever since Frege, in model-theoretic semantics, truth value of a sentence belonging to a system of formal language is perceived as the meaning of the sentence. By introducing a model, i.e., a tuple of a set of vocabularies of formal language, a domain of object, which is a part of the world (in case of possible world semantics, all the possible worlds including the real world), and a function which maps a subset of the domain of object for each vocabulary, set-theoretic structural calculation defined on vocabularies of which truth conditions are determined yields the truth condition of the sentence. If the truth condition is satisfied, the sentence is said to be true in the model (in case of possible world semantics, to be true in all the possible worlds).[1]

However, unlike formal language, natural language does not provide comprehensively amenable vocabulary and grammar. It is difficult for model-theoretic semantics to construct granular semantics notably in treatment of context- or situation-dependent vocabulary and verb representing propositional attitude such as "believe that ...," "know that ...," or "hope that ...." In other words, it cannot give different meaning to expressions with different perceptual value adequately. This is because, in model-theoretic semantics, it is necessary for the one who sets up a model of the language to define assignment of vocabulary and the part of the world so as to give different truth value to different proposition taking into consideration all the possible worlds. Therefore, if meaning of natural language is contextual- or situation-dependent, it would be almost impossible to define model taking all the possibility into consideration in advance unless someone who is omniscience.

As we have already seen in Asian disease problem, it is propositional attitude that is problematic in economics or decision theory. For instance, whatever possible world we would consider where the meaning of the word such as "save" "die" is the same as this real world, "out of 600 people, 200 will be saved and 400 will die" is the truth condition of Program A and C and their conceptual values becomes to be identical. The difficulty would consists in that there is an aspect that the judgement of cognitive agent follows her own "logic" rather than logical truth value calculation. We will see the same problem below where a new treatment of knowledge in economics or decision theory is presented.

In response to this "granularity problem," Barwise and Perry (1983) suggested "situation semantics" which views meaning of natural language by relations between number of ingredients bringing meaning to the expression. In situation semantics, what gives meaning to expression is *situation* rather than *model*. Meaning is understood as relations between the situation concerning the use of expression (such as when, where, and who utters, syntactical structure, intonation, etc.) and "situation types" as attributes of the message of the expression. The problem of extensionality is solved by these ingredients since different expressions generally correspond to different conceptual values.[2] Technically, situation type is introduced as a set. That is, attributes or relations are introduced as primitives rather than modeled as set-theoretic structures as in model-theoretic semantics. Moreover, in situation semantics, unlike model-theoretic semantics, truth value of sentence is not defined at the same time as the model is defined. Note that situation semantics models only relation between situations and situation types as a possibility of the situations.[3] A situation is the situation with a case-by-case expression and so situation type does not exhaustively describe the situation. That is, description is partial or local at any time and so omniscience is not required.

Channel theory, on which our framework stands, is a successor of both situation semantics and, as stated below, theory of flow of information. As a result, channel theory becomes a theory which can generally deal with phenomena concerning cognition, knowledge and information formally rather than a theory for linguistic analysis which inevitably entails immense complexity. At the same time, channel theory inherits the semantic stance of situation semantics. In decision making, the interpretive momentum is typically about natural language and so it could be said that our framework is methodically legitimate.

As previously noted, attributes transmitted by an expression are introduced as situation types, technically as a set. This is a lineage of Dretske (1981), a mathematical theory of flow of information

[1] Usually "sentence ϕ is true in the model M" is denoted by "$M \vDash \phi$."

[2] In relation to the notation of model-theoretic semantics, "$s \vDash \sigma$" means "a situation type σ holds in a situation s."

[3] As a matter of course, in situation semantics, truth value is assigned to a "proposition" which is a pair of situation and situation type. However, function of truth value differs vastly from that in model-theoretic semantics in which it is deemed to be meaning itself.

focused on content of information.

Theory of Flow of Information

Dretske presented a qualitative theory of information with unique semantics based on information amount and conditional probability in response to the existing information or communication theory pioneered by Shannon (1948), which is principally a quantitative theory of information, especially a theory of efficiency of information route. Introducing contents of information as a set, channel theory did not adopt Dretske's own semantics but formulated that information flows by making use of a mathematical structure, Chu space. In channel theory, that "a being α" carries information that "b being β" is stated as holding homomorphism between two spaces representing "a being α" and "b being β." The content of information such as α, β is said as *type* and the carrier of type such as a, b is said as *token*.

We interpret decision making as making homomorphism between "alternatives in real-world being some sort of type" and "some sort of token in decision maker's mind being decision maker's desire." That is, we consider decision making as an active behavior "making information flow."

A New Formulation of Knowledge

The above-referenced notion that decision making depends not necessarily on logical truth value calculation but decision maker's own "logic" would be intuitive. Although we make judgements according to our past experience and thereupon sometimes mistake, this mis-conviction would be an acceptable error indicating the limit of rationality, rather than mere irrationality, of us since we can only know the world partially. On the other hand, it sometimes becomes clue to update knowledge properly. As we will see details in later section, channel theory introduces "local logic" to capture these aspects and we formulate knowledge as one of the key components of local logic, i.e., "theory." By doing this, we can easily describe the state of affairs such as "holding mis-conviction."

The rest of this article is constructed as follows: In section 2, we introduce a roughcast of channel theory in line with certain fictitious decision making scene and ultimately give a description of constructing preference relation between alternatives. In section 3, we discuss some prospecting possibilities of our approach.

2 Construction of Preference by Channel Theory

In this section, using *channel theory* founded by Barwise and Seligman (1997), we present a brand-new framework that provides a basis for "preference," a prime structure in economics. In section 2.1, we first introduce "desire" and "knowledge" as sequents of a system of sequent calculus. Then we define the term "satisfy desire" by means of "knowledge" and preference relation between types depending on the extent to which the types satisfy the desire.

In section 2.2, we formulate "observation" by a conception of "classification" which is configured by two sets of "type" and "token" and a binary relation between them. Then, we provide a way to find out a token that "satisfy desire" from structure of the classification, and, as in section 2.1, define preference relation between tokens depending on the extent to which the tokens satisfy the desire.

From section 2.3 to 2.5, we introduce some conceptions which are necessary to constitute "representation system" in section 2.6. Concretely, we introduce "local logic" in 2.3, "infomorphism" in 2.4, "channel" in 2.5. In section 2.5, we also define preference relation between tokens on a channel. Then in section 2.6, combining these conceptions, we define "representation system" as a function of cognition of human beings that enables us to find out a token which "satisfy desire" even if the observation is insufficient to express the desire.

Lastly, in section 2.7, introducing a concept of "name," we look at a relationship between preference relations established by then.

2.1 Theories

When we think about human activities "economically," it seems almost impossible to make do without premising "preference relation" as a primitive structure. For instance, by premising some specific form of utility function, we implicitly assumes that the agent has at least a rational and, in many cases, continuous preference relation. Thus, it should be said that preference relation is indispensable for us to see human or social activities economically. Or it even could be said that preference relation enables economic thinking.

At the same time, however, it seems unrealistic in some cases that we always behave, as economics generally premises, according to some "rational preference." For we cannot always construct a preference relation between any and all goods or bundles of goods, or, alternatives. Also, a preference relation once constructed may be reversed in many cases when a situation changes.

Consider a case, for example, a desire "want something sweet" arises when you drink a cup of coffee. If you know "sugar" is "sweet" and "salt" is "not sweet," a preference relation "prefer sugar than salt" would be constructed. However, we would not always prefer "sugar" than "salt." In fact, many people will calls for salt rather than sugar when they drink tequila, and then the preference relation is reversed.

Preference relation between different commodities would often be reversed. It cannot be explained meaningfully as long as we treat preferences as fixed structures like in traditional economics and decision theory. However, by founding preference relation on desire as a more fundamental concept, we can provide a comprehensive view of this phenomenon. We think desire produces preference, that is, the more desires a commodity satisfies, the more it is preferred.

It is obvious that if the desire changes, the preference relation also changes. If we desire "sweetness," we would prefer "sugar" and if we desire "saltiness," we would prefer "salt." In this section, we are going to provide foundation to preference, long been deemed as a prime structure, on desire and in doing so, we also provide theoretical footing on which we can figure out anomalies that have ever been reported concerning preference structure.

Once we found preference relations on desire, a major aspect of decision making which was downplayed by the traditional theories emerges – knowledge. Take "salt" and "sugar" for example. It never be obvious that "sugar" is "sweet" and "salt" is "not sweet." It is the relationship held only by the one who has "knowledge" on their taste. The knowledge is essential to conclde that "sugar" is sweeter than "salt." In fact, we make use of this "knowledge" when we conclude that "sugar" is sweeter than "salt" and would regard the one who concludes like that as a person who has this knowledge.

In many cases, absence of "knowledge" about desire makes it impossible for anyone to satisfy the desire. In fact, in the situation in which you would like to sweeten up coffee, you cannot satisfy the desire by sugar unless you know "sugar is sweet." Thus, to found preference on desire, it is crucial that we take into account the structure of knowledge explicitly. We propose to express "knowledge" in *theory* as stated below.

Definition 1 (Barwise and Seligman (1997))**.** Let Σ be arbitrary set. A binary relation $\vdash$ between subsets of Σ is called a *(Gentzen) consequence relation on* Σ. A *sequent* is a pair $\langle \Gamma, \Delta \rangle$ of subsets of Σ and a sequent is called a *partition* of a set Σ' if $\Gamma \cup \Delta = \Sigma'$ and $\Gamma \cap \Delta = \emptyset$. A *theory* is a pair $T = \langle \Sigma, \vdash_T \rangle$, where $\vdash_T$ is a consequence relation on Σ of theory T. A *constraint* of the theory T is a sequent $\langle \Gamma, \Delta \rangle$ of Σ for which, $\Gamma \vdash_T \Delta$. A sequent $\langle \Gamma, \Delta \rangle$ is *T-consistent* if $\Gamma \nvdash_T \Delta$.

We consider Σ as a set of *types* where *type* means attribute of goods such as sensuous characteristics ("sweet," "salty," etc.) and nominal designation ("sugar," "salt," etc). A constraint $\Gamma \vdash \Delta$ can be interpreted intuitively as a consequence relation that if there are all types in Γ, there is at least one type of Δ. $\vdash \Delta$ represents the claim that there is at least one type in Δ and $\Gamma \vdash$ represents the claim that there is no type in Γ.[4] Let us take a constraint COFFEE, BITTER $\vdash$ TASTE BAD as an example. This constraint can be interpreted as "bitter coffee tastes bad." On the other hand, a constraint COFFEE $\vdash$ BITTER, TASTE BAD expresses "coffee is either bitter or bad taste (or both)." We also interpret $\vdash$ COFFEE as a constraint expresses "there is coffee" and SWEET $\vdash$ as "there is no sweet stuff."

Suppose a knowledge $K = \langle \Sigma, \vdash_K \rangle$ on $\Sigma = \{$ COFFEE, BITTER, TASTE BAD $\}$ has COFFEE $\vdash_K$ BITTER and BITTER $\vdash_K$ TASTE BAD as constraints. Then, from two constraints "coffee is bitter" and "bitter stuff tastes bad," it seems natural that the constraint "coffee tastes bad" is also contained by K. Below we introduce syntax rules which enable these natural constraints that should be drawn from given constraints or should be contained in the first place to be deducted. A theory satisfying the following syntax rules is called a *regular theory.*

Definition 2 (Barwise and Seligman (1997))**.** A *theory* $\mathrm{T} = \langle \Sigma, \vdash_T \rangle$ is *regular* if it satisfies the following for all types and all sets $\Gamma, \Gamma', \Delta, \Delta', \Sigma', \Sigma_0, \Sigma_1$ of types:

Identity: $\alpha \vdash_T \alpha$.

Weakening: If $\Gamma \vdash_T \Delta$, then $\Gamma, \Gamma'' \vdash_T \Delta, \Delta'$[5].

Global Cut: If $\Gamma, \Sigma_0 \vdash_T \Delta, \Sigma_1$ for each partition $\langle \Sigma_0, \Sigma_1 \rangle$ of Σ', then $\Gamma \vdash_T \Delta$.

Furthermore, if arbitrary theory T is given, we define *regular closure* of theory T, the theory which contains all the constraints derived from the constraints of theory T by regular operations above.

Definition 3 (Barwise and Seligman (1997))**.** Let arbitrary theory $T = \langle \Sigma \vdash_T \rangle$ be given. A theory whose constraints are comprised of the constraints of theory T and all the constraints derived from the constraints of theory T is called a *regular closure* of theory T and is denoted as $\bar{T} = \langle \Sigma, \vdash_{\bar{T}} \rangle$.

Let us consider the above knowledge K, which contains two constraints such as "coffee is bitter" and "bitter stuff tastes bad." The regular closure of knowledge K, that is, knowledge $\bar{K}$ contains "coffee is coffee" (COFFEE $\vdash_{\bar{K}}$ COFFEE), a trivial constraint, as well as "coffee tastes bad" (COFFEE $\vdash_{\bar{K}}$ TASTE BAD). The following proof figure shows that the latter constraint is included in knowledge $\bar{K}$.

$$\dfrac{\dfrac{\text{COFFEE} \vdash_{\bar{K}} \text{BITTER}}{\text{COFFEE} \vdash_{\bar{K}} \text{BITTER, TASTE BAD}}\ (\text{Weakening}) \qquad \dfrac{\text{BITTER} \vdash_{\bar{K}} \text{TASTE BAD}}{\text{COFFEE, BITTER} \vdash_{\bar{K}} \text{TASTE BAD}}\ (\text{Weakening})}{\text{COFFEE} \vdash_{\bar{K}} \text{TASTE BAD}}\ (\text{Global Cut})$$

[4]Note, however, we diverted here these "intuitive meanings" of theory from interpretation of the relationship between sequent and "token" (Definition 12) for descriptive purposes. Theory itself is just a syntax rule and so does not give each sequents these meanings. However, we diverted the "intuitive meaning" of constraint since in a sound local logic (Definition 15), all constraints satisfies the relationship which will be defined in Definition 12 with tokens.

Generally, we cannot equate these "intuitive meanings" with constraints since a local logic does not necessarily satisfy soundness. It is important to be aware of the difference of them in the sense that it drives "learning" which is a dynamic property of knowledge. We will argue this in section 2.3.

[5]A constraint $\Gamma, \Gamma'' \vdash_T \Delta, \Delta'$ should principally be written as $\Gamma \cup \Gamma'' \vdash_T \Delta \cup \Delta'$. $\alpha \vdash_T \alpha$ should be also written as $\{\alpha\} \vdash_T \{\alpha\}$. However, for notational convenience, we simplify the expression of constraint according to the custom. Hereafter, we will describe $\{\alpha, \beta\} \vdash \{\gamma\}$ as $\alpha, \beta \vdash \gamma$ and $\Gamma \cup \Gamma' \vdash \Delta \cup \{\alpha\}$ as$\Gamma, \Gamma' \vdash \Delta, \alpha$,too.

As above, a regular theory is a system that enables to derive satisfiable constraints from some given constraints by regular syntax rules. We see knowledges and desires are divided in two, primary ones and their regular closures, in other words, divided between knowledges or desires that are not necessarily regular and the constraints which are drivable from them by regular operations. Speaking of the example of the knowledge K, two constraints "coffee is bitter" and "bitter stuff tastes bad" are primary constraints of the knowledge K whereas "coffee tastes bad" is a consequential constraint derived from "coffee is bitter" and "bitter stuff tastes bad" by means of regular operations. While the first two of them would correspond to natural constraints of knowledge, the last one is not directly perceived as a constraint of knowledge but proved to be true by speculation.

Next we formulate desire as a theory in the same way as knowledge. Here, we define desire on a subset of types on which knowledge is defined since we think it is impossible for the individual to desire types which is not contained in her system of knowledge. Indeed, desiring something presupposes knowing the desire. Desiring unknown desire would be merely impossible.

Definition 4. A theory defined on a subset Σ_D of a set Σ_K of knowledge is said to be a *theory of desire* $D = \langle \Sigma_D, \vdash_D \rangle$.

We interpret desire as the desire for constraints of the theory. For example, a desire $\vdash_D$ SWEET means "want that there is always sweet stuff" i.e., "want sweet stuff" and BITTER $\vdash_D$ means "want that there is no bitter stuff" i.e., "not want bitter stuff." A constraint such as SWEET $\vdash_D$ BITTER can be understood as "not want that there is no sweet stuff but want that there is bitter stuff," i.e., "want sweet stuff if it is bitter."

We can define an ordering both on desire and knowledge according to the following partial order on regular theories. These orderings correspond to intensity of desire and amount of knowledge respectively.

Definition 5 (Barwise and Seligman (1997))**.** Let Σ be fixed. A natural partial order on regular theories on Σ is defined by $T_1 \sqsubseteq T_2$ if and only if each constraint of $\vdash_{T_1}$ is also a constraint of $\vdash_{T_2}$. If $T_1 \sqsubseteq T_2$ holds, we say that T_1 *is a weaker theory than* T_2 or T_2 *is a stronger theory than* T_1.

Suppose that on a set $\Sigma = \{$ COFFEE, ORANGE JUICE, SWEET, BITTER $\}$ of types, a knowledge $K = \{$ COFFEE $\vdash_K$ BITTER, ORANGE JUICE $\vdash_K$ SWEET $\}$ are provided.[6] Consider two distinct desires $D = \{ \vdash_D$ SWEET, $\vdash_D$ COFFEE $\}$ and $D_1 = \{ \vdash_{D_1}$ SWEET $\}$. It holds that $\bar{D_1} \sqsubseteq \bar{D}$ between these two regular closures of desire. That is, a desire $\bar{D}$ "want sweet coffee" is stronger than $\bar{D_1}$ "want sweet stuff" since a constraint "want coffee" is added.[7]

How about knowledge? Take a knowledge $K_1 = \{$ COFFEE $\vdash_{K_1}$ BITTER, ORANGE JUICE $\vdash_{K_1}$ SWEET, COFFEE $\vdash_{K_1}$ SWEET $\}$ as an example. It holds that $\bar{K} \sqsubseteq \bar{K_1}$ between regular closures of the knowledge K and K_1. That is, the knowledge $\bar{K_1}$ is a stronger knowledge than $\bar{K}$ since a constraint "coffee is sweet" is added.[8]

[6]The knowledge $K = \langle \Sigma_K, \vdash_D \rangle$ is comprised of a set Σ_K of types and a binary relation defined on the power set of Σ_K. Therefore, rigorous set-theoretic notation requires that the constraints included in K must be written as

$$\vdash_K = \{\langle\{\text{COFFEE}\}, \{\text{BITTER}\}\rangle, \langle\{\text{ORANGE JUICE}\}, \{\text{SWEET}\}\rangle\}.$$

However, it seems too detailed and elusive for us. So we use informal notation as in body text in describing constraints of theory hereafter.

[7]Generally, taking into account of regular structure of desire would be less necessary than knowledge. However, we sometimes contemplate the structure of desire and judge which desire is more strict. In cases for compromising, we must give up some desire and comprehend our structure of desire systematically in order to compromise on desire. Indeed, we must judge intensity of desire when we think rank of desire as in Definition 6.

[8]Most of people may think the knowledge "coffee is sweet" is invalid. However, we accept such a knowledge as far as the agents in our analysis regard it as knowledge. In fact, it is possible that an individual who has been drunk only sweet coffee believes coffee is sweet. All the knowledges we refer in this article are objectively-unfounded systems that are deemed to be knowledge by being deemed to be knowledge.

Moreover, we define a family consists of desires with different intensities and call it a *family of desires*. This concept captures magnitude of desire.

Definition 6. For any desire D, $\mathcal{F}(D) = \{D, D_1, \ldots, D_n\}$, $D_i \sqsubseteq D$ for all $i = 1, \ldots, n$ is called a *family of desires of* D.[9] In the case of $\mathcal{F}(D) = \{D\}$, it is simply denoted by D.

Let us continue the example of coffee and orange juice. Suppose that the desire $D = \{ \vdash_D \text{SWEET}, \vdash_D \text{COFFEE} \}$, that is, "want sweet coffee" arises but it cannot be satisfied. Then what will we think? In many cases we will attempt to content ourselves with weaker desire just like "sweet coffee is the best, the next best thing is a sweet stuff."

A family of desires of D describes situations like this. It shows step-by-step which constraint is too much compromise if the desire D cannot be satisfied. For instance, a family of desires $\mathcal{F}(D) = \{D, D_1\}$ can be constructed from the desire D. This family of desires expresses the following steps that firstly "want sweet coffee," but if it would not be satisfied, secondly "want sweet stuff."

According to this family of desires, we can derive preference relation on types, which is familiar to economists, from the theory of desire. To see this, we introduce a notion of relationship between types, knowledge, and desire.

Definition 7. Let a regular knowledge $K = \langle \Sigma_K, \vdash_K \rangle$, and a desire $D = \langle \Sigma_D, \vdash_D \rangle$ be given. Then, for any constraint $\Gamma \vdash_D \Delta$ in the desire D and for any type $\alpha \in \Sigma_K$, we say that α *satisfies the desire* D *under the knowledge* K if a constraint $\alpha, \Gamma \vdash_K \Delta$ is contained by the knowledge K.[10] Moreover, for a family of desires $\mathcal{F}(D)$ of the desire D and $\alpha \in \Sigma_K$, a family of desires defined as follows is called a *family of desires* $\mathcal{F}_{\alpha,K}(D)$ *satisfied by* α *under the knowledge* K.

$$\mathcal{F}_{\alpha,K}(D) = \{D_i \in \mathcal{F}(D) \mid D_i \text{ is satisfied by } \alpha \text{ under } K\}$$

Naturally, holds that $\mathcal{F}_{\alpha,K}(D) \subseteq \mathcal{F}(D)$.

Making use of $\mathcal{F}_{\alpha,K}(D)$, a preference relation between types is defined as follows.

Definition 8. Let a regular knowledge $K = \langle \Sigma_K, \vdash_K \rangle$, a desire $D = \langle \Sigma_D, \vdash_D \rangle$, and a family of desires $\mathcal{F}(D)$ of the desire D be given. Then, a preference relation $\precsim_{\langle \mathcal{F}(D), K \rangle}$ on Σ_K is defined for $\alpha, \beta \in \Sigma$ by

$$\beta \precsim_{\langle \mathcal{F}(D), K \rangle} \alpha \qquad \text{iff} \qquad \mathcal{F}_{\beta,K}(D) \subseteq \mathcal{F}_{\alpha,K}(D).$$

If a preference relation $\precsim$ on a set S is defined, we also define $\sim$ and $\prec$ on the same set S as follows.

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[9] Note that we rank the intensity of desire by the regular closure of each desire. As mentioned previously, it is less necessary to take into account regular structure of desire than knowledge. However, in the case of abandoning some desire, it is necessary for us to take into account regular structure of our desire and to judge intensity of it. For example, suppose a desire $D = \{\vdash_D \text{COFFEE}\}$ which expresses "want coffee" is not satisfied. Then we may abandon the desire D and make compromise with another desire $D' = \{ \vdash_{D'} \text{COFFEE}, \text{TEA} \}$ which expresses "want coffee or tea." Since there is no inclusive relation between theories D and D', they looks unrelated. However, considering the intuitive meaning of them, it seems natural to judge that D' is a weaker desire than D but we cannot judge which one is stronger unless taking regular closure of them. Indeed, we can judge strong-weak relation between desires if we consider the intuitive meaning, and that is what we do in everyday life. It could be said that, in doing so, we unconsciously compare their regular structures.

[10] "α satisfies the desire D under the knowledge K" means "if there is α it turns out that the desire D is satisfied by the knowledge K." Indeed, adding a constraint $\vdash_K \alpha$ to K, we obtain all constraints of desire D as the following proof figure depicts.

$$\dfrac{\dfrac{\vdash_K \alpha}{\Gamma \vdash_K \alpha, \Delta}\ (\text{Weakening}) \qquad \alpha, \Gamma \vdash_K \Delta}{\Gamma \vdash_K \Delta}\ (\text{Global Cut})$$

Definition 9. Let $\precsim$ be a preference relation on S. Then, for any $a, b \in S$, the preference relation $\sim$ is defined by

$$a \sim b \quad \text{iff} \quad a \precsim b \quad \text{and} \quad b \precsim a$$

and $\prec$ is defined by

$$a \prec b \quad \text{iff} \quad a \precsim b \quad \text{and} \quad b \not\precsim a.$$

Let us construct a preference relation $\precsim_{\langle \mathcal{F}(D), \bar{K} \rangle}$ between coffee and orange juice according to the family of desires $\mathcal{F}(D) = \{D, D_1\}$. Consider the desire $D_1 = \{ \vdash_D \text{SWEET} \}$ first. In this case, while $\text{ORANGE JUICE} \vdash_{\bar{K}} \text{SWEET}$ holds under the knowledge $\bar{K}$, $\text{COFFEE} \nvdash_{\bar{K}} \text{SWEET}$ also holds. Therefore "coffee $\precsim_{\langle \mathcal{F}(D), \bar{K} \rangle}$ orange juice" holds under the knowledge $\bar{K}$ and the family of desires $\mathcal{F}(D)$.

Since preference relation is constructed subject to desire, it varies by definition with changes in desire. Let us consider the same problem, where the desire changes as follows. Let $D' = \{ \vdash_{D'} \text{BITTER}, \vdash_{D'} \text{COFFEE} \}$ and $\mathcal{F}(D') = D'$ as the family of desires. Then while both $\text{COFFEE} \vdash_{\bar{K}} \text{BITTER}$ and $\text{COFFEE} \vdash_{\bar{K}} \text{COFFEE}$ hold under the knowledge $\bar{K}$, both $\text{ORANGE JUICE} \nvdash_{\bar{K}} \text{BITTER}$ and $\text{ORANGE JUICE} \nvdash_{\bar{K}} \text{COFFEE}$ hold. Therefore the preference relation "orange juice $\precsim_{\langle \mathcal{F}(D'), \bar{K} \rangle}$ coffee," opposite to the former case, holds under the desire D'.

Since preference relation is constructed subject not only to desire but also to knowledge, it also varies by definition with changes in knowledge. Next, we see an instance that preference relation is reversed by structure of knowledge. Let us call into account the the same problem under the family of desires $\mathcal{F}(D) = \{D, D_1\}$ and the knowledge $\bar{K_1}$. Consider the desire $D_1 = \{ \vdash_{D_1} \text{SWEET} \}$ first. This time the sequent $\langle \text{COFFEE}, \text{SWEET} \rangle$ which was $\bar{K}$-*consistent*, that is, $\text{COFFEE} \nvdash_{\bar{K}} \text{SWEET}$ under the knowledge $\bar{K}$ is included in the knowledge $\bar{K_1}$ as a constraint, that is, $\text{COFFEE} \vdash_{\bar{K_1}} \text{SWEET}$, and hence the decision maker cannot discriminate between coffee and orange juice, i.e., they are indifferent. Next consider the desire $D = \{ \vdash_D \text{SWEET}, \vdash_D \text{COFFEE} \}$. In this case, though $\text{COFFEE} \vdash_{\bar{K_1}} \text{COFFEE}$ holds under the knowledge $\bar{K_1}$, $\text{ORANGE JUICE} \nvdash_{\bar{K_1}} \text{COFFEE}$ also holds. Therefore, conversely to the first example, "orange juice $\precsim_{\langle \mathcal{F}(D'), \bar{K_1} \rangle}$ coffee" holds under the knowledge $\bar{K_1}$, too.

Below we see some properties of preference relation constructed as above.

Proposition 1. $\left(\Sigma_K, \precsim_{\langle \mathcal{F}(D), K \rangle}\right)$ *is a preordered set, i.e., a set satisfying reflexivity and transitivity.*

proof. For any desire $D_i \in \mathcal{F}(D)$ and for any $\alpha \in \Sigma_K$, $\mathcal{F}_{\alpha,K}(D_i)$ is uniquely determined and so $\alpha \precsim_{\langle \mathcal{F}(D), K \rangle} \alpha$ holds. Therefore $\precsim_{\langle \mathcal{F}(D), K \rangle}$ satisfies reflexivity.

Next, for all $\alpha, \beta, \gamma \in \Sigma$, suppose both $\alpha \precsim_{\langle \mathcal{F}(D), K \rangle} \beta$ and $\beta \precsim_{\langle \mathcal{F}(D), K \rangle} \gamma$ hold. Since $\alpha \precsim_{\langle \mathcal{F}(D), K \rangle} \beta$, $\mathcal{F}_{\alpha,K}(D_i) \subseteq \mathcal{F}_{\beta,K}(D_i)$ holds. Similarly, since $\beta \precsim_{\langle \mathcal{F}(D), K \rangle} \gamma$, $\mathcal{F}_{\beta,K}(D_i) \subseteq \mathcal{F}_{\gamma,K}(D_i)$ holds. Accordingly, $\mathcal{F}_{\alpha,K}(D_i) \subseteq \mathcal{F}_{\gamma,K}(D_i)$ holds and this yields $\alpha \precsim_{\langle \mathcal{F}(D), K \rangle} \gamma$. Thus $\precsim_{\langle \mathcal{F}(D), K \rangle}$ satisfies transitivity.

As a result, $\precsim_{\langle \mathcal{F}(D), K \rangle}$ makes a preordered set on Σ. □

Generally, it is not always possible to construct a preference relation $\precsim_{\langle \mathcal{F}(D), K \rangle}$ between arbitrary two elements of Σ_K. Nevertheless, "'abandoning" some desires enables it.

Proposition 2. *Let a knowledge K and a desire D be given. Suppose it is not possible to construct any preference relation $\precsim_{\langle \mathcal{F}(D), K \rangle}$ between $\alpha, \beta \in \Sigma_K$ under a family of desires $\mathcal{F}(D)$. However, it becomes possible to construct preference relation between α and β under some family of desires $\mathcal{F}'(D)$ made by eliminating some desires from $\mathcal{F}(D)$. The desires eliminated, i.e., $\mathcal{F}(D) \setminus \mathcal{F}'(D)$ is said to be* abandoned desires.

proof. Constitute $\mathcal{F}'(D)$ in the following manner.

$$\mathcal{F}'(D) = \mathcal{F}_{\alpha,K}(D) \cup (\mathcal{F}(D) \setminus \mathcal{F}_{\beta,K}(D))$$

Then by the definition of $\mathcal{F}'(D)$, despite the fact that α satisfies all desires of $\mathcal{F}'(D)$, there exists desires which β cannot satisfy. Thus $\beta \precsim_{\langle \mathcal{F}'(D),K \rangle} \alpha$ holds. $\square$

A simple example follows. Consider constructing a preference relation $\precsim_{\langle \mathcal{F}_2(D),\bar{K} \rangle}$ between coffee and orange juice under the knowledge $\bar{K}$ and the family of desires $\mathcal{F}_2(D) = \{D, D_1, D_2\}$ which consists of $D = \{ \vdash_D \text{SWEET}, \vdash_D \text{COFFEE} \}$, $D_1 = \{ \vdash_{D_1} \text{SWEET} \}$, and $D_2 = \{ \vdash_{D_2} \text{COFFEE} \}$. While coffee satisfies the desire D_2, orange juice does not. Contrastively, while orange juice satisfies the desire D_1, coffee does not. Furthermore, both coffee and orange juice do not satisfy the desire D. Therefore it is not possible to construct any preference relations between coffee and orange juice by $\precsim_{\langle \mathcal{F}_2(D),\bar{K} \rangle}$.

Here we take into account another family of desires $\mathcal{F}'_2(D) = \{D, D_2\}$. Then, though coffee satisfies the desire D_2, orange juice does not. Contrastively, in $\mathcal{F}'_2(D)$ there is no desire satisfied only by orange juice. Thus "orange juice $\precsim_{\langle \mathcal{F}'_2(D),\bar{K} \rangle}$ coffee" holds. In summary, $\mathcal{F}'_2(D)$ makes possible comparison of coffee with orange juice by "abandoning" the desire D_1 which was contained by $\mathcal{F}_2(D)$.

Lastly we bring up a condition for families of desires to construct a rational preference, i.e., a preference satisfying completeness and transitivity. In general, if it is possible to form monotone orderings between desires of a family of desires, a preference relation constructed under the family of desires is rational.

Proposition 3. *Suppose a family of desires $\mathcal{F}(D)$ of a desire D such that for any two elements of $D_i, D_j \in \mathcal{F}(D)$, either $\bar{D}_i \sqsubseteq \bar{D}_j$ or $\bar{D}_j \sqsubseteq \bar{D}_i$ holds. Then a preference relation constructed according to this family of desires $\mathcal{F}(D)$ is rational.*

proof. By Proposition 1, it is obvious that $\left(\Sigma_K, \precsim_{\langle \mathcal{F}(D),K \rangle}\right)$ satisfies transitivity. So we go checking completeness. Suppose that for all $\alpha, \beta \in \Sigma_K$, neither $\alpha \precsim_{\langle \mathcal{F}(D),K \rangle} \beta$ nor $\beta \precsim_{\langle \mathcal{F}(D),K \rangle} \alpha$ holds. Then there exists a desire $D_\alpha \in \mathcal{F}(D)$ satisfied by α and a desire $D_\beta \in \mathcal{F}(D)$ satisfied by β and furthermore neither $\bar{D}_\alpha \sqsubseteq \bar{D}_\beta$ nor $\bar{D}_\beta \sqsubseteq \bar{D}_\alpha$ holds. This contradicts the supposition of $\mathcal{F}(D)$. $\square$

2.2 Classifications

Theories of desire and knowledge by which preferences are constructed, however, are not sufficient to describe satisfying desire. Since, to describe satisfying desire sufficienty, it is necessary to describe how an individual who wants to satisfy her desire appropriately distiguishes the stuff she wants.

It is not true that we can always make out the stuff we want. To see this, let us return to the example of coffee. Imagine that you are in an unfamiliar room with a cup of coffee in your hand and wanting a few spoons of sugar. You find that there are two transparent containers on the table filled with white powder, one is square-shaped and the other is circular-shaped. The inhabitants of this room customarily put salt into a square container and sugar into a circular one, but you, a stranger to the room, don't know that custom. And now, you cannot get sugar, since you don't know which one is sugar.

Of course it is easy to distinguish sugar if you lick powder of each container, as you know the taste of sugar. However, can you say for sure that it is sugar only because it is sweet? Or, primarily, can you really assert that these white powders are sugar or salt? These questions would never spring up if we can perceive sugar itself intuitively and directly. But we can only judge what is at hand as "sugar" by our own "knowledge" indirectly and imperfectly each time, e.g., "what is white and sweet is sugar," "crystal with molecular formula $C_{12}H_{22}O_{11}$ is sugar," and "what is in the bottle labelled 'sugar' is sugar," etc. Therefore, we cannot detect the thing we want unless we have enough knowledge about the characteristics of the matter in front of us to judge them.

The particular in front of us, staying on as it is irrespective of one's detection. We call it *token.* It is not always detected as "sugar." It is a particular which may be detected as merely "white powder," "salt," or common "white sand." Contrastively, we say attributes of a token such as name and property as *type.* A phrase "a token is deemed to be 'sugar' " is said as "a token is classified by the type SUGAR" or "a token is of type SUGAR." And a mathematical structure called *Classification* describes relationship between *token* and *type.*

Definition 10 (Barwise and Seligman (1997))**.** A *classification* $A = \langle tok(A), typ(A), \vDash_A \rangle$ consists of

1. a set, $tok(A)$, of objects to be classified, called the *tokens of A*
2. a set, $typ(A)$, of objects used to classify the tokens, called the *types of A*, and
3. a binary relation, $\vDash_A$, between $tok(A)$ and $typ(A)$.

If $a \vDash_A \alpha$, then a is said to be *of type α in A.*

Imagine that an individual observes a token, a concrete particular, and obtains some fixed set of attributes of it. We interpret and express her observation as a classification.

Incidentally, finite classifications can be conveniently represented by a *classification table.* It is a table of types along the top side and tokens along the left side, of which fields correspond to a pair of a token and its classified type are filled with 1s and otherwise with 0s. Let us consider the above example of sugar and salt by a situation that two people observe the same tokens. The one is Mr.Naiv, a little bit naive, but plain common-sense man, the other is Mr.Chem, a chemistry teacher at a junior high school. In Naiv's observation, referred as O_n below, these two white powders are classified as Table 1.

$\vDash_{O_n}$	WHI	PWD	SQU	CIR
token1	1	1	1	0
token2	1	1	0	1

Table 1: classification table of Naiv's observation O_n

Let us take a detailed look. Token1 is of types "white (denoted as 'WHI' in the table, and similarly below)", "powder (PWD)", and "contained in the square container (SQU)." This means that token1 is observed by Naiv as a particular with these three attributes. In another words, one can interpret this classification table as meaning that Naiv observed token1 as "white powder contained in a square container." In contrast, token2 is observed as "white powder contained in a circular container." Thus we find that Naiv distinguished one token from another one only by the container they are in.

$\vDash_{O_c}$	WHI	PWD	SQU	CIR	130°C	800°C	HEX
token1	1	1	1	0	0	1	1
token2	1	1	0	1	1	0	0

Table 2: classification table of Chem's observation O_c

On the other hand, classification O_c, which is generated from Chem's observation, shows that he was able to judge two tokens by melting point (130°C and 800°C in the table) and crystal structure (regular hexahedral, HEX) besides Naiv's types. So we can see that token1 is observed by Chem as "white powder contained in the squared container, of which melting point is 800°C and crystal structure is regular hexahedral." In contrast, token2 is observed as "white powder contained in the circular container, of which melting point is 130°C." Thus we find that Chem can discriminate two tokens not only by their container but by the temperature they melt and the difference of crystal shape.

As we see above, types which constitute an observation vary in many ways. Various types, e.g., melting point, crystal structure, boiling point, etc. can be found out depending on the views of observers. Classification enables us to describe such difference of perception as different expression of classification.

Then, how do we choose particulars? How do we connect our desires to concrete particulars? To see how preference relations are constructed between particulars, here we consider a specific classification generated by desire. The following definition shows how to construct a classification from a given theory.

Definition 11 (Barwise and Seligman (1997))**.** Let a theory $T = \langle \Sigma, \vdash \rangle$ be given. Then a classification satisfying below is called the *classification generated by* T and is written $\text{Cla}^*(T)$.[11]

(a) tokens are all partitions $\langle \Gamma, \Delta \rangle$ of $\text{typ}(T)$,

(b) types are the types of T, such that

(c) $\langle \Gamma, \Delta \rangle \vDash_{\text{Cla}^*(T)} \alpha$ if and only if $\alpha \in \Gamma$ (equivalently, if and only if $\alpha \notin \Delta$).

Particulars in $\text{Cla}^*(D)$ are inextricably linked with the desire in the sense that they directly reflects the desire. But, at the same time, since they have no ex-ante relationship to existing particulars, they may have unrealistic types. Now consider the case of a desire $D = \{$ SALTY $\vdash_D$, $\vdash_D$ SWEET, $\vdash_D$ COFFEE $\}$. In this case, the classification table of $\text{Cla}^*(D)$ is as table 3.

$\vDash_{\text{Cla}^*(D)}$	SALTY	SWEET	COFFEE
#1 :$\langle\{$SALTY, SWEET, COFFEE$\}, \emptyset\rangle$	1	1	1
#2 :$\langle\{$SALTY, SWEET$\}, \{$COFFEE$\}\rangle$	1	1	0
#3 :$\langle\{$SALTY, COFFEE$\}, \{$SWEET$\}\rangle$	1	0	1
#4 :$\langle\{$SALTY$\}, \{$SWEET, COFFEE$\}\rangle$	1	0	0
#5 :$\langle\{$SWEET, COFFEE$\}, \{$SALTY$\}\rangle$	0	1	1
#6 :$\langle\{$SWEET$\}, \{$SALTY, COFFEE$\}\rangle$	0	1	0
#7 :$\langle\{$COFFEE$\}, \{$SALTY, SWEET$\}\rangle$	0	0	1
#8 :$\langle\emptyset, \{$SALTY, SWEET, COFFEE$\}\rangle$	0	0	0

Table 3: classification $\text{Cla}^*(D)$ made from the desire D

For example, #3 expresses a particular which is "not sweet but salty coffee" and #5 expresses "sweet but not salty coffee" contrary to #3. One would easily imagine a stuff corresponding to #5 because if we put sugar in coffee, we can obtain "sweet but not salty coffee." On the other hand, it might be difficult to imagine #3 because we usually don't drink "salty coffee." It is true that if we put salt in coffee, we obtain coffee with salt. But there is no guarantee that it comes to be "salty coffee," for it may happen that it tuens out to be a yucky drink which we can identify as neither "salty" nor "coffee." However, we can imagine a particular which is "salty" and at the same time "coffee" despite whether it really exists or not. Thus $\text{Cla}^*(D)$ is a classification generated by

[11] In Barwise and Seligman (1997), a classification generated by theory T is introduced as a classification with consistent partitions of $\text{typ}(T)$ as its tokens. In this article, however, we consider tokens satisfying some desire partially or satisfying none of desires as well as desire-consistent tokens and so we extend range of tokens to all partitions. Moreover, since it is not necessary to take consistent partitions, there is no need for the theory to be regular. That is why we use different notation $\text{Cla}^*(T)$ with Barwise and Seligman (1997).

exhaustive enumeration of all possible particulars which are made from given desire regardless of its physical existence.

Next, we see the relationship between classification and desire.

Definition 12 (Barwise and Seligman (1997))**.** Given a classification A, a token $a \in \text{tok}(A)$ *satisfies* a sequent $\langle \Gamma, \Delta \rangle$ of $\text{typ}(A)$ provided that if a is of every type in Γ, then it is of some type in Δ. A token not satisfying a sequent is called a *counterexample* to the sequent. A token $a \in \text{tok}(A)$ is said to satisfy a theory T, if the token a satisfies all constraints of the theory T.

Table 4 shows which of constraints of the desire D are satisfied by each of the tokens of $\text{Cla}^*(D)$.

$\vdash_D$	SALTY $\vdash$	$\vdash$ SWEET	$\vdash$ COFFEE
#1	0	1	1
#2	0	1	0
#3	0	0	1
#4	0	0	0
#5	1	1	1
#6	1	1	0
#7	1	0	1
#8	1	0	0

Table 4: constraints of the desire D satisfied by $\text{tok}(\text{Cla}^*(D))$

For example, #3 satisfies only the constraint "coffee" and neither "not salty thing" nor "sweet thing." That is, it is a counterexample to the constraint "not salty but sweet thing." On the other hand, #5 satisfies all constraints "not salty but sweet coffee," that is, it is not a counterexample to any of these constraints. Moreover, one can see that only #5 satisfies all of them. Thus only #5 satisfies the desire D.

In the meantime, the name of token #3 "$\langle\{\text{SALTY}, \text{COFFEE}\}, \{\text{SWEET}\}\rangle$" is a partition of Σ_K. Interpreting this partition as a constraint, we can see it as a constraint "SALTY, COFFEE $\vdash$ SWEET," in another words, "want salty coffee if it is sweet." This is satisfied in many cases but not satisfied only if there is "not sweet, salty coffee," and the particular #3 brings about this one and only one situation.

Each of the desires expressed by partition $\langle \Gamma, \Delta \rangle$ has a token classified by all types of Γ as a counterexample. If a desire has a counterexample, this desire is said to be *realized*. If a desire is satisfied at all times by any particulars, it does not function as a factor which construct meaningful preference relation between tokens. A desire forms meaningful distinction between what is desired and what is not desired only if a counterexample exists. Take a look at #3. Each particular other than #3 satisfies the desire "want salty coffee if it is sweet" while #3 does not. Therefore, this desire functions to construct the relation $\#3 \precsim \#i$ $(i \neq 3)$ between the particular #3 and everything else only if #3 exists.

These particulars made from desire in this way have very convenient property that makes it possible to define preference relations between these particulars easily. Here we first give a definition of the preference relation between tokens of a generalized classification A according to a desire D and a family of desires $\mathcal{F}(D)$ and then confirm this property. In advance of this, we define relationship between token and family of desires.

Definition 13. Let a classification A and a family of desires $\mathcal{F}(D)$ of desire D be given. Then, a family of desires defined below with respect to a token $a \in \text{tok}(A)$ is called a *family of desires* $\mathcal{F}_{a,A}(D)$ *satisfied by* a *under* A.

$$\mathcal{F}_{a,A}(D) = \{D_i \in \mathcal{F}(D) |\ D_i \text{ satisfied by } a \text{ under } A\}$$

Here, it naturally holds that $\mathcal{F}_{a,A}(D) \subseteq \mathcal{F}(D)$.

Making use of $\mathcal{F}_{a,A}(D)$, a preference relation between tokens is defined as follows.

Definition 14. Let a classification A, a desire D, and a family of desires $\mathcal{F}(D)$ of desire D be given. Then, if $\mathrm{typ}(D) \subseteq \mathrm{typ}(A)$ holds, a preference relation $\precsim_{\langle \mathcal{F}(D), A\rangle}$ on tokens of A is defined for $a, b \in \mathrm{tok}(A)$ by

$$b \precsim_{\langle \mathcal{F}(D), A\rangle} a \qquad \text{iff} \qquad \mathcal{F}_{b,A}(D) \subseteq \mathcal{F}_{a,A}(D).$$

We say that A *cannot express desire* D if $\mathrm{typ}(D) \nsubseteq \mathrm{typ}(A)$.

In order to check the preference relation on the tokens of $\mathrm{Cla}^*(D)$ by families of desires of D, let us see three cases. First, take the power set of $D = \{ \text{SALTY} \vdash_D, \vdash_D \text{SWEET}, \vdash_D \text{COFFEE} \}$ as a family of desires, that is, $\mathcal{F}(D) = \mathrm{Pow}(D)$. Then we obtain as a preference relation on the tokens of $\mathrm{Cla}^*(D)$ the ordering depicted as Figure 1, a kind of figure called "Hasse diagram."

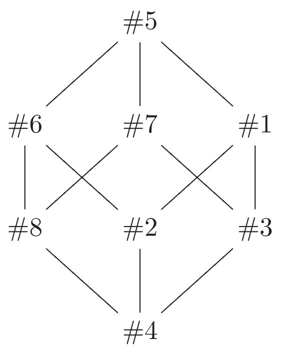


Figure 1: Hasse diagram of $(\mathrm{tok}(\mathrm{Cla}^*(D)), \precsim_{\langle \mathcal{F}(D), A\rangle})$

Let us check the preference relation between #5 and #2. The constraint #2 satisfies is only $\{ \vdash_D \text{SWEET} \}$. Contrastingly, #5 satisfies all constraints of the desire D. Therefore, though #5 satisfies all the constraints which #2 satisfies, #2 cannot satisfy most of the constraints which #5 satisfies. Thus $\#2 \precsim_{\langle \mathcal{F}(D), \mathrm{Cla}^*(D)\rangle} \#5$ holds.

Next we compare #6 and #1. Whereas #6 satisfies two constraints $\{ \text{SALTY} \vdash_D, \vdash_D \text{SWEET} \}$, #1 also satisfies two constraints $\{ \vdash_D \text{SWEET}, \vdash_D \text{COFFEE} \}$. However, #1 cannot satisfy the desire "want not salty thing and sweet thing" which #6 satisfies and #6 cannot satisfy "want sweet coffee" which #1 satisfies. Therefore, we cannot construct any preference relation between #6 and #1. In the Hasse diagram, #1 and #6 are not vertically collinear. This means that it is not possible to construct preference relation between them.

The preference relation on tokens generally forms preorder on $\mathrm{tok}(A)$ as we see it is illustrated by Hasse diagram.

Proposition 4. $\big(tok(A), \precsim_{\langle \mathcal{F}(D), A\rangle}\big)$ *is a preordered set.*

proof. It can be proved as Proposition 1. □

Next, as a family of desires of D, we take $\mathcal{F}'(D) = \{D, D_1, D_2, D_3, D_4\}$ which consists of $D_1 = \{ \vdash_{D_1} \text{SWEET}, \vdash_{D_1} \text{COFFEE} \}$, $D_2 = \{ \text{SALTY} \vdash_{D_2}, \vdash_{D_2} \text{COFFEE} \}$, $D_3 = \{ \vdash_{D_3} \text{SWEET} \}$, $D_4 = \{ \vdash_{D_4} \text{COFFEE} \}$. In this case, the Hasse diagram is given as Figure 2.

This preference relation, generated by the family of desires $\mathcal{F}'(D)$, makes it possible to construct a preference relation between #1 and #6 which was not able to be constructed by $\mathcal{F}(D)$, the case of the power set of D. Moreover, accompanied by it, it also enables to compare #1 and #8 as well as #3 and #8. This is because $\mathcal{F}'(D)$ excludes the desires $\{ \text{SALTY} \vdash \}$ and $\{ \text{SALTY} \vdash, \vdash \text{SWEET} \}$.

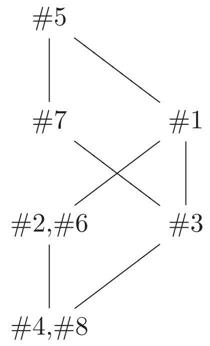


Figure 2: Hasse diagram of $(\text{tok}(\text{Cla}^*(D)), \precsim_{\langle \mathcal{F}'(D), A\rangle})$

Although #6 can satisfy both of these desires, #1 can not. And thus, "abandoning" these desires makes it possible to construct the preference relation between #1 and #6 which was not able to be constructed by $\mathcal{F}(D)$. This applies to the other pairs of token.

As just described, even if we cannot construct preference relations between any two tokens under some family of desires, it becomes to be possible by "abandoning" some desires. By modifying the set on which preference relation is defined, the same proof of Proposition 2 applies to the following proposition.

Proposition 5. *Suppose it is not possible to construct any preference relation $\precsim_{\langle \mathcal{F}(D), A\rangle}$ between $a, b \in$ tok(A) under a family of desires $\mathcal{F}(D)$. However, it becomes possible to construct preference relation between a and b under a family of desires $\mathcal{F}'(D)$ made by eliminating some desires from $\mathcal{F}(D)$. The desires eliminated, i.e., $\mathcal{F}(D) \setminus \mathcal{F}'(D)$ is said as* abandoned desires.

At last, let us consider a family of desires of D, $\mathcal{F}''(D) = \{D, D_1, D_3\}$ which consists of $D_1 = \{ \vdash_{D_1} \text{SWEET}, \vdash_{D_1} \text{COFFEE} \}$, and $D_3 = \{ \vdash_{D_3} \text{SWEET} \}$. In this case, the Hasse diagram is given as Figure 3.

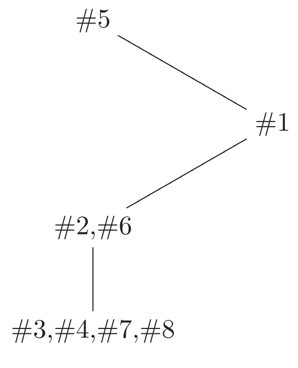


Figure 3: Hasse diagram of $(\text{tok}(\text{Cla}^*(D)), \precsim_{\langle \mathcal{F}''(D), A\rangle})$

Under this family of desires $\mathcal{F}''(D)$, it is possible to construct a preference relation between each of the tokens such as #7 and #1, #3 and #2, which we could not compare in the case of $\mathcal{F}'(D)$. We can verify the preference relation is complete by that all tokens are vertically collinear in the Hasse diagram.

It is also true in this case that excluding the desires $\{ \vdash \text{COFFEE} \}$ and $\{ \text{SWEET} \vdash , \vdash \text{COFFEE} \}$ from $\mathcal{F}''(D)$ makes it possible to construct the preference relations between #7 and #1, and #3 and #2. Whereas #7 can satisfies each of these desires, #1 can not. That is, we can construct the preference relation between #1 and #7, which were not able to be compared under the family of desires $\mathcal{F}''(D)$ by abandoning these desires. Moreover, abandoning these desires makes monotone inclusion relations among the desires contained by $\mathcal{F}''(D)$ and thus makes complete preference relations between all the tokens.

Let us take a brief look. All tokens #3, #4, #7, #8 lying at the bottom of the Hasse diagram do not satisfy the desire D_3 which expresses "want sweet thing." Tokens #2 and #6 satisfy this desire D_3 but does not the desire D_4 which expresses "want sweet coffee." Token #1 satisfies this desire D_4 but does not the desire D which expresses "want not salty but sweet coffee." Only token #5 satisfies all of the family of desires of D. Step by step, the number of desire which is to be satisfied is increasing monotonously.

As just stated, if a monotone inclusion relation exists among the sets of the family of desires, the preference relation according to it becomes to be rational. The same proof of Proposition 3 applies to the following proposition.

Proposition 6. *Suppose a family of desires $\mathcal{F}(D)$ of a desire D satisfies for any two elements of $\bar{D}_i, \bar{D}_j \in \mathcal{F}(D)$, either $\bar{D}_i \sqsubseteq \bar{D}_j$ or $\bar{D}_j \sqsubseteq \bar{D}_i$. Then any preference relation constructed according to this family of desires $\mathcal{F}(D)$ is rational.*

2.3 Local Logics

In the previous section, we presented how to construct a preference relation on tokens of a classification. But remians the question: can we always construct a preference relation between existing particulars if only an observation O is provided? Unfortunately, the answer is no. This is because classifications generated from observation cannot always inform us of the types we desire. To confirm this problem, let us revisit the example of Naiv and Chem.

The two powders were observed by Naiv and Chem as in the classification tables of Table 1 and Table 2. Let us construct preference relations on the tokens of these classifications according to, for instance, a desire $D = \{ \vdash_D \text{SWEET} \}$. First, we see Naiv's case. The type "SWEET" is not included in the types of O_n, the Naiv's observation, so that O_n cannot express the desire D. Similarly, Chem's observation O_c cannot express the desire D, too.

It would not be surprising that Naiv cannot distinguish token1 and token2 and, as a consequence, O_n cannot express the desire D, for Naiv was able to distinguish tokens only by the container they are in. However, it might be unnatural that O_c, the observation of Chem who was able to discriminate tokens by melting point and crystal structure, cannot express the desire D, for Chem must have known that "token1 is salt" and "token2 is sugar." Nevertheless, even if he knew it, the desire D still cannot be expressed by O_c since its set of types does not contain the type "SWEET."

Having knowledge of relationship between desires and existing particulars, such as "which desire is represented by which existing particular," is necessary for the one who tries to construct a preference relation. It is not until she relates each existing particular to her desires in accordance with this relationship that she can construct preference relations between these particulars. That is, the preference relation "sugar is preferred to salt" can be constructed according to the desire "want sweet stuff" only by making "sugar," the existing particular, relate to the "sweet stuff," the desired object. We call the function of perception relating the existing particular to the desired object as *representation*.

Representation would be a fundamental function lying at the base of our daily cognitive action. It is too fundamental for us to be aware of its function. For example, in everyday life we naturally pick up a particular which satisfies our desire among existing particulars. Picking up a particular, we would often equate the particular with the object of desire and, thus, be likely to think that we can always choose a particular which satisfies our desire if only alternatives are provided.

However, it is not true that we intrinsically know which of our desire can be satisfied by these particulars. In fact, there are many cases where we cannot find any types we desire in a particular only by observations, as in the case of Naiv and Chem. In these cases, we merely feel as if a particular is just the thing we desired by knowing indirectly which of desire can be satisfied by the particular through our own experiences of handling it. Therefore, it might be said that the feeling as such is nothing more than a fabrication by knowledge formed through such experiences. This kind of perceptual function, which enables us to regard an existing particular as the object of desire itself, is just what we call "representation."

In our argument, we try to understand representation through two functions. The one is the function that makes existing particulars relate to imagined particulars which are generated from desire, and the other is the function that judges whether the relation is appropriate. The former function corresponds to a broad-sense representation which enables to represent "sweet thing," the desired object, by "sugar," the existing particular. We will describe it as the function which makes connections between different particulars by knowledge, which governs the broad-sense representation, in the following sections. Contrastingly, the latter function shows the boundary of knowledge.

Knowledge we postulate in this article is not the one that is neither unique nor absolutely true. It is merely a limited system formed by each individual through her own experience. Therefore, it may happen that an observation revokes already established knowledge. For this reason, it is necessary to distinguish appropriate and inappropriate representations by assessing the boundary of the function of knowledge. *Local logics* introduced below mainly function as such in representation.

Definition 15 (Barwise and Seligman (1997))**.** A *local logic* $\mathfrak{L} = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \vDash_{\mathfrak{L}}, \vdash_{\mathfrak{L}}, N_{\mathfrak{L}} \rangle$ consists of

1. a classification $cla(\mathfrak{L}) = \langle tok(\mathfrak{L}), typ(\mathfrak{L}), \vDash_{\mathfrak{L}} \rangle$,
2. a regular theory $th(\mathfrak{L}) = \langle typ(\mathfrak{L}), \vdash_{\mathfrak{L}} \rangle$, and
3. a subset $N_{\mathfrak{L}} \subseteq tok(\mathfrak{L})$, called the *normal tokens* of $\mathfrak{L}$, which satisfy all the constraints of $\mathrm{th}(\mathfrak{L})$.

As the construction shows, local logic is a conception that makes it possible to grasp the sameness and difference between two structures, classification and theory, with which we dealt as independent conceptions made by independent principles. Viewing classification as semantics and theory as syntax, we can see it as having the same construction with the classical logic. However, unlike the classical logic, there is generally no guarantee of the conformity of classification and theory in local logic. It may occur, thus, that a particular which revokes the agent's knowledge is observed or an incomplete knowledge which cannot fully explain the agent's observation is formed. Here comes the necessity of capturing the limit of knowledge and also the possibility of arguing, in a positive manner, the dynamic property of knowledge.

A local logic is mainly characterized by two properties, soundness and completeness, according to the relationship between two components, classification and theory. We say that a local logic is *sound* if every token is normal and *complete* if every sequent satisfied by every normal token is a constraint of it. Putting soundness and completeness in line with a relationship between knowledge and observation, we regard the local logic consisting of this knowledge and observation as *sound* if the knowledge is satisfied by every token in the observation. Correspondingly, if the knowledge includes all the constraints that can be satisfied by the observation, we regard the local logic as *complete*.

As far as we consider sound and complete logics, there is no room for differences between observation and knowledge. However, we often believe in knowledges conflicting with our observations or sometimes do not even realize a simple matter which seems naturally true until someone mentions it. For example, if we were asked what temperature water boils, we would answer "100°C." But if we were in highland or put salt in the water, we could obtain observations which contradict this answer. Nonetheless we would not see this constraint "water boils at 100°C" itself to be incorrect. On the contrary, dissecting commonly-available animals, we would obtain an observation "'creature with a heart' is also 'creature with a kidney."' However, we would seldom have such knowledges as constraints and also seldom regard a lack of these constraints as a defect of knowledge. From these examples, we can say that a relationship between observation and knowledge is generally described with a neither sound nor complete local logic. This is why we focus here on generally unsound and incomplete logics, denoted as $\mathrm{Log}(O, K)$, which are constituted from an observation O and a knowledge K.

Definition 16. Let $O = \langle \mathrm{tok}(O), \mathrm{typ}(O), \vDash_O \rangle$ be an observation and $K = \langle \Sigma_K, \vdash_K \rangle$, where $\mathrm{typ}(O) \subseteq \Sigma_K$, be a regular knowledge. The *local logic generated by O and K*, written $\mathrm{Log}(O, K)$, has the classification O, the regular knowledge $K \upharpoonright \mathrm{typ}(O) \equiv \langle \mathrm{typ}(O), \vdash_K \rangle$, and all tokens which satisfy every constraint of the knowledge K are normal.[12]

Here we use $\mathrm{Log}(O, K)$ as a concept capturing the divergence between knowledge and observation. Let us take an example. Suppose a knowledge K_2 and an observation O are given as in Table 5.

$$\begin{aligned} K_2 \;=\; \{&\text{COFFEE} \vdash_{K_2} \text{BITTER}, \quad \text{COFFEE} \vdash_{K_2} \text{SMELL GOOD}, \quad \text{COFFEE} \vdash_{K_2} \text{SWEET}, \\ &\text{ORANGE JUICE, BITTER} \vdash_{K_2}, \quad \text{ORANGE JUICE} \vdash_{K_2} \text{SWEET}.\} \end{aligned}$$

$\vDash_O$	COF	ORA	BIT	SWE	SMG
token1	1	0	1	0	1
token2	0	1	0	1	0
token3	0	1	1	1	0
token4	1	0	1	1	1
token5	1	0	1	1	0

Table 5: knowledge K_2 and observation O

In this case, $\mathrm{Log}(O, \bar{K_2})$ are written as follows.

$$\begin{aligned} &\mathrm{cla}(\mathrm{Log}(O, K_2)) = O, \\ &\mathrm{th}(\mathrm{Log}(O, K_2)) = \bar{K_2}, \\ &N_{\mathrm{Log}(O,K_2)} = \{\text{token2, token4}\}. \end{aligned}$$

$\mathrm{Log}(O, \bar{K_2})$ consists of the classification O of the observation, the knowledge $\bar{K_2}$, and the normal tokens token2 and token4. Token1, 3, and 5 are not included in the normal token of $\mathrm{Log}(O, \bar{K_2})$, and thus we find that $\mathrm{Log}(O, \bar{K_2})$ is not a sound local logic. They are excluded from normal token since token1 cannot satisfy the constraint "coffee is sweet," token3 cannot satisfy the constraint "orange juice is not bitter," and token5 cannot satisfy the constraint "coffee smells good." These tokens are incompatible with the knowledge $\bar{K_2}$ and so not deemed to be normal. This discrepancy between the knowledge $\bar{K_2}$ and the observation O violates the soundness of $\mathrm{Log}(O, \bar{K_2})$.

[12]Note technically that knowledges contained by $\mathrm{Log}(O, K)$ are confined to the constraints on $\mathrm{typ}(O)$, the set of types of observation. If K is regular, $K \upharpoonright \mathrm{typ}(O)$ is regular too.

Let us consider the meaning of tokens not satisfying constraints of knowledge. In our argument, we treat knowledge as a limited system which has been formed through a certain series of experience. If we regard knowledge as such a system, tokens not satisfying constraints of knowledge can be seen as totally impossible particulars demolishing the knowledge itself. For example, for an individual who has a constraint "coffee smells good," "bad smelling coffee" is recognized as a particular which revokes the past belief.

We have stated above that representation is a function making some particular relate to some object of desire in accordance with knowledge. Stating like this, we implicitly presume an appropriate knowledge about the particular. But how about if the particular in front of us does not satisfy the constraint of our knowledge? The representation of this particular would no longer be adequate. It is clear that "bad smelling coffee" is not the coffee in our mind. It truly is coffee, but we would no longer be able to judge properly whether this coffee is a particular which satisfy our desire. Then wavers the adequacy of representation.

Of course, such "waver of adequacy" would cause the cognitive agent to update her knowledge. We consciously or unconsciously update our knowledge in everyday life through experiences of consumimg something unexpected, e.g., "bad smelling coffee possibly exists." Although the adequacy of knowledge often wavers, we can stabilize it by updating the system of knowledge to be more adequate each time. This is enabled by a dynamic property of knowledge, which would be adequately called as "learning."

On the other hand, if the discrepancy between knowledge and observation were venial, we would regard the experience of discrepancy as "exceptional" and try to preserve the structure of the knowledge rather than to update it. This is because knowledge forms a foundation of perceiving the world and so doubting it may cause significant damage to cognitive action by making it inappropriate. What preserves the structure of knowledge and supports its static stability by screening the tokens conflicting with the knowledge is, then, normal token.[13] This function of normal token allows knowledge to construct appropriate representation in unsound local logics.

2.4 Infomorphisms

The narrow-sense representation is predominantly comprised of two conceptions, *infomorphism* and *channel*. *Infomorphism* is a concept capturing a certain kind of homomorphism between two classifications. Contrastingly, *channel* is a concept creating a connection between two distinct classifications by the intermediary of a tertiary classification, called *core*, and makes it possible to capture what kind of relationship the two classifications have. We can figure out sameness and difference between two or more classifications by these two concepts. In this section, we introduce *infomorphism*.

Definition 17 (Barwise and Seligman (1997)). An *infomorphism* $f : A \rightleftarrows B$ from A to B is a contravariant pair of functions $f = \langle f^{\wedge}, f^{\vee} \rangle$ satisfying the following *Fundamental Property of Infomorphisms*:

$$f^{\vee}(b) \vDash_A \alpha \quad \text{iff} \quad b \vDash_B f^{\wedge}(\alpha)$$

for each token $b \in \text{tok}(B)$ and each type $\alpha \in \text{typ}(A)$. Classification A is called *domain* of f and classification B is called *codomain* of f.

Infomorphism is depicted in a commutative diagram as Figure 4.

One of the main feature of infomorphism is that it is defined as a pair of two mappings in opposite directions. To understand the relation these mappings capture, it is helpful to imagine

[13]These duality, that is, the dynamic and static property of the structure of knowledge may be grasped by W.V.O. Quine's "conservatism." We will not deal with a dynamic property of knowledge, or "learning," in this article, but local logics and infomorphisms, which will be introduced in the following section, play a prominent role in understanding this property. We will cover this issue in another article.

$$\begin{array}{ccc} \mathrm{typ}(A) & \xrightarrow{f^{\wedge}} & \mathrm{typ}(B) \\ \Big| \vDash_A & & \Big| \vDash_B \\ \mathrm{tok}(A) & \xleftarrow[f^{\vee}]{} & \mathrm{tok}(B) \end{array}$$

Figure 4: diagram of infomorphism

an individual A and B mutually confirm their classifications. Let us revisit the previous example of salt and sugar and verify whether the infomorphism holds according to the observations of Naiv and Chem expressed by Table 1 and Table 2. Suppose Naiv is the counterpart of individual A.

As in Table 1, classification O_n, obtained by Naiv's observation, shows that token1 and token2 are distinguished only by the container they are in. On the other hand, as in Table 2, classification O_c, obtained by Chem's observation, shows that they are discriminated by melting point and crystal structure. Note that the types included in O_n are also included in O_c.

First of all, we confirm whether a type-token-identical infomorphism from Naiv's observation O_n to Chem's observation O_c exists. Naiv casts the type "white" from his classification to Chem, i.e., asks Chem "which one is white?" Chem replies to this question, pointing a finger at token1 and token2 which are of type "white" in his classification, "these are white." In response to this reply, Naiv verifies whether token1 and token2 are also classified by the type "white" in his classification. If they are, it is verified that the Fundamental Property is satisfied in respect of the type "white." Then, Naiv continues to verify the Fundamental Property in respect of the type "powder" in the same way. If the Fundamental Property is verified in respect of each type of Naiv's classification, he comes to complete the confirmation process from him successfully.

The confirmation process is followed by Chem. Chem casts the token "token1" from his classification to Naiv, i.e., asks Naiv pointing a finger at token1 in front of them, "what is this?" Naiv lists the types classified with token1 and replies, "this is white powder in a square-shaped container." In response to this reply, Chem verifies whether "white," "powder," and "in a square-shaped container" are of types of token1 in his classification. If they are, it is verified that the Fundamental Property is satisfied in respect of the "token1." Then, Chem continues to verify the Fundamental Property in respect of the token "token2" in the same way. If the Fundamental Property is verified in respect of each token of Chem's classification, he comes to complete the confirmation process from him successfully, too. This completes the confirmation of the Fundamental Property.

As we saw above, one can easily verify that a type-token-identical infomorphism from O_n to O_c exists in this example. On the other hand, it is all impossible to construct any type-token-identical infomorphism from O_c to O_n since Chem uses types which Naiv does not know.

Generally, infomorphism needs to be neither type-identical nor token-identical, since it is possible to construct an infomorphism if only there exists some similarlity between the way tokens of different classifications are classified. For example, an infomorphism from O_n to O_c exists even if Naiv constitutes his classification in Japanese, since the structure of classification remains the same except the type is translated from English to Japanese.

Even though the meanings of types are different, the two types are regarded as the same if they are assigned to the same token. For instance, the type "melting point 130°C" of O_c is assigned to the same token as the type "in a circular-shaped container" of O_c. Therefore we can see them as the same type and the Fundamental Property is satisfied if Naiv maps the type "melting point 130°C" of O_c to "in a circular-shaped container" of O_n by $f^{\wedge}$. ($f^{\wedge}(130°\mathrm{C}) = \textsc{cir}.$) Additionally, mapping both "melting point 800°C" and "crystal of hexahedron" of O_c to "in a square-shaped container" constructs a token-identical infomorphism from O_c to O_n. Of course, in this case, Naiv maps Chem's type to his own type by "mistake." However, even if he maps types by "mistake," it can be said that this mapping satisfies the Fundamental Property together with token-identical mapping and makes the infomorphism hold.

A case where any infomorphism cannot be constructed is as follows. Suppose Chem has a type "black" besides the types above. Then, "BLACK" is added to the type of classification table of O_c and 0s are assigned to the fields of both token1 and token2. However, Naiv does not have a type as such pattern and therefore cannot map types so as to make infomorphism hold. Thus infomorphism cannot be constructed.

Whether infomorphism can be constructed or not depends on whether the tokens classified as the same pattern exist or not. Therefore, for example, even though Naiv adds types such that both token1 and token2 are of these types, there is no case that infomorphisms from O_n to O_c does not exist. On the other hand, any infomorphism from O_n to O_c does not exists if only a type "black" is added to O_n.

Incidentally, the infomorphisms we will take up hereafter are confined to the type-identical. This is because "representation," which is the main topic of this article, originally functions to make a connection between different particulars by assuming the identity of types. Note, however, that one of the most interesting feature of infomorphism, we think, resides in its ability to form a relationship which is neither type-identical nor token-identical and to allow "mistakes" preserving a certain kind of homomorphism between classifications. With this ability, infomorphism enables us to ask questions such as how to misunderstand or how to notice a misunderstanding. We will deal with these subjects in another article.

2.5 Channels

Let us recall the function of "representation." Representation is a function to view some particular in connection with another particular. That is, we regard an existing particular as a representation of some desire if it is deemed to be as close as an object of the desire itself. Therefore, in order to give a description of representation system, formalizing a concept which connects different objects that have no relation with each other in advance is inevitable.

Infomorphism, introduced in the previous section, is a concept which holds between a kind of homomorphic classifications and hence we can say that it is impossible to describe relationships between different particulars only by infomorphism. However, what we need is not a new kind of mapping which can directly connect two distinct classifications. What is important is rather to construct a viewpoint to look over inclusively the difference of the two distinct classifications. Only by acquiring this viewpoint, we can describe the function that relates both sides as different things and at the same time equates them. What makes it possible is *channel*.

Definition 18 (Barwise and Seligman (1997))**.** A *channel* $\mathcal{C}$ is an indexed family $\{f_i : A_i \rightleftarrows C\}_{i \in I}$ of infomorphisms with a common codomain C, called the *core* of $\mathcal{C}$. The tokens of C are called *connections*; a connection c is said to *connect* the tokens $f_i(c)$ for $i \in I$. A channel with index set $\{0, \ldots, n-1\}$ is called an *n-ary* channel.

A channel forms a core between two or more classifications as a common codomain. Being mediated by this core, the channel constructs infomorphisms between multiple classifications and relates them each other. Figure 5 puts channel into a diagram.

Let us see the example of Naiv and Chem. Their classifications of observation were given in Table 1 and Table 2. As was verified in the previous section, there exists a type-token identical infomorphism from O_n to O_c, but does not from O_c to O_n. Here we consider a classification of Table 6.

As easily seen, this is a classification made by indexing each type of O_n and O_c by the observer's name and having them line up along the rows of token. This simple classification comes to be a core between O_n and O_c and functions as an intermediary for two distinct classifications.

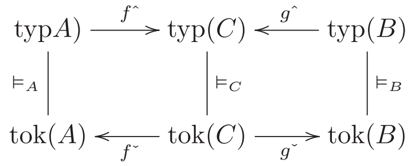


Figure 5: channel and core

$\models_C$	WHI_n	POW_n	SQU_n	CIR_n	WHI_c	POW_c	SQU_c	CIR_c	$130°\text{C}_c$	$800°\text{C}_c$	HEX_c
token1	1	1	1	0	1	1	1	0	0	1	1
token2	1	1	0	1	1	1	0	1	1	0	0

Table 6: core C between O_n and O_c

Let us verify this classification becomes a core of channel. First, construct a contravariant pair of type-token-identical mappings from O_c to C which maps each type to the counterpart with subscript c. This clearly satisfies the definition of infomorphism by the definition of C since, in fact, this is the same as constructing infomorphism between the same classifications. Similarly, constructing a contravaiant pair of type-token-identical mappings from O_n to C satisfies the definition of infomorphism. Thus it is verified that C is a core of channel which is constituted by these infomorphisms.

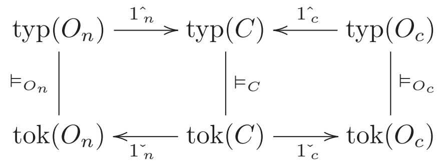


Figure 6: channel by O_n and O_c

In this way, a channel enables to connect different classifications through the intermediary of a core having both construction of the classifications and hence also enables us to investigate correspondence between types and tokens of these classifications.

Let us see how this channel functions. As we have seen, the two classifications O_n and O_c, which are connected by the channel, map their constructions as they are into the core of the channel. The core of channel, expressing two classifications in one, makes it possible to compare them and comprehend the relationship between them. For example, we find that the type "melting point 130°C" of Chem has the same meaning as "in a circular-shaped container" of Naiv since both "$130°\text{C}_c$" and "CIR_n" classify tokens exactly the same way. We can also think that the types such as "in a square-shaped container" and "white" of Chem has the same meaning as Naiv's.

Such a function of channel could be interpreted as Naiv's cognitive action. That is, Naiv could not have understood Chem's observation, so that he objectified both Chem's and his own observation, compared them, and, as a result, came up with the core above.

Next, let us consider a classification C' as in Table 7. C' is a classification made by removing subscription of types common to O_n and O_c and unifying them. In other words, it can be interpreted as the classification constructed based on the view that the types common to Naiv and Chem are used, as was seen in C, exactly the same way, so that each common type is unified. As well as C, C' is able to be a core of a channel between O_n and O_c.

As previous argument on C makes clear, C' and C have a kind of same construction. In other words, we can construct a infomorphism between C and C' from both directions since C' is a classification made by unifying types common to O_n and O_c such as "in a square-shaped container" or "white" and hence the pattern of the way each token classified is the same.

$\models_{C'}$	WHI	POW	SQU	CIR	130°C_c	800°C_c	HEX_c
token1	1	1	1	0	0	1	1
token2	1	1	0	1	1	0	0

Table 7: core C' between O_n and O_c

What is to be noted is that the sameness of the construction of C and C' does not imply the uniqueness of the construction of the core. In general, construction of core is not uniquely determined and can be varied depending on the way it is formed. In fact, even if we follow the way C and C' are formed respectively, different cores can be yielded. For instance, consider the case where some token is perceived "taste good" by Naiv but "not taste good" by Chem. As far as we form the core by the way C is formed, i.e., distinguishing Naiv and Chem's types, we can comprehend from the core how the type "taste good" is used by each of them. However, if we form the core according to the way C' is formed, i.e., presuming that the type "taste good" is the same for both observations, we cannot figure out the gap between the way Naiv and Chem use the type. This is because the way token is classified is not uniquely determined on the core, since to the token Naiv assigned "taste good" Chem did not. As a result, the core cannot be a core of the channel consisting of type-token-identical infomorphisms and thus cannot be of the same construction as C.

Let us see the gap between the way C and C' is formed as the gap between perception of the agents, say, a careful agent and a simple-minded agent who form these cores respectively. An attentive agent, for example, would be able to notice the gap between the way "taste good" is used. If the same tokens are classified differently, she would see it as the difference between the way of classifying the tokens without failing. Such an agent would form C, in which the way Naiv and Chem use "taste good" is distinguished, as a core. Contrarily, an agent who believes without a doubt that the way of using types are always the same might overlook the gap between the way of using the type "taste good." In other words, she might not take it as the difference between the way of using them but negate the fact that the observation was made on the same token, say, "we might eat different things." In this case, the simple-minded agent would not distinguish the types they use and form a core like C', in which tokens are classified differently than C. As just described, we can model the gap betweem perception of each agent via the difference betweem the construction of cores of channels by regarding it as the reflection of the perception of the agents.

Let us revisit again the issue of representation. Recall that representation is a function to view some particular in connection with another particular. Besides it, we could also say generally that representation is a function that has a vast room for various formation depending on the agent's perception and so a huge variety of structures. In other words, it is determined by each agent's perception how should we make connections between one particular and another, and so is open to diversity. This diversity of core, which makes it possible to connect distinct classifications in various ways, can be said to be suitable for modeling this diversity of function of representation. We attempt to describe each agents' function of representation by making use of these flexibility of core in a positive way. That is, we deem the core to be a conceptual constitution as a result of a kind of the agent's cognitive action and hence formulate modalities of representation as they are without losing their diversity. [14]

[14] In this section, we have interpreted the cores C and C' as the classifications formed by Naiv's cognitive action. However, of course, there is no need to think of the agent who formed these cores is Naiv. It is definitely possible

The following is an example of core of channel representing a desire by an observation. We are going to confirm it according to the Chem's case in section 2. Consider the observation O_c and a desire $D = \langle \Sigma_D, \vdash_D \rangle$ such that $D = \{ \vdash_D \text{SWEET} \}$ given in Table 2. The classification $\text{Cla}^*(D)$ made from D is as follows. (This classification will sometimes be written simply as "desire" below.)

$\vDash_{\text{Cla}^*(D)}$	SWE
$\langle \{\text{SWEET}\}, \emptyset \rangle$	1
$\langle \emptyset, \{\text{SWEET}\} \rangle$	0

Table 8: classification $\text{Cla}^*(D)$ generated from the desire D

Recall that the observation O_c cannot express the desire D. This is because the type of O_c does not contain the type of D. However, Chem can connect the tokens of the observation to the types of the desire by forming a channel of which core is, for instance, C_{c1} as follows between the observation and the desire.

$\vDash_{C_{c1}}$	WHI	POW	SQR	CIR	130°C	800°C	HEX	SWE
$\text{token1}_{\text{SWE}}$	1	1	1	0	0	1	0	1
$\text{token1}_{\text{NOT SWE}}$	1	1	1	0	0	1	0	0
$\text{token2}_{\text{SWE}}$	1	1	0	1	1	0	1	1
$\text{token2}_{\text{NOT SWE}}$	1	1	0	1	1	0	1	0

Table 9: core C_{c1} between O_c and $\text{Cla}^*(D)$

It is clear that this classification C_{c1} comes to be a core of binary channel which connects O_c and $\text{Cla}^*(D)$ since there are type-identical infomorphisms from both O_c and $\text{Cla}^*(D)$ to C_{c1}. That is, with respect to the infomorphism from O_c to C_{c1}, it relates $\text{token1}_{\text{SWE}}$ and $\text{token1}_{\text{NOT SWE}}$ to token1, while $\text{token2}_{\text{SWE}}$ and $\text{token2}_{\text{NOT SWE}}$ to token2. With respect to the infomorphism from $\text{Cla}^*(D)$ to C_{c1}, it relates $\text{token1}_{\text{SWE}}$ and $\text{token2}_{\text{SWE}}$ to $\langle \{\text{SWEET}\}, \emptyset \rangle$, while $\text{token1}_{\text{NOT SWE}}$ and $\text{token2}_{\text{NOT SWE}}$ to $\langle \emptyset, \{\text{SWEET}\} \rangle$. These mappings constitute infomorphisms. We denote hereafter the infomorphism from O_c to C_{c1} as f_{c1}, the infomorphism from $\text{Cla}^*(D)$ to C_{c1} as g_{c1}, and define the channel comprised of f_{c1} and g_{c1} as $\mathcal{C}_{c1} = \{f_{c1}, g_{c1}\}$.

Then, what perception of Chem does this channel reflect? Let us check token1 first. Being intermediated by the core, token1 is connected to two tokens of desire $\langle \{\text{SWEET}\}, \emptyset \rangle$ and $\langle \emptyset, \{\text{SWEET}\} \rangle$ simultaneously. That is, it is simultaneously connected to both tokens which satisfies the desire "want sweet stuff" and which does not. This can be considered as the situation Chem vacillates on judgement whether token1 is "sweet" or not. Since he is not sure whether token1 is "sweet" or not, he connects token1 to both tokens. A similar argument applies to token2. That is, we can see that Chem is in situation in which he cannot judge whether token1 and token2 are "sweet" or not respectively under this core of channel.

We would like to refrain here from asking whether this core of channel C_{c1} represents Chem's perception properly. Because, although every channel can be understood as expressing Chem's some perception, the channel itself gives no criteria to judge whether the channel is proper or not.

to interpret that these cores are formed by Chem as well as by third person. For instance, the situation that two people talk about the observation on the same token, and a third person listens to their conversation and interprets their way of classifying the tokens. In this case, a listener who distinguishes their words carefully would form C and a listener who believes that the same words have the same meaning and equates them would form C'. Then the former would notice the differences of meaning of their words and the latter would not. Indeed, we are ordinarily familiar with a kind of experience that two person talk past each other, but the third person judges it "objectively" and points out the gap. Assuming the third person who forms the core C makes it possible to describe these kinds of situations.

The core of channel between observation O_c and classification $\mathrm{Cla}^*(D)$ is not unique up to C_{c1}. There are in principle $2^4 = 16$ cores of channels which consist of type-identical infomorphisms. One core connects the tokens of the observation only to the tokens of "sweet" desire, another does not neither of them to any desire, and each of these cores can be considered as an expression of some distinct perceptions. However, each core per se is not the very essence of one's perception. Each core is rather an expression given as it is by this essence. We attempt to capture this essence which gives its expression to each core by means of knowledge.

We judge some core is proper if the core is consistent with the knowledge of the agent who forms the core. In other words, if the connections of the core have some proper relationship with the constraints of knowledge, we regard the core as a proper expression of perception. In the following section, we will give a definition of proper core within the framework of representation system consisting of a channel and a local logic including its core.

If a channel is once constructed, the preference relation between the tokens of the observation can be defined by means of its core. Even if the observation itself cannot express the desire, it is possible to specify desires which the observation can satisfy since they are connected by the core. At the last of this section, we give a definition of preference relation under a core of channel.

To begin with, a family of desires that is satisfied by tokens of an observation under a channel is defined as follows.

Definition 19. Let an observation O, a desire D, a binary channel between a classification $\mathrm{Cla}^*(D)$ generated from D and the observation O of which core is C, written $\mathcal{C} = \{f, g\}$ such that $f : O \rightleftarrows C$, $g : \mathrm{Cla}^*(D) \rightleftarrows C$, and a family of desires $\mathcal{F}(D)$ of D be given. Then, relative to $\mathcal{F}(D)$ and a token a of the observation O, the family of desires which all the connections of a can satisfy is said as *a family of desires satisfied by token a under channel* $\mathcal{C}$, written $\mathcal{F}_{a,\mathcal{C}}(D)$ and is defined as follows.

$$\mathcal{F}_{a,\mathcal{C}}(D) = \bigcap_{c \in \{b \in \mathrm{tok}(C) \,|\, f\check{}(b) = a\}} \mathcal{F}_{c,C}(D)$$

where

$$\mathcal{F}_{c,C}(D) = \{D_i \in \mathcal{F} \,|\, c \text{ satisfies all the constraints } \langle g\hat{}[\Gamma], g\hat{}[\Delta]\rangle \text{ of desire } D_i \text{ under core } C\}.$$

By means of this family of desires, we define a preference relation on $\mathrm{tok}(O)$ mediated by channel $\mathcal{C}$.

Definition 20. Let an observation O, a desire D, a binary channel between classification $\mathrm{Cla}^*(D)$ generated from D and the observation O of which core is C, and a family of desires $\mathcal{F}(D)$ of D be given. Then, a preference relation between tokens a and b of the observation O, written $\precsim_{\langle \mathcal{F}(D), \mathcal{C}\rangle}$ is defined as follows.

$$b \precsim_{\langle \mathcal{F}(D), \mathcal{C}\rangle} a \quad \text{iff} \quad \mathcal{F}_{b,\mathcal{C}}(D) \subseteq \mathcal{F}_{a,\mathcal{C}}(D).$$

Let us check the preference relation between token1 and token2 under the channel $\mathcal{C}_{c1}$. Take $\mathcal{F}(D) = \{D\}$ as a family of desires of D expressing "want sweet stuff." Recall that the both tokens token1 and token2 of the observation O are connected to the two tokens of desire $\langle\{\text{SWEET}\}, \emptyset\rangle$ and $\langle\emptyset, \{\text{SWEET}\}\rangle$ simultaneously. Therefore, the family of desires that each of tokens can satisfy mediated by this channel is provided as follows.

$$\mathcal{F}_{\text{token1},\mathcal{C}_{c1}} = \mathcal{F}_{\text{token2},\mathcal{C}_{c1}} = \emptyset.$$

Since both token1 and token2 are connected to the both tokens that satisfies the desire D and does not satisfy by the core C_{c1}, they cannot satisfy the desire "want sweet stuff." That is, each of them is not deemed to satisfy the desire D under this core and channel since they have a connection which relates them to the token of desire which does not satisfy D.

Thus the following preference relation is derived.

$$\text{token1} \sim_{\langle \mathcal{F}(D), \mathcal{C}_{c1} \rangle} \text{token2}.$$

With regard to preference relation under channel, propositions which hold in the case of preference relation between types and tokens of classification also hold. Proofs are similar to Proposition 1, 2, and 3 respectively.

Proposition 7. $\left(\text{tok}(O), \precsim_{\langle \mathcal{F}(D), \mathcal{C} \rangle}\right)$ *is a preordered set.*

Proposition 8. *Let an observation O, a regular knowledge K, and a desire D be given. Moreover, a channel $\mathcal{C}$ between the observation O and the classification $Cla^*(D)$ generated from D, of which core is C, is given. Suppose it is not possible to construct any preference relation $\precsim_{\langle \mathcal{F}(D), \mathcal{C} \rangle}$ between $a, b \in tok(O)$ under a family of desires $\mathcal{F}(D)$. However, there exists a family of desires $\mathcal{F}'(D)$ made by eliminating some desires from $\mathcal{F}(D)$ such that it becomes possible to construct preference relation between a and b under $\mathcal{F}'(D)$. The desires eliminated, i.e., $\mathcal{F}(D) \setminus \mathcal{F}'(D)$ is said as* abandoned desires*.*

Proposition 9. *Suppose a family of desires $\mathcal{F}(D)$ of a desire D satisfies for any two elements of $\bar{D}_i, \bar{D}_j \in \mathcal{F}(D)$, either $\bar{D}_i \sqsubseteq \bar{D}_j$ or $\bar{D}_j \sqsubseteq \bar{D}_i$. Then any preference relation constructed according to this family of desires $\mathcal{F}(D)$ is rational.*

2.6 Representation System

Let us begin with our model of representation system.

Definition 21 (Barwise and Seligman (1997))**.**

1. A *representation system* $\mathcal{R} = \langle \mathcal{C}, \mathfrak{L} \rangle$ consists of a binary channel $\mathcal{C} = \{f : A \rightleftarrows C, g : B \rightleftarrows C\}$, with one of the classifications designated as *source* (say A) and the other as *target*, together with a local logic $\mathfrak{L}$ on the core C of this channel.

2. The *representations* of $\mathcal{R}$ are the tokens of A. If $a \in \text{tok}(A)$ and $b \in \text{tok}(B)$, a is a representation of b, written $a \leadsto_{\mathcal{R}} b$, if a and b are connected by some $c \in \text{tok}(C)$. The token a is an *accurate representation of* b if a and b are connected by some normal token, that is, some $c \in N_{\mathfrak{L}}$.

3. For any token $a \in \text{tok}(A)$, the *set of types* of a is denoted as $\text{typ}(a) = \{\alpha \in \text{typ}(O) | a \vDash_O \alpha\}$, and the *complement* of the set of types of a is denoted as $\text{typ}^c(a) = \{\beta \in \text{typ}(O) | a \nvDash_O \beta\}$. The representation *a represents* $b \in \text{tok}(B)$ as satisfying a sequent $\langle \Gamma, \Delta \rangle$ if the translations of the set of types of a, $\text{typ}(a)$, the complement of it, $\text{typ}^c(a)$, and the sequent $\langle \Gamma, \Delta \rangle$ which is satisfied by b constitute a constraint of the logic $\mathfrak{L}$, that is, if $f[\text{typ}(a)], g[\Gamma] \vdash_{\mathfrak{L}} g[\Delta], f[\text{typ}^c(a)]$.[15]

We can depict a representation system as below:

The conception of representation system can be regarded as that of a channel enriched with a local logic which includes the core of the channel. It works as follows: First, it forms a binary channel between a source and a target which makes representation possible. Second, it determines whether every representation is accurate or not and checks out which sequents it satisfies according

[15] This third condition is different from that of Barwise and Seligman (1997). Since we focus on the function of a representation system which makes it possible for representations to represent targets as satisfying a sequent, we define a representation system in a more general form. If we focus our attentions only on the sequents where the left-hand side is empty and right-hand side is a singleton, that is, $\langle \emptyset, \{\beta\} \rangle$ and omit $\text{typ}^c(a)$, our condition coincides with that of Barwise and Seligman (1997).

$$
\begin{array}{ccccc}
\mathrm{typ}(A) & \xrightarrow{f^{\wedge}} & \mathrm{typ}(\mathfrak{L}) & \xleftarrow{g^{\wedge}} & \mathrm{typ}(B) \\
\models_A \Big| & & \Big| \models_{\mathfrak{L}} & & \Big| \models_B \\
\mathrm{tok}(A) & \xleftarrow[f^{\vee}]{} & \mathrm{tok}(\mathfrak{L}) & \xrightarrow[g^{\vee}]{} & \mathrm{tok}(B)
\end{array}
$$

Figure 7: diagram of representation system

to its local logic. In short, representation system is the system which determines each representation's characteristics with its own local logic. Let us see its construction and function by checking four different representation systems of Chem's.

Recall the channel $\mathcal{C}_{c1}$ and its core C_{c1} linking the observation O_n to the classification $\mathrm{Cla}^*(D)$ which is generated by the desire D, that is, "want sweet stuff." We shall deal with the representation system consisting of a channel $\mathcal{C}_{c1}$ and its core C_{c1} as a first example. To define the representation system, we start with defining the tokens of the core C_{c1} by *realizable states* given as below.

Definition 22 (Barwise (1997))**.** Let $A_1, \ldots, A_n$ be sets of types, classifications or theories. Given $A_1, \ldots, A_n$, the set of all partitions of $\bigcup_{i \in \{1,\ldots,n\}} \mathrm{typ}(A_i)$ is said as the set of *states* generated by $A_1, \ldots, A_n$, written $\Omega_{\langle A_1,\ldots,A_n \rangle}$. Furthermore, given a set of tokens $B \subseteq \bigcup_{i \in \{1,\ldots,n\}} \mathrm{tok}(A_i)$, the set

$$\Omega^R_{\langle A_1,\ldots,A_n \,|\, B \rangle} = \{\langle \Theta, \Lambda \rangle \in \Omega_{\langle A_1,\ldots,A_n \rangle} \mid \exists a \in B, \quad \mathrm{typ}(a) \subseteq \Theta \ \text{ and } \ \mathrm{typ}^c(a) \subseteq \Lambda\}$$

is said as the set of *realizable states* generated by $A_1, \ldots, A_n$ under B.[16] When $B = \bigcup_{i \in \{1,\ldots,n\}} \mathrm{tok}(A_i)$, we may omit B and write the set of realizable states simply as $\Omega^R_{\langle A_1,\ldots,A_n \rangle}$.

With the definition, given an observation O and a desire D, we can define the set of all states generated by D and O as $\Omega_{\langle D,O \rangle}$. We can also define the set of all realizable states generated by D and O under $\mathrm{tok}(O)$ (or simply under O) as $\Omega^R_{\langle D,O \rangle}$.

State can be thought of as a special kind of types of a classification for which each token is of exactly one type. For example, the state $\langle \Theta, \Lambda \rangle$ where $\Theta = \{\text{WHI, POW, SQU, } 800°\text{C, SWE}\}$ and $\Lambda = \{\text{CIR}, 130°\text{C}, \text{HEX}\}$ represents the state which is of all types of Θ and is not of any types of Λ. This type can be interpreted as the type which classifies the only token a such that $\mathrm{typ}(a) = \Theta$ and $\mathrm{typ}^c(a) = \Lambda$ in a classification of which space of type is $\mathrm{typ}(O) \cup \mathrm{typ}(D)$. Thus, we can specify every token uniquely by the state which corresponds to the token. So, we describe here every possible connections between an observation and a desire by states.

Suppose the classification $A_1 = \langle \Omega^R_{\langle D,O_c \rangle}, \mathrm{typ}(O_c) \cup \mathrm{typ}(D), \models_{A_1} \rangle$, where $\models_{A_1}$ between $\langle \Theta, \Lambda \rangle \in \Omega^R_{\langle O_c,D \rangle}$ and $\alpha \in \mathrm{typ}(O_c) \cup \mathrm{typ}(D)$ is defined as

$$\langle \Theta, \Lambda \rangle \models_{A_1} \alpha \quad \text{iff} \quad \alpha \in \Theta.$$

This classification A_1 coincides with C_{c1}. Let us check, for example, $\mathrm{token1}_{\mathrm{SWEET}}$. The token of A_1 which corresponds to the token $\mathrm{token1}_{\mathrm{SWEET}}$ is given by the state $\langle \Theta, \Lambda \rangle$, where $\Theta =$

[16] "The set of all states", as defined in Barwise (1997), Definition 4.7.1., coincides with $\Omega_{\langle \mathfrak{L} \rangle}$ where $\mathfrak{L}$ is an arbitrary local logic $\mathfrak{L}$. And the set of "realized" states, as defined in Barwise (1997), Definition 4.7.3., coincides with $\Omega^R_{\langle \mathfrak{L} \,|\, \mathrm{tok}(\mathfrak{L}) \rangle}$.

To model representation systems, however, we must define the states which link a given observation to some desire, that is, the states which include types of the desire. This is why we define here the realizable states, being different from Barwise (1997), on extended set $\mathrm{typ}(O) \cup \mathrm{typ}(D)$ of types. Of course, we don't know whether these states are really realized or not by a given observation, as the observation tells us only observable information. We only know that these states are realizable, and this leads us to call these states as "realizable."

$\{\text{WHI}, \text{POW}, \text{SQU}, 800°\text{C}, \text{SWE}\}$ and $\Lambda = \{\text{CIR}, 130°\text{C}, \text{HEX}\}$. Clearly, this token is of exactly the same types as $\text{token1}_{\text{SWEET}}$. By the definition of $\Omega^R_{\langle D, O_c\rangle}$, it also clearly holds that $\text{tok}(C_{c1}) = \Omega^R_{\langle D, O_c\rangle}$.[17]

Next, suppose that Chem has a knowledge $\bar{K}_c$, a regular closure of K_c where

$$\begin{aligned} K_c \;=\; & \{\text{SALT} \vdash_{K_c} \text{SALTY}, \quad \text{SUGAR} \vdash_{K_c} \text{SWEET}, \\ & \text{MELT AT } 130°\text{C} \vdash_{K_c} \text{SUGAR}, \quad \text{MELT AT } 800°\text{C} \vdash_{K_c} \text{SALT}, \\ & \text{HEX CRYSTAL} \vdash_{K_c} \text{MELT AT } 800°\text{C}\}. \end{aligned}$$

We can, then, define a local logic $\mathfrak{L}_{c1}$ as below, consisting of the knowledge $\bar{K}_c$ and the core C_{c1} of the channel which is previously given. The reason why the token $\text{token2}_{\text{NOT SWEET}}$ is not normal is that the token falsifies the constraint MELT AT $130°\text{C} \vdash_{\bar{K}_c}$ SWEET of $\bar{K}_c$.

$$\begin{aligned} &\text{cla}(\mathfrak{L}_{c1}) = C_{c1}, \\ &\text{th}(\mathfrak{L}_{c1}) = \bar{K}_c \upharpoonright \text{typ}(O_c) \cup \text{typ}(D), \\ &N_{\mathfrak{L}_{c1}} = \{\text{token1}_{\text{SWEET}},\ \text{token1}_{\text{NOT SWEET}},\ \text{token2}_{\text{SWEET}}\}. \end{aligned}$$

Now, we can define the representation system $\mathcal{R}_{c1} = \langle \mathcal{C}_{c1}, \mathfrak{L}_{c1}\rangle$, consisting of the channel $\mathcal{C}_{c1}$, with the observation O_{c1} as a source and the classification $\text{Cla}^*(D)$ generated by the desire D as a target, and the local logic $\mathfrak{L}_{c1}$.

Let us look into the construction of the representation system $\mathcal{R}_{c1}$. Let us see token2 first. We can see that token2 is connected to the tokens $\langle\{\text{SWEET}\}, \emptyset\rangle$ and $\langle\emptyset, \{\text{SWEET}\}\rangle$ through the connections $\text{token2}_{\text{SWEET}}$ and $\text{token2}_{\text{NOT SWEET}}$, so token2 can be regarded as a representation of $\langle\{\text{SWEET}\}, \emptyset\rangle$ and $\langle\emptyset, \{\text{SWEET}\}\rangle$. From the fact that the connection $\text{token2}_{\text{SWEET}}$ is normal we can conclude that token2 can be regarded as an accurate representation of $\langle\{\text{SWEET}\}, \emptyset\rangle$. Moreover, token2 represents $\langle\{\text{SWEET}\}, \emptyset\rangle$ as satisfying the desire D since MELT AT $130°\text{C} \vdash_{\bar{K}_c}$ SWEET is a constraint of the knowledge $\bar{K}_c$. On the other hand, the connection $\text{token2}_{\text{NOT SWEET}}$ which connects token2 to $\langle\emptyset, \{\text{SWEET}\}\rangle$ is not normal and hence token2 cannot be regarded as an accurate representation of $\langle\emptyset, \{\text{SWEET}\}\rangle$. This is because the connection falsifies the constraint MELT AT $130°\text{C} \vdash_{\bar{K}_c}$ SWEET of knowledge $\bar{K}_c$. In other words, while Chem recognizes that token2 is "SWEET" through his observation and knowledge, the connection $\text{token2}_{\text{NOT SWEET}}$ contradicts with this Chem's perception. Thus the connection $\text{token2}_{\text{NOT SWEET}}$ is excluded from normal tokens and so the representation which is mediated by this connection is not regarded as accurate.

Let us next turn to token1. The connections which connect token1 to the tokens of the desire are all normal. Token1, thus, is an accurate representation of both $\langle\{\text{SWEET}\}, \emptyset\rangle$ and $\langle\emptyset, \{\text{SWEET}\}\rangle$. But, at the same time, token1 cannot represent $\langle\{\text{SWEET}\}, \emptyset\rangle$ as satisfying the desire "want sweet stuff," since the sequent $\text{typ}(\text{token1}) \nvdash_{\bar{K}_c} \text{SWEET}, \text{typ}^c(\text{token1})$ about token1 is consistent with the knowledge $\bar{K}_c$ and, thus, the condition 3. of the definition 21 cannot be satisfied. On the other hand, however, the sequent being consistent with the knowledge $\bar{K}_c$ makes token1 to be an accurate representation of $\langle\emptyset, \{\text{SWEET}\}\rangle$, a token of the desire classified as "not sweet." Since token1 does not represent the desire "want sweet stuff," it can also represent accurately the token $\langle\emptyset, \{\text{SWEET}\}\rangle$, which does not satisfy the desire "want sweet stuff." That is, unlike token2, token1 can also be an accurate representation of the token which does not satisfy the desire "want sweet stuff," by being incapable of satisfying the desire.

When a representation represents some token of a target as satisfying some desire, it cannot represent generally any token which does not satisfy the desire. In fact, the reason why token2 cannot

[17] The classification C_{c1}, constructed above, can be given generally by the *sum* of two different classifications A and B, that is, $A + B$. (Barwise and Seligman (1997), Definition 5.1.) The classification A_1, on the other hand, can be given by the sum of their *quotient* classifications (Barwise and Seligman (1997), Example 5.12), that is, $\text{Sep}(A) + \text{Sep}(B)$. In this case, these two sums, C_{c1} and A_1, coincides with each other since O_c and $\text{Cla}^*(D)$ don't have any pair of tokens satisfying $\text{typ}(a) = \text{typ}(b)$. In general, however, A_1 contains less tokens than C_{c1}.

be an accurate representation of the token $\langle \emptyset, \{\text{SWEET}\}\rangle$ is that token2 represents $\langle \{\text{SWEET}\}, \emptyset\rangle$ as satisfying the desire "want sweet stuff." Therefore, when representations, conversely, don't represent any token as satisfying the desire, they can represent accurately every token of the desire. In other words, when we cannot find out what these representations represent, we cannot accurately represent any token of the desire as satisfying them in a positive manner by these representations, but the very possibility itself to satisfy them cannot be rejected.[18]

Now let us look more closely into our representation constructed above and check whether representation system $\mathcal{R}_{c1}$ fits our purpose to describe these functions of our own perception and is plausible. We can regard token1 as appropriate, since we may sometimes confront with particulars which we cannot judge whether they are sweet or not. Token2, on the other hand, may seem a little bit bizarre. We would rarely conscious of the possibility of some particular being "not sweet" if we deem it as "sweet." However, $\mathcal{R}_{c1}$ implies the situation in which the agent denies the possibility of being "not sweet" after she realizes that. We think it unrealistic, since we may think the possibility only when we are not sure whether it is "sweet" or "not," like token1 but not token2.

The reason why this representation system contains such a connection is that the core C_{c1} of the representation system is formed without any regard for constraints of the knowledge. We can actually confirm that the representation system is formed without any regard to the knowledge from the fact that the core of the representation system consists only of the set of the tokens $\Omega^{R}_{\langle D, O_c\rangle}$, the set of realizable states which contains all the realizable states regardless of the construction of the knowledge. This is the source of the problem that makes a representation system $\mathcal{R}_{c1}$ unreal.

So, let us next try to form another core reflecting the construction of the knowledge and, thus, not including tokens such as "token$2_{\text{NOT SWEET}}$." We first define below *possible and realizable state* as all possible partitions of the set of types of given observations and desires, satisfying all the constraints of the knowledge.

Definition 23 (Barwise (1997))**.** Let $A_1, \ldots, A_n$ be sets of types, classifications or theories. Given $A_1, \ldots, A_n$ and a regular theory T, the set

$$\Omega^{IP}_{\langle A_1,\ldots,A_n \mid T\rangle} = \{\langle \Theta, \Lambda\rangle \in \Omega_{\langle A_1,\ldots,A_n\rangle} \mid \langle \Theta, \Lambda\rangle \in \vdash_T\}$$

is said as the set of *impossible states* under the theory T. Moreover, the set

$$\Omega^{P}_{\langle A_1,\ldots,A_n \mid T\rangle} = \Omega_{\langle A_1,\ldots,A_n\rangle} \setminus \Omega^{IP}_{\langle A_1,\ldots,A_n \mid T\rangle}$$

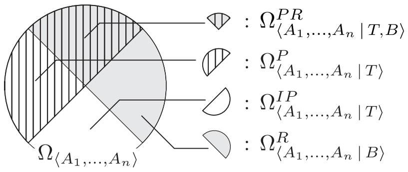
is said as the set of *possible states* under the theory T, which is not impossible under T. Furthermore, given a set of tokens $B \subseteq \bigcup_{i\in\{1,\ldots,n\}} \text{tok}(A_i)$, the set

$$\Omega^{PR}_{\langle A_1,\ldots,A_n \mid T,B\rangle} = \Omega^{P}_{\langle A_1,\ldots,A_n \mid T\rangle} \cap \Omega^{R}_{\langle A_1,\ldots,A_n \mid B\rangle}$$

is said as the set of *possible and realizable states* under the theory T and the tokens of B.[19]

[18] We will discuss this issue in another article as the problem of inability to represent.

[19] The set $\Omega^{R}_{\langle A_1,\ldots,A_n \mid B\rangle}$ of realizable states and the set $\Omega^{P}_{\langle A_1,\ldots,A_n \mid T\rangle}$ of possible states are mutually independent. So, it may happen that the set $\Omega^{PR}_{\langle A_1,\ldots,A_n \mid T,B\rangle}$ of possible and realizable states is empty. The inclusion relation between these sets can be drawn, for instance, on the set $\Omega_{\langle A_1,\ldots,A_n\rangle}$ of all states as follows.



Here, we can form the core of which tokens are all normal, i.e., consistent with the knowledge, by defining a classification of which tokens are possible and realizable states. At this time, we suppose another situation in which Chem found another token "token3," which is also white powder in a square container. Accordingly, Chem's observation O'_c changes as in Table 10. From this observation table, we can see that token3 is the same as token1 except not being melted at 800°C. We assume that Chem's knowledge remains the same as before but his desire changes to $D' = \{\vdash_{D'} \text{SALTY}\}$.

$\vDash_{O'_c}$	WHI	POW	SQU	CIR	130°C	800°C	HEX
token1	1	1	1	0	0	1	1
token2	1	1	0	1	1	0	0
token3	1	1	1	0	0	0	1

Table 10: Chem's observation O'_c

First, let us construct, as in the case of classification C_{c1}, the classification $C_{c21} = \langle \Omega^R_{\langle D', O'_c \rangle}, \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'), \vDash_{C_{c21}} \rangle$, which contains the set of realizable states under the tokens of the observation O'_c, generated by the observation O'_c and the desire D' as the set of tokens. The classification table of C_{c21} is given as below where SAL is an abbreviation of SALTY and NSAL is of NOT SALTY.

$\vDash_{C_{c21}}$	WHI	POW	SQU	CIR	130°C	800°C	HEX	SAL
token1$_{\text{SAL}}$	1	1	1	0	0	1	1	1
token1$_{\text{NSAL}}$	1	1	1	0	0	1	1	0
token2$_{\text{SAL}}$	1	1	0	1	1	0	0	1
token2$_{\text{NSAL}}$	1	1	0	1	1	0	0	0
token3$_{\text{SAL}}$	1	1	1	0	0	0	1	1
token3$_{\text{NSAL}}$	1	1	1	0	0	0	1	0

Table 11: classification table of C_{c21}

This classification C_{c21} contains all possible connections between every token of the observation and desire, as C_{c1} does. On the other hand, classification $C_{c2} = \langle \Omega^{PR}_{\langle D', O'_c \,|\, \bar{K}_c \rangle}, \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'), \vDash_{C_{c2}} \rangle$, which contains as tokens the set $\Omega^{PR}_{\langle D', O'_c \,|\, \bar{K}_c \rangle}$ of states that are possible under the knowledge $\bar{K}_c$ and realizable under the tokens of the observation O'_c generated by the observation O'_c and the desire D', contains less tokens than C_{c21} and might fit our purpose better than C_{c21} to some extent. The classification table of C_{c2} is given as follows.

$\vDash_{C_{c2}}$	WHI	POW	SQU	CIR	130°C	800°C	HEX	SAL
token1$_{\text{SAL}}$	1	1	1	0	0	1	1	1
token2$_{\text{SAL}}$	1	1	0	1	1	0	0	1
token2$_{\text{NSAL}}$	1	1	0	1	1	0	0	0

Table 12: classification table of C_{c2}

As easily seen by the classification table of C_{c2}, unlike C_{c21}, this classification does not contain any connections that connect token1 to $\langle \emptyset, \{\text{SALTY}\} \rangle$ and token3 to any desire. The reason why it does not have the connection between token1 and $\langle \emptyset, \{\text{SALTY}\} \rangle$ is that the constraint MELT AT 800°C $\vdash_{\bar{K}_c}$ SALTY is contained by the knowledge $\bar{K}_c$. We can regard it as the connection reasonably excluded by the knowledge. Contrastingly, all the connections between token3 and the desire are excluded since token3 itself does not satisfy the constraint HEX CUBE $\vdash_{\bar{K}_c}$ MELT AT 800°C of $\bar{K}_c$, and thus all the partitions $\langle \Gamma, \Delta \rangle$ which satisfy typ(token3) $\subseteq \Gamma$ and typ^c(token3) $\subseteq \Delta$ are excluded from the set $\Omega^{PR}_{\langle D', O'_c \mid \bar{K}_c \rangle}$ of possible and realizable states. Despite the fact of having no connection between token3 and the desire, this classification C_{c2} still can be a core of a binary channel between O'_c as a source and $\mathrm{Cla}^*(D')$ as a target. Let us check this.

We start with defining type-identical infomorphisms. As for the infomorphism f_{c2} from O'_c to C_{c2}, we define $\check{f_{c2}}(\mathrm{token1}_{\mathrm{SAL}}) = \mathrm{token1}$ and $\check{f_{c2}}(\mathrm{token2}_{\mathrm{SAL}}) = \check{f_{c2}}(\mathrm{token2}_{\mathrm{NSAL}}) = \mathrm{token2}$. As for the infomorphism g_{c2} from $\mathrm{Cla}^*(D')$ to C_{c2}, we define $\check{g_{c2}}(\mathrm{token1}_{\mathrm{SAL}}) = \check{g_{c2}}(\mathrm{token2}_{\mathrm{SAL}}) = \langle \{\mathrm{SALTY}\}, \emptyset \rangle$ and $\check{g_{c2}}(\mathrm{token2}_{\mathrm{NSAL}}) = \langle \emptyset, \{\mathrm{SALTY}\} \rangle$. It is obvious that these are actually infomorphisms. We define the channel consisting of this pair of infomorphisms as $\mathcal{C}_{c2} = \{f_{c2}, g_{c2}\}$.

We can construct a local logic with the core C_{c2} and the knowledge $\bar{K}_c$ as follows.

$$
\begin{aligned}
&\mathrm{cla}(\mathfrak{L}_{c2}) = C_{c2},\\
&\mathrm{th}(\mathfrak{L}_{c2}) = \bar{K}_c \upharpoonright \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'),\\
&N_{\mathfrak{L}_{c2}} = \{\mathrm{token1}_{\mathrm{SAL}},\ \mathrm{token2}_{\mathrm{SAL}},\ \mathrm{token2}_{\mathrm{NSAL}}\}.
\end{aligned}
$$

By the above, we can construct a representation system $\mathcal{R}_{c2} = \langle \mathcal{C}_{c2}, \mathfrak{L}_{c2} \rangle$, consisting of the local logic $\mathfrak{L}_{c2}$ and the channel $\mathcal{C}_{c2}$, with O'_c as a source and $\mathrm{Cla}^*(D')$ as a target.

Now, let us check again whether this representation system $\mathcal{R}_{c2}$ fits our purpose and is plausible or not. We can regard all connections contained in the core of this channel as plausible, since all connections are normal. A shortcoming of this representation system $\mathcal{R}_{c2}$ is, however, not present in the existing connections but in lacking some necessary connections. That is, it excludes all the connections which falsify some constraints of knowledge from the core.

It should be recalled that in everyday life we often come across goods different from what we have expected. We sometimes regard them as "abnormal" and drop them off from the list of a choice. But we wouldn't always treat them as that way. In fact, we do sometimes buy some goods which is abnormal and unexpected, for what it's worth. So, we don't regard this core which ignores these tokens as plausible and appropriate for describing the function of our perception.

The reason why $\mathcal{R}_{c2}$ does not contain the connections of token3 is that all the connections falsifying at least one constraint of the knowledge are excluded from the core C_{c2} of the representation system. However, as we stated above, representation system that deems some tokens of the observation as impossible and does not connect them to any tokens of desire at all, would not be appropriate. Thus, let us lastly try to form the two different representation systems each of which enables token3 to represent the desire.

Let us start with specifying tokens which are counterexamples to some constraints of knowledge K. These tokens can be specified as the set $AN_{\mathrm{Log}(O,K)}$ of abnormal tokens of the local logic $\mathrm{Log}(O, K)$ generated by the observation O and the knowledge K. So we specify at first the set of abnormal tokens and then specify the set of states realizable with respect to these tokens.

Definition 24. Let an observation O, a regular knowledge K, and a desire D be given. We denote the set of abnormal tokens of the local logic $\mathrm{Log}(O, K)$ generated by O and K by $AN_{\mathrm{Log}(O,K)}$. By definition, $AN_{\mathrm{Log}(O,K)}$ is the set of tokens which falsify some of the constraints of K. The knowledge K which satisfies $AN_{\mathrm{Log}(O,K)} = \emptyset$ is especially called the *sound knowledge with respect to O*.

Next, we construct a set of *unreasonable states* by removing from the set of impossible states under the knowledge K, the states realizable with abnormal tokens of the local logic $\mathrm{Log}(O, K)$.

Definition 25. Let $A_1, \ldots, A_n$ be sets of types, classifications, or theories. Given $A_1, \ldots, A_n$, where A_ℓ is a classification, and a regular theory T, the set

$$\Omega^{U}_{\langle A_1, \ldots, A_n \mid T, A_\ell \rangle} = \Omega^{IP}_{\langle A_1, \ldots, A_n \mid T \rangle} \setminus \Omega^{R}_{\langle A_1, \ldots, A_n \mid AN_{\mathrm{Log}(A_\ell, T)} \rangle}$$

is said as the set of *unreasonable states* under the theory T and the classification A_ℓ.

When $A_1, \ldots, A_n$ contains only one classification A_ℓ, then we may omit A_ℓ, write the set of unreasonable states simply as $\Omega^U_{\langle A_1,\ldots,A_n \,|\, T\rangle}$, and call it simply the set of unreasonable states under T generated by $A_1, \ldots, A_n$.[20]

All of the states generated by the observation O and the desire D are partitions of $\mathrm{typ}(O) \cup \mathrm{typ}(D)$. Moreover, any partition of a given set of types can also be regarded as a sequent. Thus, by regarding each state as a sequent, we can construct a regular knowledge K' which contains all of the unreasonable states as constraints. The knowledge K' constructed as this way, in general, is weaker than K in terms of excluding the constraints that some tokens of the observation falsify. The core constructed by this knowledge K', therefore, can ensure at least one corresponding connection for all tokens of the observation, which was excluded from the core generated by the knowledge K.

Definition 26. Let an observation O, a regular knowledge K, and a desire D be given. Suppose the regular knowledge whose all constraints are constructed from the set of unreasonable states. That is, let $\vdash_T = \Omega^U_{\langle D,O \,|\, K\rangle}$, the set of sequents consisting of unreasonable states, be the theory T and take its regular closure. This $\bar{T}$ is said as the *minimum sound knowledge with observation O generated by K*, written $K^{min}_{\langle D,O \,|\, K\rangle}$. Moreover, the regular knowledge K' that satisfies $K^{min}_{\langle D,O \,|\, K\rangle} \sqsubseteq K' \sqsubseteq K \restriction \mathrm{typ}(O) \cup \mathrm{typ}(D)$ and $AN_{\mathrm{Log}(O,K')} = \emptyset$ is said as *sound knowledge with the observation O which can be generated by K*.

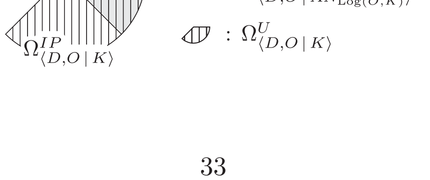
Now, we can construct a core between the observation O and the classification $\mathrm{Cla}^*(D)$ generated by the desire D, with at least one connection for every token of the observation, by the sound knowledge K' with the observation O. Let us construct representation systems between the observation O and the classification $\mathrm{Cla}^*(D)$.

First, we construct a representation system by the knowledge $\bar{K}'_{c3} = K^{min}_{\langle D',O'_c \,|\, \bar{K}_c\rangle}$, which is the minimum sound knowledge $K^{min}_{\langle D',O'_c \,|\, \bar{K}_c\rangle}$ with the observation O'_c generated by $\bar{K}_c$. Recall token3 is the token of the observation O'_c which falsifies the constraints of the knowledge $\bar{K}_c$. Then the states which are realizable with token3, that is, realizable with abnormal tokens of the local logic generated by O'_c and $\bar{K}_c$ are:

$$\langle \mathrm{typ}(\text{token3}), \{\textsc{salty}\} \cup \mathrm{typ}^c(\text{token3})\rangle, \quad \langle \mathrm{typ}(\text{token3}) \cup \{\textsc{salty}\}, \mathrm{typ}^c(\text{token3})\rangle.$$

Thus, we can construct the minimum sound knowledge $K^{min}_{\langle D',O'_c \,|\, \bar{K}_c\rangle}$ with the observation O'_c generated by $\bar{K}_c$, by taking a regular closure of the theory whose constraints are the set $\Omega^U_{\langle D',O'_c|\bar{K}_c\rangle}$ which is obtained by removing $\Omega^R_{\langle D',O'_c \,|\, AN_{\mathrm{Log}(O'_c,\bar{K}_c)}\rangle}$, i.e., the realizable states with token3 above, from $\Omega^{IP}_{\langle D',O'_c \,|\, \bar{K}_c\rangle}$, i.e., the set of all the partitions on $\mathrm{typ}\,(O'_c) \cup \mathrm{typ}\,(D')$ contained by $\bar{K}_c$. This is because we can construct a sound knowledge with the observation about token3, "hexahedral crystal and melt at 800°C," by removing at least one of these two constraints. Then the classification C_{c3}, whose set of tokens is the set $\Omega^{PR}_{\langle D',O'_c \,|\, \bar{K}'_{c3}\rangle}$ of possible and realizable states under the

[20] Suppose the set of states generated by an observation O and a desire D. The set $\Omega^U_{\langle D,O \,|\, K\rangle}$ of unreasonable states can be divided into two parts: the one is the set of the states which are beyond both the observation and the knowledge, the other is the set of the states which are realizable under the observation but excluded from the set of possible states reasonably by works of the knowledge. Relationship between these sets are depicted, for instance, on the set of the impossible states under the knowledge K, that is, $\Omega^{IP}_{\langle D,O \,|\, K\rangle}$, as follows.



knowledge $\bar{K}'_{c3}$ generated by the observation O'_c and the desire D', is given as:

$$C_{c3} = \left\langle \Omega^{PR}_{\langle D', O'_c \,|\, \bar{K}'_{c3} \rangle}, \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'), \models_{C_{c3}} \right\rangle .$$

$\models_{C_{c3}}$	WHI	POW	SQU	CIR	130°C	800°C	HEX	SAL
token1$_{\mathrm{SAL}}$	1	1	1	0	0	1	1	1
token2$_{\mathrm{SAL}}$	1	1	0	1	1	0	0	1
token2$_{\mathrm{NSAL}}$	1	1	0	1	1	0	0	0
token3$_{\mathrm{SAL}}$	1	1	1	0	0	0	1	1
token3$_{\mathrm{NSAL}}$	1	1	1	0	0	0	1	0

Table 13: classification table of core C_{c3}

Let us, next, construct a representation system generated by the knowledge $\bar{K}'_{c4}$, whose set of constraints are constructed by removing only one state

$$\langle \mathrm{typ}(\mathrm{token3}) \cup \{\mathrm{SAL}\}, \mathrm{typ}^c(\mathrm{token3}) \rangle$$

from $\Omega^{IP}_{\langle D', O'_c \,|\, \bar{K}_c \rangle}$ and taking its regular closure. This knowledge $\bar{K}'_{c4}$ is also sound with the observation O'_c, since it satisfies $AN_{\langle D', O'_c, \bar{K}'_{c4} \rangle} = \emptyset$. It is also the knowledge generated by $\bar{K}_c$, since it satisfies $K^{min}_{\langle D', O'_c \,|\, \bar{K}_c \rangle} \sqsubseteq \bar{K}'_{c4} \sqsubseteq \bar{K}_c$ obviously from the way it is constructed. Then the classification C_{c4}, whose set of tokens is the set $\Omega^{PR}_{\langle D', O'_c \,|\, \bar{K}'_{c4} \rangle}$ of possible and realizable states under the knowledge $\bar{K}'_{c4}$ generated by the observation O'_c and the desire D', is given as:

$$C_{c4} = \left\langle \Omega^{PR}_{\langle D', O'_c \,|\, \bar{K}'_{c4} \rangle}, \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'), \models_{C_{c4}} \right\rangle .$$

$\models_{C_{c4}}$	WHI	POW	SQU	CIR	130°C	800°C	HEX	SAL
token1$_{\mathrm{SAL}}$	1	1	1	0	0	1	1	1
token2$_{\mathrm{SAL}}$	1	1	0	1	1	0	0	1
token2$_{\mathrm{NSAL}}$	1	1	0	1	1	0	0	0
token3$_{\mathrm{SAL}}$	1	1	1	0	0	0	1	1

Table 14: classification table of core C_{c4}

Both classifications C_{c3} and C_{c4} we constructed above are the cores of the channels between the observation O'_c and the classification $\mathrm{Cla}^*(D)$ generated by the desire D. In fact, we can construct binary channels between them as the same manner as above, consisting of type-identical infomorphisms. We indicate these channels by $\mathcal{C}_{c3}$ and $\mathcal{C}_{c4}$.

We can also define local logics generated by each core and the knowledge $\bar{K}_c$, as above. We denote these local logics by $\mathfrak{L}_{c3}$ and $\mathfrak{L}_{c4}$ respectively. The local logic $\mathfrak{L}_{c3}$ is given as:

$$\begin{aligned} &\mathrm{cla}(\mathfrak{L}_{c3}) = C_{c3}, \\ &\mathrm{th}(\mathfrak{L}_{c3}) = \bar{K}_c \upharpoonright \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'), \\ &N_{\mathfrak{L}_{c3}} = \{\mathrm{token1_{SAL}},\ \mathrm{token2_{SAL}},\ \mathrm{token2_{NSAL}}\}, \end{aligned}$$

and the local logic $\mathfrak{L}_{c4}$ is given as:

$$\begin{aligned} &\mathrm{cla}(\mathfrak{L}_{c4}) = C_{c4}, \\ &\mathrm{th}(\mathfrak{L}_{c4}) = \bar{K}_c \upharpoonright \mathrm{typ}(O'_c) \cup \mathrm{typ}(D'), \\ &N_{\mathfrak{L}_{c4}} = \{\mathrm{token1_{SAL}},\ \mathrm{token2_{SAL}},\ \mathrm{token2_{NSAL}}\}. \end{aligned}$$

From these channels and local logics we can construct representation systems $\mathcal{R}_{c3} = \langle \mathcal{C}_{c3}, \mathfrak{L}_{c3} \rangle$ and $\mathcal{R}_{c4} = \langle \mathcal{C}_{c4}, \mathfrak{L}_{c4} \rangle$ respectively.

Note that these two local logics are the same except for their classifications. Moreover, these local logics also have the same structure in common with $\mathfrak{L}_{c2}$ except for their classifications. This is because these local logics are constructed from the same knowledge $\bar{K}_c$, and all the tokens satisfying all the constraints of $\bar{K}_c$, i.e., possible and realizable states generated by O'_c and $\mathrm{Cla}^*(D')$, are exhausted by $N_{\mathfrak{L}_{c2}}$. We constructed C_{c3} and C_{c4} not to add more normal tokens but to construct at least one connection to every token of the observation. Thus, it is not surprising that the difference between these local logics appears only in the classification, but an evidence that our purpose is accomplished satisfactorily.

Now, let us once again check whether the representation systems $\mathcal{R}_{c3}$ and $\mathcal{R}_{c4}$ fit our purpose and are plausible. The cores of the channels of these representation systems, being different from that of $\mathcal{R}_{c2}$, contain connections which are not normal: "token3_{SAL}" and "token3_{NSAL}" of $\mathcal{R}_{c3}$ and "token3_{SAL}" of $\mathcal{R}_{c4}$. It is these connections that enable token3 to represent some tokens of the desire, while they are excluded from the representation system $\mathcal{R}_{c2}$. In fact, token3 is the representation of the tokens satisfying both desire "want salty stuff" and "not want salty stuff" in $\mathcal{R}_{c3}$, and is the representation of a token satisfying the desire "want salty stuff" in $\mathcal{R}_{c4}$. Of course, these representations are not accurate, since token3 per se falsifies constraints of the knowledge. However, we may conclude that these representation systems appropriately model our common behavior such as purchasing "unknown" goods, in a sense that they enable every representation to represent some tokens of desire in some way.

Recall here that we supposed two different knowledges $\bar{K}'_{c3}$ and $\bar{K}'_{c4}$ in constructing the classifications C_{c3} and C_{c4}. These two knowledges are both sound with the observation O'_c because they are constructed by excluding all (or some of) the constraints which token3 falsifies. It is this exclusion of these constraints and resulting soundness of these knowledges that enable the cores to contain the connections of token3. Of course, as well as $\bar{K}'_{c3}$ and $\bar{K}'_{c4}$, we can construct another sound knowledge with O'_c by removing only one state below from $\Omega^{IP}_{\langle D', O'_c \mid \bar{K}_c \rangle}$:

$$\langle \mathrm{typ}(\mathrm{token3}), \{\mathrm{SAL}\} \cup \mathrm{typ}^c(\mathrm{token3}) \rangle.$$

In general, there always exist multiple sound knowledges with each observation. This is because the observation does not determine the sound knowledge with it uniquely. To see this multiplicity, for instance, let us suppose a situation in which an observation containing a token a which falsifies some constraints of a knowledge is given. This token a of the observation falsifies the constraint $\mathrm{typ}(a) \vdash \mathrm{typ}^c(a)$ of the knowledge. To construct a sound knowledge with the observation, therefore, we must make it satisfy $\mathrm{typ}(a), \Gamma \nvdash \Delta, \mathrm{typ}^c(a)$ for at least one partition $\langle \Gamma, \Delta \rangle$ of the types of the desire.[21] However, the observation per se does not determine the partition to satisfy $\mathrm{typ}(a), \Gamma \nvdash \Delta, \mathrm{typ}^c(a)$, and besides there always exist multiple partitions for any set of types generally. Thus, we can conclude that there always exist multiple sound knowledges with any observation.

From the fact that there always exist multiple sound knowledges with any observation, we can conclude that representation system is not always uniquely determined even if we followed exactly the way we have described so far. That is, whenever we observe tokens falsifying some constraints of the knowledge, there always remain multiple interpretations about what these tokens represent. This is, then, what we have just examined here through the examples of the representation systems $\mathcal{R}_{c3}$ and $\mathcal{R}_{c4}$.

We do not need, however, to think of this multiplicity of the representation systems as a shortcoming. Firstly, because every representation system distinguishes these kinds of tokens as something beyond their knowledge. Secondly, because we do not need to regard it for representation

[21] If the knowledge satisfies $\mathrm{typ}(a), \Gamma \vdash \Delta, \mathrm{typ}^c(a)$ for all partitions $\langle \Gamma, \Delta \rangle$, it has to also satisfy $\mathrm{typ}(a) \vdash \mathrm{typ}^c(a)$ by Global Cut.

system's role to decide how to represent such tokens. It is rather a role of "learning," which in some way reflects dynamic nature of knowledge, to determine how to represent the tokens of which existence we do not even think at all until facing them. Probably, it would be a common situation in which different people study different things from the same observation. Some might think, for example, cautiously token3 as uncertain whether it is "salty" or "not salty." Some might think it as "salty" since token3 satisfies a constraint "HEX CRYSTAL $\vdash_{\bar{K}_c}$ SALTY." Accordingly, each sound knowledge with observation O'_c, such as $\bar{K}'_{c3}$ or $\bar{K}'_{c4}$, may be constructed by each corresponding judgement. We can think of this multiplicity of knowledge and representation system obtained by learning as a key to capture differences between our manners of learning.[22]

Now, let us call the classification constructed above as the *cognizance classification* and define as follows.

Definition 27. Let an observation O, a desire D, and a regular knowledge K be given. Let K' be a sound knowledge with the observation O generated by K, and let $\Omega^{PR}_{\langle D,O\,|\,K'\rangle}$ be a set of possible and realizable states under the knowledge K' generated by the observation O and the desire D. The *cognizance classification* $C_{\langle D,O,K'\rangle}$ is the classification generated by O, K', and D as:

$$C_{\langle D,O,K'\rangle} = \left\langle \Omega^{PR}_{\langle D,O\,|\,K'\rangle},\ \mathrm{typ}(O) \cup \mathrm{typ}(D),\ \models_{C_{\langle D,O,K'\rangle}} \right\rangle$$

where $\models_{C_{\langle D,O,K'\rangle}}$ is defined as:

$$\langle \Theta, \Lambda \rangle \models_{C_{\langle D,O,K'\rangle}} \alpha \qquad \text{iff} \qquad \alpha \in \Theta.$$

Every cognizance classification is a core of channel between the observation O and the classification of desire $\mathrm{Cla}^*(D)$. Let us next examine it.

Proposition 10. *Let an observation O, a desire D, and a regular knowledge K be given. Let K' be a sound knowledge with the observation O generated by the knowledge K. Then there always exists a channel between O and $\mathrm{Cla}^*(D)$ with the cognizance classification $C_{\langle D,O,K'\rangle}$ as the core and the type-identical infomorphisms. This channel is especially called* cognizance channel*, written $\mathcal{C}_{\langle D,O,K'\rangle}$.*

proof. Construct infomorphisms as follows: let f be a type-identical infomorphism from O to $C_{\langle D,O,K'\rangle}$ and $f^{\check{}}(\langle \Theta, \Lambda \rangle) = a$ for each $\langle \Theta, \Lambda \rangle \in \mathrm{tok}(C_{\langle D,O,K'\rangle})$, where $a \in \mathrm{tok}(O)$ satisfies $\mathrm{typ}(a) \subseteq \Theta$ and $\mathrm{typ}^c(a) \subseteq \Lambda$. By the definition of $C_{\langle D,O,K'\rangle}$, f satisfies the biconditional properties of infomorphism, and such token $a \in \mathrm{tok}(O)$ always exists for every token of cognizance classification clearly by the definition of the set $\Omega^{PR}_{\langle D,O\,|\,K'\rangle}$ of possible and realizable states.

Next, let g be a type-identical infomorphism from $\mathrm{Cla}^*(D)$ to $C_{\langle D,O,K'\rangle}$ and $g^{\check{}}(\langle \Theta, \Lambda \rangle) = \langle \Gamma, \Delta \rangle$ for each $\langle \Theta, \Lambda \rangle \in \mathrm{tok}(C_{\langle D,O,K'\rangle})$, where $\langle \Gamma, \Delta \rangle \in \mathrm{tok}(Cla^*(D))$ satisfies $\Gamma \subseteq \Theta$ and $\Delta \subseteq \Lambda$. By the definition of $C_{\langle D,O,K'\rangle}$, g satisfies the biconditional properties of infomorphism, and such token $\langle \Gamma, \Delta \rangle \in \mathrm{tok}(\mathrm{Cla}^*(D))$ always exists for every token of cognizance classification clearly by the definition of the classification $\mathrm{Cla}^*(D)$. $\square$

Let us next construct a local logic with the cognizance classification $C_{\langle D,O,K'\rangle}$ and the knowledge K, as follows.

[22] Note that the constraint "HEX CUBE $\vdash_{\bar{K}_c}$ MELT AT 800°C" of the knowledge $\bar{K}_c$ is not valid, in general. In fact, there exists at least one matter which is typed exactly the same way as token3, that is ammonium chloride. Ammonium chloride has almost the same feature in common with sodium chloride, white or colorless crystal cube etc..., except for that it sublimates at 337.8°C. Ammonium chloride is used as fertilizer and is edible too, but tastes bitter, not salty. Salmiakki, a Finnish candy, contains ammonium chloride. Salmiakki is popular in Nordic and northern European countries, but is also notorious for its "worst taste." The resident of this room might be a Finnish. Anyway, it seems certain that curiosity leads Chem to trouble.

Definition 28. Let an observation O, a regular knowledge K, and a desire D be given. Let K' be a sound knowledge with the observation O generated by K. The *local logic on the cognizance classification* $C_{\langle D,O,K'\rangle}$, written $\mathrm{Log}(C_{\langle D,O,K'\rangle}, K)$, is the local logic generated with cognizance classification $C_{\langle D,O,K'\rangle}$ and the knowledge K, given as:

$$\mathrm{cla}(\mathrm{Log}(C_{\langle D,O,K'\rangle}, K)) = C_{\langle D,O,K'\rangle},$$
$$\mathrm{th}(\mathrm{Log}(C_{\langle D,O,K'\rangle}, K)) = K \restriction \mathrm{typ}(C_{\langle D,O,K'\rangle}),$$

and the normal tokens of $\mathrm{Log}(C_{\langle D,O,K'\rangle}, K)$, written $N_{\mathrm{Log}(C_{\langle D,O,K'\rangle},K)}$, is the set of the tokens which satisfy all the constraints of $\mathrm{th}(\mathrm{Log}(C_{\langle D,O,K'\rangle}, K))$.

With this definition, we can define one representation system, written $\mathcal{R}_{\langle D,O,K,K'\rangle}$, by an observation O, knowledges K, K', and a desire D.

Definition 29. Let an observation O, a regular knowledge K, and a desire D be given. Let K' be a sound knowledge with the observation O generated by K. A representation system $\mathcal{R}_{\langle D,O,K,K'\rangle}$ is a pair of the cognizance channel $\mathcal{C}_{\langle D,O,K'\rangle}$ and the local logic $\mathrm{Log}(C_{\langle D,O,K'\rangle}, K)$, that is,

$$\mathcal{R}_{\langle D,O,K,K'\rangle} = \langle \mathcal{C}_{\langle D,O,K'\rangle}, \mathrm{Log}(C_{\langle D,O,K'\rangle}, K)\rangle,$$

where the core of cognizance channel is the cognizance classification $C_{\langle D,O,K'\rangle}$ generated by the observation O, the classification $\mathrm{Cla}^*(D)$ of the desire D, and the knowledge K'. We call it as a *representation system representing a perception* generated by an observation O, knowledges K, K', and a desire D. When $K' = K$, we may omit K' and write it simply as $\mathcal{R}_{\langle D,O,K\rangle}$.

Finally, let us define preference relations between these representations of the representation system. Recall that representation systems are binary channels equipped with local logics. So the preference relations between representations can be defined by preference relations constructed by its channel of the representation system. Let us denote preference relations constructed by a channel of a representation system especially as follows.

Definition 30. Let a representation system $\mathcal{R}_{\langle D,O,K,K'\rangle}$ and a family of desires $\mathcal{F}(D)$ of D be given. Then a preference relation between representations $a, b \in \mathrm{tok}(O)$ of the representation system is defined as:

$$b \precsim_{\langle \mathcal{F}(D), \mathcal{R}_{\langle D,O,K,K'\rangle}\rangle} a \quad \text{iff} \quad \mathcal{F}_{b,\mathcal{C}_{\langle D,O,K'\rangle}}(D) \subseteq \mathcal{F}_{a,\mathcal{C}_{\langle D,O,K'\rangle}}(D).$$

The same preference relation can be obtained by different and easier way with a family of desires which will be defined below. Let us begin with defining the family of desires which each token of an observation O satisfies under a knowledge K.

Definition 31. Let an observation O, a regular knowledge K, and a desire D be given. Take arbitrarily a token $a \in \mathrm{tok}(O)$ of the observation O and fix it. The token a is said to satisfy the desire D under the knowledge K if for any constraint $\Gamma \vdash_D \Delta$ of the desire D, the knowledge K contains a constraint $\mathrm{typ}(a), \Gamma \vdash_K \Delta, \mathrm{typ}^c(a)$. The family of desires with respect to $\mathcal{F}(D)$ and a token a defined below is said as the *family of the desires which the token a satisfies under a knowledge* K, written $\mathcal{F}_{a,O,K}(D)$.

$$\mathcal{F}_{a,O,K}(D) = \{D_i \in \mathcal{F}(D) \mid \mathrm{typ}(a), \Gamma \vdash_K \Delta, \mathrm{typ}^c(a) \quad \text{for all } \Gamma \vdash_{D_i} \Delta\}.$$

A family of desires which a token of an observation O satisfies under a knowledge K coincides with a family of desires which the token satisfies through the channel with a cognizance classification as its core. Let us examine this relationship.

Proposition 11. *Let an observation O, a regular knowledge K, a sound knowledge K' with the observation O generated by K, and a family of desires $\mathcal{F}(D)$ of D be given. Then the following relation holds.*

$$\mathcal{F}_{a,O,K'}(D) = \mathcal{F}_{a,\mathcal{C}_{\langle D,O,K'\rangle}}(D).$$

The LHS is the family of desires that a token $a \in \mathrm{tok}(O)$ of the observation satisfies under the sound knowledge K' with O generated by K. On the other hand, the RHS is the family of desires $\mathcal{F}_{a,\mathcal{C}_{\langle D,O,K'\rangle}}(D)$ that a representation a satisfies under the cognizance channel.

proof. $D_i \in \mathcal{F}_{a,O,K'}(D)$ coincides with $\mathrm{typ}(a), \Gamma \vdash_{K'} \Delta, \mathrm{typ}^c(a)$ for all constraints $\langle \Gamma, \Delta\rangle$ of D_i, by definition. By the definition of $\Omega^{PR}_{\langle D,O\,|\,K'\rangle}$, the knowledge K' containing the constraint $\mathrm{typ}(a), \Gamma \vdash_{K'} \Delta, \mathrm{typ}^c(a)$ coincides with $\Omega^{PR}_{\langle D,O\,|\,K'\rangle}$ not containing the state $\langle \mathrm{typ}(a) \cup \Gamma, \Delta \cup \mathrm{typ}^c(a)\rangle$. $\Omega^{PR}_{\langle D,O\,|\,K'\rangle}$ not containing the state $\langle \mathrm{typ}(a) \cup \Gamma, \Delta \cup \mathrm{typ}^c(a)\rangle$ coincides with all tokens $\langle \Theta, \Lambda\rangle$ of the cognizance classification, each of which satisfies $f^{\check{}}(\langle \Theta, \Lambda\rangle) = a$, where $f : O \rightleftarrows C_{\langle D,O,K'\rangle}$ is the infomorphism of the cognizance channel $\mathcal{C}_{\langle D,O,K'\rangle} = \{f, g\}$ of the representation system $\mathcal{R}_{\langle D,O,K,K'\rangle}$, satisfy all the constraints $\Gamma \vdash_{K'} \Delta$ of the desire D_i. This coincides with $D_i \in \mathcal{F}_{a,\mathcal{C}_{\langle D,O,K'\rangle}}(D)$, by definition. □

By this proposition, we can construct a preference relation between representations of a representation system, easily by an observation O and a sound knowledge K' with the observation O generated by a knowledge K, as follows.

Corollary 1. *Let an observation O, a regular knowledge K, and a family of desires $\mathcal{F}(D)$ of D be given. Let K' be a sound knowledge with the observation O generated by K. Then, by Proposition 11, the following relation holds.*

$$a \precsim_{\langle \mathcal{F}(D), \mathcal{R}_{\langle D,O,K,K'\rangle}\rangle} b \quad \textit{iff} \quad \mathcal{F}_{a,O,K'}(D) \subseteq \mathcal{F}_{b,O,K'}(D).$$

Let us now ask ourselves what it means that a preference relation between representations constructed on a representation system can be defined only by an observation and a knowledge. Recall that a representation satisfying a desire coincides with its cognizance classification not including the corresponding connections which do not satisfy the desire at all. Furthermore, the cognizance classification not including the connections which do not satisfy the desire coincides with the knowledge including the constraints which correspond to the connections. From these arguments, the cognizance classification can be considered as the classification which preserves the structure of the knowledge in the sense that all the states regarded as impossible by the knowledge are excluded from its set of tokens. In other words, we can regard the cognizance classification as the classification which represents the structure of the knowledge explicitly with its set of tokens. Therefore, we can think that every state in which some desire is regarded as not satisfied by some representation through the cognizance channel is already contained in the knowledge as a constraint. This is why the preference relation between any representation system can be defined solely by a knowledge and an observation.

Let us, finally, go back to Chem's case and construct preference relations on two different representation systems $\mathcal{R}_{c3}$ and $\mathcal{R}_{c4}$. We start with the representation system $\mathcal{R}_{c3}$. The sound knowledge, which we used to construct $\mathcal{R}_{c3}$, with the observation O'_c generated by $\bar{K}_c$ is $\bar{K}'_{c3}$. So, the set of the desires which each representation satisfies can be given as:

$$\begin{aligned}
\mathcal{F}_{\mathrm{token1},O'_c,\bar{K}'_{c3}}(D') &= \{D'\},\\
\mathcal{F}_{\mathrm{token2},O'_c,\bar{K}'_{c3}}(D') &= \emptyset,\\
\mathcal{F}_{\mathrm{token3},O'_c,\bar{K}'_{c3}}(D') &= \emptyset.
\end{aligned}$$

Let us first look at the accurate representations. We can see that these representations can satisfy all the desires which are represented as satisfied, as we expect. As for the non-accurate representations, however, it seems a little bit strange. Recall that token3 represents two contradictory tokens of the desire, that is, one as satisfying the desire “want salty thing” and the other as satisfying the desire “not want salty thing” at the same time. Since token3 is regarded by the knowledge $\bar{K}_c$ as what should not exist, the knowledge $\bar{K}_c$ contains following contradictory constraints $\mathrm{typ}(\mathrm{token3}), \textsc{salty} \vdash_{\bar{K}_c} \mathrm{typ}^c(\mathrm{token3})$ and $\mathrm{typ}(\mathrm{token3}) \vdash_{\bar{K}_c} \textsc{salty}, \mathrm{typ}^c(\mathrm{token3})$. Therefore, token3 has to satisfy the desire D', but it is not contained in the set of the desires which token3 satisfies. Why this happened?

This is because when we construct the set of the desires which token3 satisfies, we do it by making use of the sound knowledge $\bar{K}'_{c3}$ with the observation O'_c, which we also used in constructing the cognizance classification C_{c3}. The knowledge $\bar{K}'_{c3}$ is generated by excluding just the two constraints, that is, $\mathrm{typ}(\mathrm{token3}), \textsc{salty} \vdash_{\bar{K}_c} \mathrm{typ}^c(\mathrm{token3})$ and $\mathrm{typ}(\mathrm{token3}) \vdash_{\bar{K}_c} \textsc{salty}, \mathrm{typ}^c(\mathrm{token3})$, of knowledge $\bar{K}_c$. This is why the set of the desires which token3 satisfies does not include the desire D'. Recall here that $\bar{K}'_{c3}$ enables token3 to have corresponding connections of the cognizance classification C_{c3} and to represent tokens of the desire, just by excluding these constraints. Thus, as long as token3 works as a representation, it would be reasonable to use the knowledge $\bar{K}'_{c3}$ to determine the desires to be satisfied by token3. We should think the situation in $\mathcal{R}_{c3}$, where token3 is regarded as the representation satisfying multiple contradictory desires, as only the evidence that it does not work as an accurate representation.

The preference relation between representations of $\mathcal{R}_{c3}$ can be constructed as:

$$\mathrm{token2} \prec_{\langle D', \mathcal{R}_{c3} \rangle} \mathrm{token1},$$
$$\mathrm{token3} \prec_{\langle D', \mathcal{R}_{c3} \rangle} \mathrm{token1},$$
$$\mathrm{token2} \sim_{\langle D', \mathcal{R}_{c3} \rangle} \mathrm{token3}.$$

Similarly, we can construct a preference relation on the representation system $\mathcal{R}_{c4}$. The sound knowledge, which we used to construct $\mathcal{R}_{c4}$, with the observation O'_c generated by $\bar{K}_c$ is $\bar{K}'_{c4}$. So, the set of the desires which each representation satisfies can be given as:

$$\begin{aligned}
\mathcal{F}_{\mathrm{token1}, O'_c, \bar{K}'_{c4}}(D') &= \{D'\}, \\
\mathcal{F}_{\mathrm{token2}, O'_c, \bar{K}'_{c4}}(D') &= \emptyset, \\
\mathcal{F}_{\mathrm{token3}, O'_c, \bar{K}'_{c4}}(D') &= \{D'\}.
\end{aligned}$$

The preference relation between representations of $\mathcal{R}_{c4}$ can be constructed as:

$$\mathrm{token1} \sim_{\langle D', \mathcal{R}_{c4} \rangle} \mathrm{token3},$$
$$\mathrm{token2} \prec_{\langle D', \mathcal{R}_{c4} \rangle} \mathrm{token1},$$
$$\mathrm{token2} \prec_{\langle D', \mathcal{R}_{c4} \rangle} \mathrm{token3}.$$

2.7 Interchangeability of Preference

In the last of this section, we take a closer look at the relationship between preference relations on types, on tokens, and on representations.

We have constructed preference relations so far under three independent schemes, i.e., theory, classification, and representation system. The first one is on types constructed by knowledge. The second one is on tokens constructed by observation. The third one is on representations constructed by representation system. A preference relation by knowledge is constructed based only on the types of the objects without observing them directly. In mail-order or online shopping, say, we choose articles only by referring to catalogs, and it would be a typical scene where we construct a preference relation by knowledge. Contrastingly, in ordinary shopping, we pick up the stuffs firsthand and examine which one is best for us, and it would be a typical scene where we construct a preference relation by observation. Moreover, when we cannot observe what desires these stuffs can satisfy, we construct a preference relation by representation system which enables us to detect it. Then, what is the relationship between preference relations on types, on tokens, and on representations? In this last subsection, we investigate this relationship by way of "local logic" and "name."

As we have already seen, local logic is a conception that makes it possible to capture the relationship between classification and theory by soundness and completeness. That is, if a classification and a theory constitute a sound and complete local logic, we regard the classification and the theory as having a proper correspondence. Therefore, by checking local logic consisting of observation and knowledge, we can determine the relationship between preference relations constructed by observation, knowledge and representation system.

Name, on the other hand, is a conception that functions to fix the relationship between types and tokens and thus enables us to compare between preference relations on types, on tokens, and on representations. Type and token are originally distinct conceptions and hence the relationship between them is not fixed a priori. This leads to the necessity of the concept which fixes it and indicates each token itself. "Name" is introduced for this reason. To begin with, we define for arbitrary classification A the *named classification* $A^\dagger$ as follows.

Definition 32. Let a classification A be given. The classification which is made as follows is called *named classification* of A, written $A^\dagger$.

$$A^\dagger = \langle \mathrm{tok}(A), \mathrm{typ}(A) \cup \mathrm{tok}(A), \vDash_{A^\dagger} \rangle$$

where $\vDash_{A^\dagger}$ is defined as follows. For any $a \in \mathrm{tok}(A)$, if a type $\alpha \in \mathrm{typ}(A)$,

$$a \vDash_{A^\dagger} \alpha \qquad \text{iff} \qquad a \vDash_A \alpha,$$

and if $\alpha \in \mathrm{tok}(A)$,

$$a \vDash_{A^\dagger} \alpha \qquad \text{iff} \qquad a = \alpha.$$

Moreover, a theory $T = \langle \Sigma, \vdash_T \rangle$ which contains the set of the names of classification A, that is, $\mathrm{tok}(A) \subseteq \Sigma$, is said as a *theory with the names of* A. We call $\mathrm{typ}(O^\dagger) \setminus \mathrm{typ}(O)$ as a set of names.

Introducing name to classification enables us to comprehend the relationship between classification and theory much easier since it makes it easier to describe by theory how tokens are classified. In other words, replacing $\vDash$, the binary relation between types and tokens on classification, by $\vdash$, the binary relation between types on theory, enables us to construct a theory which constitutes a sound and complete local logic with the classification. The following proposition states this.

Proposition 12. *Let an observation O be given. Then the local logic $Log(O^\dagger, \bar{K})$, consisting of the named classification $O^\dagger$ and the regular closure of a knowledge K with the name of O, is sound and complete, where $K = \langle typ(O^\dagger), \vdash_K \rangle$ is defined as follows. For any pair of $a \in tok(O^\dagger)$ and $\alpha \in typ(O^\dagger)$, it holds that*

$$\begin{aligned} &a \vdash_K \alpha \quad \textit{iff} \quad a \vDash_{O^\dagger} \alpha, \\ &a, \alpha \vdash_K \quad \textit{iff} \quad a \nvDash_{O^\dagger} \alpha, \\ &\textit{and} \\ &\vdash_K tok(O^\dagger). \end{aligned}$$

proof. (**completeness**) Suppose all tokens of $\mathrm{tok}(O^\dagger)$ satisfy $\langle \Gamma, \Delta \rangle$.

Let $X = \{a \in \mathrm{tok}(O^\dagger) \,|\, \forall \alpha \in \Gamma,\ a \vDash_{O^\dagger} \alpha\}$. Then $a \in X$ satisfies $a \vDash_{O^\dagger} \beta$ for some $\beta \in \Delta$. By the definition of $\vdash_{\bar{K}}$, it follows that $a \vdash_{\bar{K}} \beta$. By Weakening, $\Gamma, a \vdash_{\bar{K}} \Delta$ holds.

Let $Y = \{a \in \mathrm{tok}(O^\dagger) \mid \exists \alpha \in \Gamma,\ a \nvDash_{O^\dagger} \alpha\}$. Then $a \in Y$ satisfies $a \nvDash_{O^\dagger} \alpha$ for some $\alpha \in \Gamma$. By the definition of $\vdash_{\bar{K}}$, it follows that $\alpha, a \vdash_{\bar{K}}$. By Weakening, $\Gamma, a \vdash_{\bar{K}} \Delta$ holds.

Clearly, $X \cup Y = \mathrm{tok}(O^\dagger)$. Thus $\Gamma, a \vdash_{\bar{K}} \Delta$ holds for all $a \in \mathrm{tok}(O^\dagger)$. Furthermore, by the definition of $\vdash_{\bar{K}}$, it holds that $\vdash_{\bar{K}} \mathrm{tok}(O^\dagger)$. By Weakening, $\Gamma \vdash_{\bar{K}} \Delta, \mathrm{tok}(O^\dagger)$ holds.

One can confirm that for any partition $\langle \Theta, \Lambda \rangle$ of $\mathrm{tok}(O^\dagger)$, it always holds that $\Gamma, \Theta \vdash_{\bar{K}} \Lambda, \Delta$. Since, for any $a \in \Theta$, there exists a constraint $\Gamma, a \vdash_{\bar{K}} \Delta$ and by Weakening, $\Gamma, \Theta \vdash_{\bar{K}} \Lambda, \Delta$ follows. Even if $\Theta = \emptyset$, clearly, $\Gamma \vdash_{\bar{K}} \Delta, \mathrm{tok}(O^\dagger)$ holds. As a result, by Global Cut, $\Gamma \vdash_{\bar{K}} \Delta$ holds.

(**soundness**) Suppose $\Gamma \vdash_{\bar{K}} \Delta$ is a constraint of the knowledge $\bar{K}$.

Suppose there exists some $a \in \mathrm{tok}(O^\dagger)$ which does not satisfy $\langle \Gamma, \Delta \rangle$. In other words, for all $\alpha \in \Gamma$, $a \vDash_{O^\dagger} \alpha$ and for all $\beta \in \Delta$, $a \nvDash_{O^\dagger} \beta$.

Then for any partition $\langle \Theta, \Lambda \rangle$ of $\Gamma \cup \Delta$, it always holds that $a, \Theta \vdash_{\bar{K}} \Lambda$. For if $\Theta \setminus \Gamma \neq \emptyset$, there exists $\beta \in \Theta$ such that $\beta \in \Delta$. Since this β is an element of Δ, it satisfies $a \nvDash_{O^\dagger} \beta$ and so, by the definition of $\vdash_{\bar{K}}$, also satisfies $a, \beta \vdash_{\bar{K}}$. By Weakening, $a, \Theta \vdash_{\bar{K}} \Lambda$ follows. Next, if $\Lambda \setminus \Delta \neq \emptyset$, there exists $\alpha \in \Lambda$ such that $\alpha \in \Gamma$. Since this α is an element of Γ, it satisfies $a \vDash_{O^\dagger} \alpha$ and so, by the definition of $\vdash_{\bar{K}}$, also satisfies $a \vdash_{\bar{K}} \alpha$. By Weakening, $a, \Theta \vdash_{\bar{K}} \Lambda$ follows. Moreover, if $\Theta \setminus \Gamma = \Lambda \setminus \Delta = \emptyset$, that is, $\Theta = \Gamma$, by the supposition and Weakening, $a, \Gamma \vdash_{\bar{K}} \Delta$ holds. Finally, by Global Cut, $a \vdash_{\bar{K}}$ holds but this contradicts the definition of the knowledge $\bar{K}$. □

Consequently, with respect to a name $a \in \mathrm{typ}(O^\dagger)$ of the observation $O^\dagger$, we find that the following relationships hold between $\mathcal{F}_{a,K}(D)$, $\mathcal{F}_{a,O^\dagger}(D)$, and $\mathcal{F}_{a,\mathcal{R}_{\langle D,O^\dagger,K\rangle}}(D)$, the families of desires which a satisfies under a knowledge K, under observation $O^\dagger$, and under a representation system $\mathcal{R}_{\langle D,O^\dagger,K\rangle}$ respectively.

Proposition 13. *Let an observation O, a regular knowledge K, a desire D, and a family of desires $\mathcal{F}(D)$ of D be given. Moreover, suppose the local logic* $\mathrm{Log}(O,K)$ *consisting of O and K is sound and complete.*

1. *If* $\mathrm{typ}(D) \subseteq \mathrm{typ}(O)$, *then it holds that*

$$\mathcal{F}_{a,O}(D) = \mathcal{F}_{a,\mathcal{R}_{\langle D,O,K\rangle}}(D).$$

2. *If the knowledge K contains the names of the observation O and, additionally,* $\mathrm{Log}(O^\dagger,K)$ *is sound and complete, then it holds that*

$$\mathcal{F}_{a,K}(D) = \mathcal{F}_{a,\mathcal{R}_{\langle D,O^\dagger,K\rangle}}(D).$$

3. *If both 1. and 2. are satisfied, then it holds that*

$$\mathcal{F}_{a,O^\dagger}(D) = \mathcal{F}_{a,K}(D) = \mathcal{F}_{a,\mathcal{R}_{\langle D,O^\dagger,K\rangle}}(D).$$

proof. (**To show 1.**) Suppose a token $a \in \mathrm{tok}(O)$ satisfies a desire $D_i \in \mathcal{F}(D)$. Since $b \nvDash_O a$ for all $b \neq a$, every sequent $\langle \{a\} \cup \Gamma_i, \Delta_i \rangle$ for each constraint $\Gamma_i \vdash_{D_i} \Delta_i$ of D_i is satisfied not only by a but any token b. Since $\mathrm{Log}(O,K)$ is complete, K contains constraints such that $a, \Gamma_i \vdash_K \Delta_i$ for each constraint $\Gamma_i \vdash_{D_i} \Delta_i$ of D_i. By Weakening, K obviously includes constraints such that $\mathrm{typ}(a), \Gamma_i \vdash_K \Delta_i, \mathrm{typ}^c(a)$. Therefore, $D_i \in \mathcal{F}_{a,O,K}(D)$. By Proposition 11, $\mathcal{F}_{a,\mathcal{R}_{\langle D,O,K\rangle}}(D) = \mathcal{F}_{a,O,K}(D)$ holds, so that $D_i \in \mathcal{F}_{a,\mathcal{R}_{\langle D,O,K\rangle}}(D)$ follows.

Suppose a representation $a \in \mathrm{tok}(O)$ satisfies a desire $D_i \in \mathcal{F}(D)$. A representation a satisfying the desire D_i is equivalent to that there exist constraints of the knowledge K such that $\mathrm{typ}(a), \Gamma_i \vdash_K \Delta_i, \mathrm{typ}^c(a)$ for each constraint $\Gamma_i \vdash_{D_i} \Delta_i$ of the desire D_i, since $\mathcal{F}_{a,\mathcal{R}_{\langle D,O,K\rangle}}(D) = \mathcal{F}_{a,O,K}(D)$. Since $\mathrm{Log}(O,K)$ is sound, the token a satisfies the sequent $\langle \mathrm{typ}(a) \cup \Gamma_i, \Delta_i \cup \mathrm{typ}^c(a) \rangle$ on $\mathrm{typ}(O)$. By the definition of $\mathrm{typ}(a)$, both $\forall \alpha \in \mathrm{typ}(a),\ a \vDash_O \alpha$ and $\forall \beta \in \mathrm{typ}^c(a),\ a \nvDash_O \beta$ hold. Moreover, since a

satisfies $\langle \text{typ}(a) \cup \Gamma_i, \Delta_i \cup \text{typ}^c(a) \rangle$, if $a \vDash_O \gamma$ for all $\gamma \in \Gamma_i$, then $a \vDash_O \delta$ for some $\delta \in \Delta_i$, and thus, a satisfies $\langle \Gamma_i, \Delta_i \rangle$. Even if $a \nvDash_O \gamma$ for some $\gamma \in \Gamma_i$, a trivially satisfies $\langle \Gamma_i, \Delta_i \rangle$. Thus a satisfies $\langle \Gamma_i, \Delta_i \rangle$ and hence $D_i \in \mathcal{F}_{a,O}(D)$.

(**To show 2.**) Suppose a name $a \in \Sigma_K$ satisfies a desire $D_i \in \mathcal{F}(D)$. By this supposition, there exist constraints of the knowledge K such that $a, \Gamma_i \vdash_K \Delta_i$ for each constraint $\Gamma_i \vdash_{D_i} \Delta_i$ of D_i. Then, by Weakening, it follows that K contains constraints such that $\text{typ}(a), \Gamma_i \vdash_K \Delta_i, \text{typ}^c(a)$. Therefore, $D_i \in \mathcal{F}_{a,O^\dagger,K}(D)$. By $\mathcal{F}_{a,\mathcal{R}_{\langle D,O^\dagger,K \rangle}}(D) = \mathcal{F}_{a,O^\dagger,K}(D)$, $D_i \in \mathcal{F}_{a,\mathcal{R}_{\langle D,O^\dagger,K \rangle}}(D)$ follows.

Suppose a representation $a \in \text{tok}(O^\dagger)$ satisfies a desire $D_i \in \mathcal{F}(D)$. Since a satisfies the desire D_i and $\mathcal{F}_{a,\mathcal{R}_{\langle D,O^\dagger,K \rangle}}(D) = \mathcal{F}_{a,O^\dagger,K}(D)$ holds, there exist constraints of the knowledge K such that $\text{typ}(a), \Gamma_i \vdash_K \Delta_i, \text{typ}^c(a)$ for each constraint $\Gamma_i \vdash_{D_i} \Delta_i$ of the desire D_i. Besides this, both $\forall \alpha \in \text{typ}(a),\ a \vdash_K \alpha$ and $\forall \beta \in \text{typ}^c(a),\ a, \beta \vdash_K$ hold, since $a \in \text{typ}(O^\dagger)$ is a name of $O^\dagger$. By Weakening, therefore, we find that $a, \Gamma_i, \Theta \vdash_K \Lambda, \Delta_i$ is contained by the knowledge K for each partition $\langle \Theta, \Lambda \rangle$ of $\text{typ}^c(a) \cup \text{typ}(a) \setminus \{a\}$. By Global Cut, $a, \Gamma_i \vdash_K \Delta_i$ holds and hence $D_i \in \mathcal{F}_{a,K}$ follows.

3. is automatically satisfied from 1. and 2.. □

The proposition above obviously yields the following theorem.

Theorem 1. *Let an observation O, a regular knowledge K, a desire D, and a family of desires $\mathcal{F}(D)$ of D be given. Moreover, suppose a local logic* $\text{Log}(O, K)$ *is sound and complete.*

1. *If* $\text{typ}(D) \subseteq \text{typ}(O)$, *then* $\precsim_{\langle \mathcal{F}(D),O \rangle}$ *and* $\precsim_{\langle \mathcal{F}(D),\mathcal{R}_{\langle D,O,K \rangle} \rangle}$ *coincide with each other.*
2. *If the knowledge K contains the names of the observation O and, additionally,* $\text{Log}(O^\dagger, K)$ *is sound and complete, then* $\precsim_{\langle \mathcal{F}(D),K \rangle}$ *and* $\precsim_{\langle \mathcal{F}(D),\mathcal{R}_{\langle D,O^\dagger,K \rangle} \rangle}$ *coincide with each other.*
3. *If both 1. and 2. are satisfied, then* $\precsim_{\langle \mathcal{F}(D),O^\dagger \rangle}$, $\precsim_{\langle \mathcal{F}(D),K \rangle}$, *and* $\precsim_{\langle \mathcal{F}(D),\mathcal{R}_{\langle D,O^\dagger,K \rangle} \rangle}$ *coincide with each other.*

We say that the preference relations are interchangeable *if they coincide with each other.*

Let us verify this theorem with an example. To verify the statement 1. of Theorem 1, we revisit the example in section 2.2. We considered there the classification $\text{Cla}^*(D)$ which was made from the desire $D = \{\ \text{SALTY} \vdash_D,\ \vdash_D \text{SWEET},\ \vdash_D \text{COFFEE}\ \}$. Note, however, that for simplicity we consider here the classification $\text{Cla}'(D)$ such that the tokens are confined to the ones shown in the following classification table 15.

$\vDash_{\text{Cla}'(D)}$	SALTY	SWEET	COFFEE
#1 :⟨{SALTY, SWEET, COFFEE}, ∅⟩	1	1	1
#2 :⟨{SALTY, SWEET}, {COFFEE}⟩	1	1	0
#5 :⟨{SWEET, COFFEE}, {SALTY}⟩	0	1	1
#6 :⟨{SWEET}, {SALTY, COFFEE}⟩	0	1	0

Table 15: classification $\text{Cla}'(D)$ made from desire D

One can easily confirm that the knowledge which constitutes a sound and complete local logic with this classification $\text{Cla}'(D)$ is the regular closure $\bar{K}$ of the knowledge $K = \{\ \vdash_K \text{SWEET}\ \}$.

Since this classification $\text{Cla}'(D)$ is made from the desire D, it satisfies $\text{typ}(\text{Cla}'(D)) \subseteq \text{typ}(D)$. Therefore, by 1. of Theorem 1, the preference relation constructed on the tokens of $\text{Cla}'(D)$ coincides with the preference relation on the representations of the representation system $\mathcal{R}_{\langle D,\text{Cla}'(D),\bar{K} \rangle}$ which is generated by $\text{Cla}'(D)$, the desire D, and the knowledge $\bar{K}$. Let us verify that.

First, we construct a preference relation on the tokens of $\mathrm{Cla}'(D)$. We consider $\mathcal{F}(D) = \mathrm{Pow}(D)$ as a family of desires. As we have seen in section 2.2, the ordering of the preference relation on the tokens of $\mathrm{Cla}^*(D)$ is depicted by the Hasse diagram 1. To write it explicitly, the following preference relation holds.

$$\#2 \prec_{\langle \mathcal{F}(D), \mathrm{Cla}'(D) \rangle} \#6, \quad \#6 \prec_{\langle \mathcal{F}(D), \mathrm{Cla}'(D) \rangle} \#5,$$
$$\#2 \prec_{\langle \mathcal{F}(D), \mathrm{Cla}'(D) \rangle} \#1, \quad \#1 \prec_{\langle \mathcal{F}(D), \mathrm{Cla}'(D) \rangle} \#5.$$

Next, let us construct a preference relation on representations in a similar way. Representation #1 is of each type of $\mathrm{typ}(\#1) = \{\ \text{SALTY}, \text{SWEET}, \text{COFFEE}\ \}$. By Identity and Weakening, the knowledge $\bar{K}$ contains $\mathrm{typ}(\#1) \vdash_{\bar{K}} \text{SWEET}$ and $\mathrm{typ}(\#1) \vdash_{\bar{K}} \text{COFFEE}$. Meanwhile, $\text{SALTY}, \mathrm{typ}(\#1) \nvdash_{\bar{K}}$ holds. Thus, among the family of desires $\mathcal{F}(D)$ of D, #1 can satisfy only desires that do not contain $\text{SALTY} \vdash_D$. On the other hand, #5 is not of type SALTY and hence $\mathrm{typ}(\#5) = \{\ \text{SWEET}, \text{COFFEE}\ \}$ and $\mathrm{typ}^c(\#5) = \{\ \text{SALTY}\ \}$ hold. Meanwhile, by Identity and Weakening, the knowledge $\bar{K}$ contains $\text{SALTY}, \mathrm{typ}(\#5) \vdash_{\bar{K}} \mathrm{typ}^c(\#5)$. Thus #5 satisfies all the constraints of the desire D. Similarly, verifying all the representations, we obtain the following preference relation which coincides with the preference relation on the tokens of $\mathrm{Cla}'(D)$.

$$\#2 \prec_{\langle \mathcal{F}(D), \mathcal{R}_{\langle D, \mathrm{Cla}'(D), \bar{K} \rangle} \rangle} \#6, \quad \#6 \prec_{\langle \mathcal{F}(D), \mathcal{R}_{\langle D, \mathrm{Cla}'(D), \bar{K} \rangle} \rangle} \#5,$$
$$\#2 \prec_{\langle \mathcal{F}(D), \mathcal{R}_{\langle D, \mathrm{Cla}'(D), \bar{K} \rangle} \rangle} \#1, \quad \#1 \prec_{\langle \mathcal{F}(D), \mathcal{R}_{\langle D, \mathrm{Cla}'(D), \bar{K} \rangle} \rangle} \#5.$$

Finally, let us verify whether the preference relations on the types of the knowledge $\bar{K}$ coincide with the preference relations on the tokens and on the representations. However, one would find immediately that the knowledge K does not contain any sort of types that identify tokens uniquely. In other words, since K does not have any names, tokens are not identified by corresponding types and hence the preference relations on tokens and on representations above cannot be expressed by any preference relation on types.

By setting up a knowledge that constitutes a sound and complete local logic with the named classification $\mathrm{Cla}^\dagger(D)$ as below, however, the preference relation on the tokens of $\mathrm{Cla}'(D)$ can be expressed by the preference relation on the corresponding names.

$\vDash_{\mathrm{Cla}^\dagger(D)}$	SALTY	SWEET	COFFEE	#1	#2	#5	#6
#1 :$\langle\{$SALTY, SWEET, COFFEE$\}, \emptyset\rangle$	1	1	1	1	0	0	0
#2 :$\langle\{$SALTY, SWEET$\}, \{$COFFEE$\}\rangle$	1	1	0	0	1	0	0
#5 :$\langle\{$SWEET, COFFEE$\}, \{$SALTY$\}\rangle$	0	1	1	0	0	1	0
#6 :$\langle\{$SWEET$\}, \{$SALTY, COFFEE$\}\rangle$	0	1	0	0	0	0	1

Table 16: named classification $\mathrm{Cla}^\dagger(D)$ generated from desire D

Constructing a theory $K^\dagger$ with the name of $\mathrm{Cla}^\dagger(D)$ according to Proposition 12, we obtain the regular theory $\bar{K}^\dagger$ with the name of $\mathrm{Cla}^\dagger(D)$ from this classification $\mathrm{Cla}^\dagger(D)$ as follows.

$$
\begin{array}{llll}
K^\dagger = \{ & \#1 \vdash_{K^\dagger} \text{SALTY}, & \#1 \vdash_{K^\dagger} \text{SWEET}, & \#1 \vdash_{K^\dagger} \text{COFFEE}, \\
& \#2 \vdash_{K^\dagger} \text{SALTY}, & \#2 \vdash_{K^\dagger} \text{SWEET}, & \#2, \text{COFFEE} \vdash_{K^\dagger}, \\
& \#5, \text{SALTY} \vdash_{K^\dagger}, & \#5 \vdash_{K^\dagger} \text{SWEET}, & \#5 \vdash_{K^\dagger} \text{COFFEE}, \\
& \#6, \text{SALTY} \vdash_{K^\dagger}, & \#6 \vdash_{K^\dagger} \text{SWEET}, & \#6, \text{COFFEE} \vdash_{K^\dagger}, \\
& & & \\
& \#1, \#2 \vdash_{K^\dagger}, & \#1, \#5 \vdash_{K^\dagger}, & \#1, \#6 \vdash_{K^\dagger}, \\
& \#2, \#5 \vdash_{K^\dagger}, & \#2, \#6 \vdash_{K^\dagger}, & \#5, \#6 \vdash_{K^\dagger}, \\
& & \vdash_{K^\dagger} \#1, \#2, \#5, \#6 & \}.
\end{array}
$$

Since this knowledge $\bar{K}^{\dagger}$ includes constraints such that $\#1 \vdash_{\bar{K}^{\dagger}}$ SWEET and $\#1 \vdash_{\bar{K}^{\dagger}}$ COFFEE, the name #1 of the token #1 satisfies desires including constraints such that $\vdash_D$ SWEET and $\vdash_D$ COFFEE. Similarly, verifying satisfiable desires about all the names, we can obtain the following preference relation on the names of $\mathrm{Cla}^{\dagger}(D)$, which coincides with the preference relations on the tokens of $\mathrm{Cla}'(D)$ and on the representations of $\mathcal{R}_{\langle D,\mathrm{Cla}(D),\bar{K}\rangle}$.

$$\begin{array}{ll} \#2 \prec_{\langle \mathcal{F}(D),\bar{K}^{\dagger}\rangle} \#6, & \#6 \prec_{\langle \mathcal{F}(D),\bar{K}^{\dagger}\rangle} \#5, \\ \#2 \prec_{\langle \mathcal{F}(D),\bar{K}^{\dagger}\rangle} \#1, & \#1 \prec_{\langle \mathcal{F}(D),\bar{K}^{\dagger}\rangle} \#5. \end{array}$$

Finally, let us consider the meaning of the theorem. Having stated at the beginning of this subsection, we can regard the decision making in, say, mail-order shopping as an example of constructing preference relation between commodities only by knowledge. Contrastingly, we can regard our everyday shopping in stores as an example of constructing preference relation between commodities in front of us by picking them up firsthand, observing them, and sometimes representing what desires these commodities can satisfy. The first statement of the theorem is relevant to the latter case, that is, if the types the agent desires are observable, the preference relation derived only from the observation and that made by representation system coincide with each other. On the other hand, the second statement is on the relationship between the former and the latter forms of choice behavior, that is, if the agent's knowledge contains the name of the observed tokens, in other words, if she knows the names of the tokens, the preference relations derived from real shopping and mail-order shopping coincide with each other. These two claims are assured by the primary assumption requiring conformity between knowledge and observation, that is, the soundness and completeness of local logic consisting of knowledge and observation.

However, as we already have referred in connection with local logic, our knowledge and observation do not always construct a sound and complete local logic. This is because we may actually find things that revoke our ex-ante knowledge occasionally or cannot integrate all informations contained in observations inadequately into our knowledge quite often. In fact, most of us might have disappointed several times by articles bought via mail-order or found unexpected articles being sold in shops. What surfaces in these experiences is the discrepancy between knowledge and observation, or, unsoundness and incompleteness of local logic.

It could be said that Theorem 1 deals with the situation in which such unsoundness or incompleteness of the local logic does not, maybe yet, surface. In fact, we would act implicitly presupposing conformity between observation and knowledge in everyday life. Indeed, we purchase ordinary salt and ordinary sugar at grocery stores without checking the details or, in some cases, willingly use mail-order or group buying. Especially, when we buy homogeneous articles such as books or CDs, we would actively use mail-order. In these instances, we implicitly presuppose that preference relation on tokens and types are interchangeable, that is, our knowledge and observation constitute a sound and complete local logic.

The point is that our knowledge and observation are in many cases congruent, but may differ in some situations. This gap sometimes causes unsoundness and incompleteness of local logic and, in some cases, makes preference relations on tokens and types reverse. Through such gaps between preference relations, then, we realize inadequacy or fallacy of our knowledge and are forced to update the constraints of our knowledge. We will argue this dynamic property of knowledge, to be called "learning," in another article as mentioned previously.

3 Concluding Remarks

Finally, we will give some prospects of our approach.

The outstanding feature of our approach can be said to reside mainly in its radical relativism. That is, our approach regard not only preference relations as the constitution that is relative to the agent's knowledge and desire, but also the knowledge and desire themselves as the constituents that is relative in the sense that they remain only temporal. In our approach, there is no room for absolute concepts such as the "true state" to exist unless we put specific structures such as the "God's view" into our approach, since the truth of states is relative to agent's judgement. And this is why we think our approach as radically relative.

This radical relativity enables us to treat subjects that have long been regarded as impossible to treat formally and neglected in many social sciences. We will examine the possibilities our approach opens up in two specific contexts here, i.e., the process of decision and the knowledge.

"How do we make our decisions?" This is one of the questions that have not been answered adequately. It can be answered, though, meaningfully and formally with the help of our framework. In our framework, desires are constructed according to some criteria contained in the agent's knowledge. When we want the lottery that maximizes its expected value among others, for example, the desire itself assumes the criterion which maximizes expected value. Or when we want the lottery that maximizes the prize that can be won for sure, on the other hand, the desire assumes another criterion than that of previous one. In short, the criteria of decision are not an objective constitutuent that are given regardless of the agent inside the model, but a subjective one that is constructed according to the agent's own knowledge. Thus, it is important to know, in our framework, according to what knowledge do the agents construct criteria of decision. On the other hand, it will end up in asking which criteria are "objectively true," when we, as the model builders or researchers, give specific criteria to the agents inside the models, as we did for long time in the standard approach. Consequently, the process of agents' own decision, such as, "according to what kind of criteria do we satisfy our desires?" are neglected.

It is not our original idea, of course, to investigate the decision process itself. On this subject, in fact, psychologists and behavioral economists have already accumulated many interesting studies. A variety of characteristics found in human behaviors in these studies, called heuristics or biases, shows us eloquently that we do not make our decisions according to the uniquely determined, say, omnipotent criterion. Our approach enables us to formalize these heuristics and biases as knowledge and desire in the same way as we did for rational preference. In other words, our approach enables us within a unified formal language to formalize various criteria of decision regardless of that of standard modern economics or that of behavioral economics. Such formalizations have not been adequately accomplished by neither psychology nor behavioral economics.

The decision process itself might also have been the problem of more practical fields, such as marketing. Provided that we always have a rational preference on all commodities, there remains little room for marketing. However, if someone makes an appeal to turn our mind to some commodity, we cannot afford not to be conscious of it, or might actually feel a desire for it. Thus, it comes to be one of the core problems of marketing how to appeal to consumer's desire. Our approach will give one basis for formalizing and analyzing consumers' desire and knowledge, lying in the core of the problems of marketing.

It is not only important for sellers to know buyers' decision process. But also important for buyers themselves to know ones' own decision process. When we purchase some sort of goods, such as a car or a house, for example, or when we are faced with the problems that require technical knowledge to solve them, such as health, financial or judicial problems, we are also faced with the decision process itself. Because we don't have enough knowledge in these cases, we can't decide at all what to choose, or even don't know according to what criteria should we choose. That is why we need specialists, such as doctors, financial advisers or lawyers, as agents who give information about what options are available, and give advices to us which options satisfy our desires or how to satisfy our desires with what. In short, in these cases we buy the decision process itself.

To satisfy clients' desires appropriately, agents need appropriate method to catch clients' situations since agents can't offer appropriate options to clients unless they know details of situations the clients are in. Our approach will give one basis for formalizing the relationship between clients' situations and agents' technical knowledge and constructing a system which enables them to satisfy clients' desires efficiently.

Let us move on to knowledge, the next subject. We have already seen that in our approach the world each agent represents is relative in the sense that it is constructed by an observation and a knowledge of the agent. Thus, we can't assume, in general, that the set of states is common to all agents in our framework. But our framework, at the same time, has the conceptions which enable us to grasp relations between different classifications such as informophisms or channels. These conceptions, of course, don't always guarantee the sameness between two classifications, as we examined in Section 2, that is, infomorphisms may exist between two different classifications, and channels can always be defined between any pair of classifications, and that is why we can use channels to grasp relations between any two classifications. Thus this feature of our approach, where we can grasp subjective structures between knowledges or classifications without introducing any objective criteria, opens up a possibility of capturing constructions of knowledge or communications in a new perspective.

Let us consider, for example, the construction of common knowledge, which is one of the basic assumptions of game theory and is needed so frequently to define Nash equilibrium or other key concepts. Since Aumann (1976), the state-space approach has been used to describe the structure of common knowledge. In this approach, common knowledge is defined as the event that includes the finest common coarsening of all agents' partitions. But this definition neglects some basic question about knowledge, that is, "how on earth can we know that other players see the world with exactly the same state-space as ourselves?"

The question cannot be neglected, however, in our approach, and may not be possible to answer positively, since we do not always see the world in the same way. Recall here that infomorphisms do not guarantee the isomorphisms between two different classifications. Thus infomorphisms do not guarantee the sameness of agents' viewpoint. This feature of the infomorphisms, however, would be said to fit our feeling better since we do sometimes believe that we know other's knowledge almost groundlessly. It is not because we get objective proof that we know other's knowledge, but because we regard it almost the same as ours according to some subjective basis. Our knowledge or understanding, we think, must have an ever-changing structure that is ambiguous to some extent but at the same time fairly robust. And the conceptions such as infomorphisms and channels, which themselves have also ambiguous and robust structures, can grasp these structure as they are.

It might be already apparent that the most suitable subject for infomorphisms and channels to describe is communication. It is not because we know the unique proper language that we can communicate with each other. We simply communicate with someone, without knowing what the unique proper language is. Communication is flexible enough to the extent that in every conversation the new meanings of words are invented. Therefore, the person who knows the unique proper language may not be possible to communicate with anyone. This flexibility of communication is surprisingly similar to that of infomorphisms and channels, which can be constructed between multiple different classifications in many ways. Our approach enables us to grasp this flexibility, which any methodologies that rely on some kind of objectivity or absoluteness cannot grasp or even see, just as it is.

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