Hyperbolic Discounting: It's Just Allais Paradox

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Abstract

This paper demonstrates that present bias may be viewed as a systematic violation of expected utility theory. I propose a model in which a well known failure of expected utility captured by the Allais paradox is also equivalent to also a well-known failure of exponential discounting captured by hyperbolic discounting. This result also implies a connection between two well known behavioral regularities, one in intertemporal choice and another in choice under uncertainty.

KEYWORDS: Allais Paradox, Hyperbolic Discounting, Preference over lotteries.

1 Introduction

Animals and humans have been shown to gain pleasure more immediately but to defer paint until later. O'Donoghue and Rabin (1999) call such tendencies *present*

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biased preferences: When considering trade-offs between two future moments, present biased preferences give stronger relative weight to the earlier moment as it gets closer[1]. The most salient example of present bias is preference reversal: If we are offered the choice on February 1 between $10 on April 1 and $11 on April 2, then many of us may choose the $11 on April 2 over accepting $10 on April 1. However, on April 1, given the same choice, it is no wonder that most of us come to choose $10 on April 1 over $11 on April 2. The same story goes for the case of negative payoffs: If we are offered the choice on February 1 between seven hours of work on April 1 and eight hours of work on April 2, then most of us may prefer seven hour work on April 1. However, on April 1, given the same choice, many of us would be inclined to postpone the pain of working.

These behavior can be explained as follows: Animals and humans discount future rewards and pains. Especially, the more they discount, the sooner those payoffs realize. In other words, they *hyperbolically* discount the future[2]. Since the present bias is regarded as key to understanding important economic behaviors such as undersaving or addiction, a growing number of literature use hyperbolically discounting utility function (see ,e.g., Laibson (1997) and Harris and Laibson (2003)). However, there are small number of studies trying to answer the question: "What is the cause of present bias?".

Behavioral ecology suggests that discounting the future may well be an adaptive response to the risks associated with waiting for delayed payoff. As realization of outcome delays, the probability of receiving payoff usually decreases. For example, an animal which defers foraging may risk the food getting rotten. or being stolen by other animals. As well, the effort of searching for water may be foregone if the animal waits long enough for a rainfall. This story is convincing as an explanation of discounting the future but it does not explain the cause of *hyperbolic* discounting.

The purpose of the present paper is to explore the origin of present bias. I propose a model in which *a well-known failure of expected utility-captured by the Allais paradox is also equivalent to a well-known failure of exponential discounting captured by hyperbolic discounting.* The the Allais paradox is observed regularly in human behavior, which has been a driving force in the development of alternate

[1]Stroz (1956) first suggested the present bias.

[2]Green and Myerson (1996) reports experimental results on birds. See Ainslie (1992) and Frederick et al. (2002) for reviews of the evidence for and against hyperbolic discounting.

theories to the expected utility theory. Allais (1953) provides experimental results that in the situation where people have to choose between a safe option that offers a smaller payoff with a higher probability and a risky option that offers a larger payoff with the lower probability, as winning probabilities goes down in the both options by the same ratio, most people come to choose the risky option. This effect, called the*the common ratio effect*, is regarded as a cause of the Allais paradox. The main idea of the paper shows that if people regard a future payoff as uncertain, then they will exhibit present bias if and only if they also exhibit the common ratio effect. A Key is an equivalence between delaying realization and reducing winning probability. This observation implies that *even if a decision maker is an exponential discounter, the risk to lose future payoffs may make the person present biased.* Emphasizing the certainty of future costs or rewards would attenuate present bias. My conjecture matches evidence from the Neuroscience. By using functional MRI, McClure et al. (2004) confirm that the prefrontal cortex is activated when subjects make intertemporal choices. Huettel et al. (2005) shows independently that a similar area of the brain is working when subjects make a choice under uncertainty. Neither mentions the relationship between intertemporal choice and choice under uncertainty. However, it is possible that the prefrontal cortex may cause the common ratio effect as well as present bias. As well, support for my conjecture can be found in Keren and Roelofsma (1995), which provides experimental evidences that present bias is not apparent when the present payoff becomes risky. They imply equivalence between delaying realization of payoffs and increasing risk of losing payoffs. Equivalence is suggested in animal experiments by Benjamin et al.(2007). Also, several empirical papers also report the positive correlation between risk aversion and impatience (see e.g., Anderson et al. (2006) and Anderhub (2001)), which can be explained along with my conclusion: Risk averse subjects tend to dislike the larger and later payoff because waitings translates into a higher risk of losing the payoff.

This paper is organized as follows. Section 2 provides an example in order to clarify the relationship between intertemporal choice and its helps develop an intuitive understanding of the main theorem. Section 3 discusses related literature. Section 4 reviews the experimental result of Kahneman and Tversky (1979) which captures the common ratio effects and formally defines the common ratio effect for a preference of lotteries. Section 5 defines intertemporal preferences based on preference of lotteries. Then, a definition of present bias for the intertemporal preferences is defined. Section 6 provides the main theorem that a preference of lotteries produces the common ratio effects if and only if the generated intertemporal preference exhibits present bias. Section 7 concludes.

2 Example

In this section, I show a motivating example in order to clarify the relationship between intertemporal choice and choice under uncertainty and to help intuitive understanding of the main theorem. In particular, I will illustrate that present biased behavior can be regarded to be a dynamic inconsistent choice caused by the common ratio effect. I assume future payoffs may be lost independently at random and at a uniform rate γ per unit of time. I do not take into account any other things that would deteriorate future payoffs other than the hazard rate. For an easy explanation, I assume discrete time structure only in this section. Then, at the moment of date 0, the probability that the decision maker actually obtains the payoff V at date T is $(1-\gamma)^T$. Hence, $[V, T]$, the payoff V at date T, should be regarded as a prospect $(V, (1-\gamma)^T)$ which gives the payoff V, with the probability $(1-\gamma)^T$, otherwise it gives nothing. Suppose that the decision maker has to choose between [$10, April 1] and [$11, April 2]. On February 1, these future payoffs should be regarded as lotteries $s_{Feb.1} \equiv (\$10, (1-\gamma)^{60})$ and $r_{Feb.1} \equiv \big(\$11, (1-\gamma)^{61}\big)$, respectively. However, on April 1, these future payoffs come to be regarded as lotteries $s_{Apr.1} \equiv (\$10, 1)$ and $r_{Apr.1} \equiv (\$11, 1-\gamma)$, respectively. Note that under the assumption of the reduction of lotteries, $s_{Feb.1} = (s_{Apr.1}, (1-\gamma)^{60})$. Similarly, since the decision maker would consider $11 after 61 days is equivalent to $11 after 60 days plus 1 day, it should be established that $r_{Feb.1} = (r_{Apr.1}, (1-\gamma)^{60})$. Therefore, $s_{Apr.1}$ and $r_{Apr.1}$ are sub lotteries of $s_{Feb.1}$ and $r_{Feb.1}$, respectively. Figure 1 shows a present biased choice, [$11, April 2] over [$10, April 1], on February 1, but [$10, April 1] over [$11, April 2] on April 1, is equivalent to the dynamically inconsistent choice, $r_{Feb.1}$ over $s_{Feb.1}$, but $s_{Apr.1}$ over $r_{Apr.1}$. It is well known that such dynamic inconsistency would be caused by the common ratio effect.

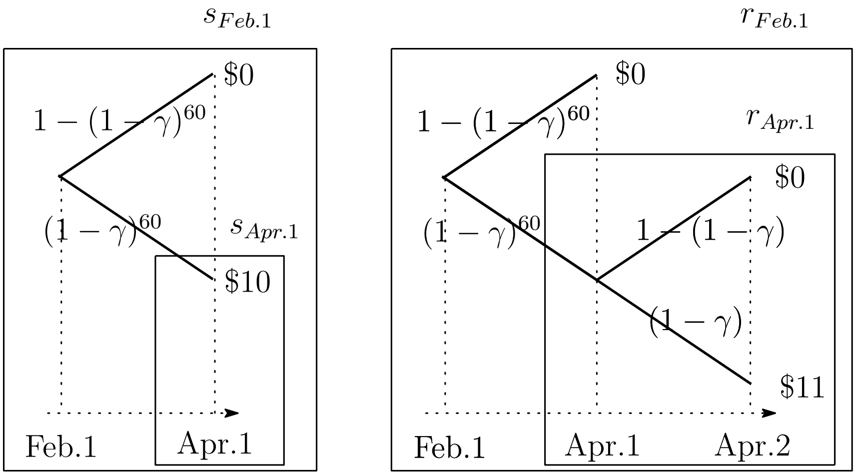

Figure 1: Intertemporal Choice and Choice under Uncertainty

3 Related Literature

Although, as noted in the previous section, non-expected utility theory has suggested that failure of the independence axiom might produce dynamic inconsistency, few authors have explored the direct link between intertemporal choice and choice under uncertainty (see, e.g., Cubitt, Starmer, and Sugden (1998)). Recently, Halvey (forthcoming) shows that *diminishing impatience* can be induced by a risk attitude which is compatible with the common ratio effect. However, since his purpose is rather axiomatic, Halvey's result is restricted in the class of rank-dependent utility, which is originally axiomatized by Quiggin (1982) and Yarri (1987), satisfying several other axioms. Furthermore, strictly speaking, what he calls diminishing impatience is not general hyperbolic discounting, in the sense that his concept of diminishing impatience does not capture the important fact that that people gradually becomes impatient as a promised date is approaching. So his definition only implies quasi-hyperbolic discounting: People are impatient when the promised day is close at hand. This captures only part of general hyperbolic discounting. As we will see in the following section, my definition of present bias allows general hyperbolic discounting as well as quasi-hyperbolic discounting. Hence my contribution is to show a *more general equivalence* between present bias and the common ratio effect *without* any other assumptions.

Apart from economics, behavioral ecology and zoology have explored the cause of discounting the future. Although they suggests hat risks associated with waiting for delayed payoffs may be a key to explaining discounting the future from the view point of adaptive response (see, e.g., Green and Myerson (1996)), they cannot formally explain the reason why animals and humans *hyperbolically* discount. An illuminating paper, Dasgupta and Maskin (2005) advance and formalizes the idea. They rationalize hyperbolic discounting in an environment in which payoffs may be lost or realized early. They show that the decision maker becomes more impatient as the horizon is shortened since the likelihood of early realization diminishes in time. Similarly with my model, Dasgupta and Maskin assume a constant hazard rate γ. Then the probability that a decision maker actually obtains payoff V after time T is $e^{-\gamma T}$, so that, they suggest, expected payoff V at time T is $e^{-\gamma T}V$. In this point, however, they implicitly assume the *independence axiom*. Numerous studies suggest the axiom is not without difficulties as a descriptive theory. The present paper induces hyperbolic discounting without the independence axiom as well as the probability of early realization. The present paper rather show that a failure of the independence axiom is enough to induce hyperbolic discounting in an environment where payoffs may be lost.

To see this, in this paragraph only, I will assume the rank-dependent generalization of expected utility theory. Then expected payoff V at time T becomes $g(e^{-\gamma T})V$. If g is the identity function, then the model reduces down to expected utility theory. Suppose that g's elasticity is strictly increasing, which is known to be compatible with the common ratio effect (see Segal (1987)). Then it can be shown that the following result is established:

PROPOSITION (Dasgupta and Maskin (2005)): *Assume that there exists* $t^* < T$ *at which the decision maker is indifferent between* $[V, T]$ *and* $[V', T']$, *with* $0 < V < V'$ *and* $T < T'$. *Then the decision maker prefers* $[V', T']$ *to* $[V, T]$ *at all* $t < t^*$, *but* $[V, T]$ *to* $[V', T']$ *at all* t *such that* $t^* < t < T$ (see Appendix for the proof).

This proposition is more extensive than the main theorem (Proposition 1) of Dasgupta and Maskin (2005) in the sense that the result above does not assume possibility of early realization. The main theorem of the present paper shows *far* *more general equivalence result between the common ratio effect and present bias without assuming a specific form of the utility function.*

As an application, Bommier (2007) discusses the impact of longevity extension on aggregate wealth accumulation. He considers a class of preference that is more general than the one introduced by Yarri (1965).

4 Allais Paradox

In this section, I formally define the common ratio effect, which is regarded as a cause of the Allais paradox. First, I show Table 1 of Kahneman and Tversky (1979) as a typical experimental result capturing the common ratio effect.

Problem 3 N=95	(4,000,.80) [20%]	$<$	(3,000,1.0) [80%]	Problem 3′ N=95	(-4,000,.80) [92%]	$>$	(-3,000,1.0) [8%]
Problem 4 N=95	(4,000,.20) [65%]	$>$	(3,000,.25) [35%]	Problem 4′ N=95	(-4,000,.20) [42%]	$<$	(-3,000,.25) [58%]
Problem 7 N=66	(3,000,.90) [86%]	$>$	(6,000,.45) [14%]	Problem 7′ N=66	(-3,000,.90) [8%]	$<$	(-6,000,.45) [92%]
Problem 8 N=66	(3,000,.002) [27%]	$<$	(6,000,.001) [73%]	Problem 8′ N=66	(-3,000,.002) [70%]	$>$	(-6,000,.001) [30%]

Table 1: The Common Ratio Effect

For example, in Problem 3, 80% of ninety five subjects prefered the option of giving $\$3,000$ for sure to the option giving $\$4,000$ with a probability of 0.8, otherwise giving nothing. However, increasing risks by multiplying 0.25 to the winning probabilities in both options reversed their preference: In Problem 4, 65% of the same subjects as before prefered an option of giving $\$4,000$ with a probability of $0.2(= 0.8 \times 0.25)$ to an option of giving $\$3,000$ with a probability of $0.25(= 1 \times 0.25)$. This shows at least 45% of the subjects simultaneously chose the safe option in Problem 3 and the risky option in Problem 4. This pair of choices is inconsistent with the expected utility theory. If u is the individual's von Neumann-Morgenstern utility function, then the choice in Problem 3 means that $u(\$3,000) > .8\ u(\$4,000)$. Similarly, the choice in Problem 4 means that $.2\ u(\$4,000) > .25\ u(\$3,000)$, or $.8\ u(\$4,000) > u(\$3,000)$. These two choices are inconsistent. Choices in Problem 7 and 8 have the similar tendency: By increasing the risk to get nothing, at least 59% of another sixty six subjects switched their choices from the safe option in Problem 7 to the risky option in Problem 8.

This tendency can be generalized as follows: Suppose that subjects choose either a safe option which gives the smaller gain x with the higher probability $\eta \in [0,1]$ or a risky option gives the larger gain y with the lower probability $\eta\mu$, where $\mu < 1$. As η falls, subjects switch their choice from the safe option to the risky option. Note that reducing η means increasing risk to get nothing in both options. Kahneman and Tversky (1979) performed similar experiments, changing the prizes from gains into losses. For example, in Problem $3'$, subjects chose either a certain loss of $\$3,000$ or an 80% risk of losing $\$4,000$. By this switch into the negative prospects, the tendency of choice was also reversed: As η falls, subjects shift their choice from the risky option which gives the larger loss y with the lower probability $\eta\mu$ to the safe option which gives the smaller loss x with the higher probability η.

These regularities, called the common ratio effect, are found by numerous studies and are regarded as a cause of the Allais paradox. To define the regularities formally and as simple as possible, I assume the set of prizes to be $X = \{x, y, \varnothing\}$, where $\varnothing$ means getting nothing. The decision maker is supposed to have the preference $\succsim$ on the set of all probability measures on X. Based on the setting, the common ratio effect can be defined as follows:

DEFINITION: $\succsim$ is said to exhibit *the common ratio effect* if there exists $\mu \in [0,1]$ satisfying the following conditions:
(i) Let $\delta_y \succ \delta_x \succ \delta_\varnothing$. Suppose that there exists $\tilde{\eta} \in [0,1]$ such that $(x,\tilde{\eta}) \sim (y,\tilde{\eta}\mu)$. Then

$$(x,\eta) \prec (y,\eta\mu) \text{ if and only if } \eta < \tilde{\eta},$$

(ii) Let $\delta_y \prec \delta_x \prec \delta_\varnothing$. Suppose that there exists $\tilde{\eta} \in [0,1]$ such that $(x,\tilde{\eta}) \sim (y,\tilde{\eta}\mu)$. Then

$$(x,\eta) \succ (y,\eta\mu) \text{ if and only if } \eta > \tilde{\eta}.$$

5 Present Bias

Consider a decision maker who has to perform a task (consuming goods or bads) by the time $\bar{t}$ at the latest. I assume continuous time structure and constant hazard rate γ, or the task is lost independently at random and at a uniform rate γ per unit of time. Then the probability for the decision maker at time d to obtain a payoff at time $s > d$ is $e^{-\gamma(s-d)}$. Only for simplicity, I assume that no riskless factors would deteriorate a future payoff. Therefore, the decision maker at time

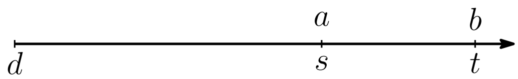

Figure 2: Time Structure

d regards prize a at time s, denoted by $[a, s]$, as a prospect $(a, e^{-\gamma(s-d)})$ which yields a with the probability $e^{-\gamma(s-d)}$, otherwise yielding nothing. Formally, the decision maker's time preference $\succsim_d^*$ at time d can be defined as follows:

Definition: For all $a, b \in X$ and $d, s, t \in [0, \bar{t}]$ such that $d < s, t$,

$$[a, s] \succsim_d^* [b, t] \Leftrightarrow (a, e^{-\gamma(s-d)}) \succsim (b, e^{-\gamma(t-d)}).$$

We are now in a position to define *present bias* for the time preference. When a task gives positive payoff, present bias appears as *preproperation*, or the desire for more rapid realization. When a task gives negative payoff, it appears as *procrastination*:

Definition: $\succsim^*$ is said to exhibit *present bias* if there exists $t \in [0, \bar{t}]$ satisfying the following conditions:
(i) (*Preproperation*:) Let $[y, 0] \succ_0^* [x, 0] \succ_0^* [\varnothing, 0]$. Suppose that there exists $\tilde{d} \in [0, t]$ such that $[x, t] \sim_{\tilde{d}}^* [y, \bar{t}]$. Then

$$[x, t] \prec_d^* [y, \bar{t}] \text{ if and only if } d < \tilde{d},$$

(ii) (*Procrastination*:) Let $[y, 0] \prec_0^* [x, 0] \prec_0^* [\varnothing, 0]$. Suppose that there exists $\tilde{d} \in [0, t]$ such that $[x, t] \sim_{\tilde{d}}^* [y, \bar{t}]$. Then

$$[x, t] \succ_d^* [y, \bar{t}] \text{ if and only if } d > \tilde{d}.$$

Note that the definition (i) of *preproperation* is an ordinal manifestation of the proposition proved by Dasgupta and Maskin (2005). As the following figure

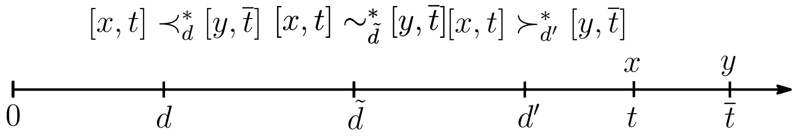

Figure 3: Present Bias (Preproperation)

shows, the definition captures key property of general hyperbolic discounting: People are impatient when the promised date is close at hand. As a special case where $\tilde{d} = t$, the definition reduces into $[x, t] \sim_t^* [y, \bar{t}]$ and $[x, t] \prec_d^* [y, \bar{t}]$ for all $d < t$. This captures the property of quasi-hyperbolic discounting: People become very impatient in the very short run.

6 Main Theorem: Equivalence

We are now in a position to state the equivalence result.

THEOREM: $\succsim$ *exhibits the common ratio effect if and only if* $\succsim^*$ *exhibits present bias.*
PROOF OF THEOREM:

STEP 1: If $\succsim$ exhibits the common ratio effect, then $\succsim^*$ exhibits present bias.
PROOF OF STEP 1: Assume that $\succsim$ exhibits the common ratio effect. Define $t \equiv \bar{t} - \frac{\log \mu}{-\gamma}$[3], so that $\mu \equiv e^{-\gamma(\bar{t}-t)}$. Choose any $\tilde{d} \in [0, \bar{t}]$ such that $[x, t] \sim_{\tilde{d}}^* [y, \bar{t}]$, or $(x, e^{-\gamma(t-\tilde{d})}) \sim (y, e^{-\gamma(\bar{t}-\tilde{d})})$. Define $\tilde{\eta} \equiv e^{-\gamma(t-\tilde{d})}$. Therefore

$$\begin{aligned}
d > \tilde{d} &\Leftrightarrow e^{-\gamma(t-d)} > \tilde{\eta} && (\because \tilde{\eta} \equiv e^{-\gamma(t-\tilde{d})}) \\
&\Leftrightarrow (x, e^{-\gamma(t-d)}) \succ (y, e^{-\gamma(t-d)}\mu) && (\because \text{the Allais paradox}) \\
&\Leftrightarrow (x, e^{-\gamma(t-d)}) \succ (y, e^{-\gamma(\bar{t}-d)}) && (\because \mu \equiv e^{-\gamma(\bar{t}-t)}) \\
&\Leftrightarrow [x, t] \succ_d^* [y, \bar{t}].
\end{aligned}$$

□

STEP 2: If $\succsim^*$ exhibits present bias, then $\succsim$ exhibits the common ratio effect.
PROOF OF STEP 2: Assume that $\succsim$ exhibits present bias. Define $\mu \equiv e^{-\gamma(\bar{t}-t)}$.

[3] I assume $\bar{t}$ is large enough to hold $\bar{t} - \frac{\log \mu}{-\gamma} \geq 0$.

Choose any $\tilde{\eta} \in [0,1]$ such that $(x, \tilde{\eta}) \sim (y, \tilde{\eta}\mu)$. Define $\tilde{d} \equiv t - \frac{\log \tilde{\eta}}{-\gamma}$[4] so that $\tilde{\eta} \equiv e^{-\gamma(t-\tilde{d})}$. Then $[x,t] \sim^*_{\tilde{d}} [y, \bar{t}]$.

For all $\eta \in [0,1]$, define $d(\eta) \equiv t - \dfrac{\log \eta}{-\gamma}$. Then $\eta \equiv e^{-\gamma(t-d(\eta))}$. Hence

$$\begin{aligned}
\eta > \tilde{\eta} \quad &\Leftrightarrow \quad e^{-\gamma(t-d(\eta))} > e^{-\gamma(t-\tilde{d})} && (\because \eta \equiv e^{-\gamma(t-d(\eta))} \text{ and } \tilde{\eta} \equiv e^{-\gamma(t-\tilde{d})}) \\
&\Leftrightarrow \quad d(\eta) > \tilde{d} \\
&\Leftrightarrow \quad [x,t] \succ^*_{d(\eta)} [y, \bar{t}] && (\because \text{present bias}) \\
&\Leftrightarrow \quad \left(x, e^{-\gamma(t-d(\eta))}\right) \succ \left(y, e^{-\gamma(\bar{t}-d(\eta))}\right) \\
&\Leftrightarrow \quad (x, \eta) \succ (y, \eta\mu). && (\because \eta \equiv e^{-\gamma(t-d(\eta))} \text{ and } \mu \equiv e^{-\gamma(\bar{t}-t)})
\end{aligned}$$

□

■

The theorem shows that the phenomenon of present bias may be viewed as a preference reversal *that is caused by the same failure of the independence axiom that causes the common ratio effect.*[5] This observation implies that *even if a decision maker is an exponential discounter, risk to lose future payoffs may make the person present biased.* Many authors try to design incentives when people have present bias. However, most of them do not consider the risks associated with future payoffs or implicitly assume the independence axiom when they account for with the risks. Hence, the possibility for the risks to induce present bias has been dismissed. Not considering the risks may cause some of their recommendations for present bias ineffective. Considering the risks may improve the incentive designs. For example, O' Donoghue and Rabin (2005) provides several general principles. Among those, they claim the usefulness of "magnify"ing future costs or rewards as Principle#2, because present biased people put too little weight on future payoffs. As well as "magnify"ing future costs or rewards, emphasizing the certainty of future costs or rewards would also be effective. That is because the certainty of future costs may stop a present biased person from postponing the costs and the certainty of future rewards may encourage him to wait for the rewards.

[4] I assume t is large enough to hold $t - \frac{\log \tilde{\eta}}{-\gamma} \geq 0$. This assumption would be satisfied if $x \& y$ are sufficiently close and $\bar{t}$ is sufficiently large.

[5] The cause of the failure might be *similarity* between choices as Rubinstein (2003) suggests.

7 Conclusion

This paper suggests a parsimonious explanation for why some people may be present biased. An important conclusion is that preproperation or procrastination that people exhibit when performing intertemporal tasks may be just another display of the well known Allais paradox. When designing incentives for present biased people, this conclusion is important: Present biased behavior can often be improved by emphasizing the certainty of future payoffs.

An important extension would be to built a more general relationship between intertemporal choice and choice under *subjective* uncertainty. Considering preparation for retirement, subjective forecast about future commodity prices or one's life expectancy would affect the decision. With the extension, it may become possible to design appropriate incentives for such problems.

8 Appendix: Proof of the Proposition

Since $[V, T]$ and $[V', T']$ are indifferent at t^*,

$$\frac{g(e^{-\gamma(T-t^*)})}{g(e^{-\gamma(T'-t^*)})} = \frac{V'}{V}.$$

For all $t \leq T$, define

$$f(t) = \frac{g(e^{-\gamma(T-t)})}{g(e^{-\gamma(T'-t)})}.$$

Since for all $t \leq T$, $g(e^{-\gamma(T-t)})V < g(e^{-\gamma(T'-t)})V' \Leftrightarrow f(t) < f(t^*)$. Hence it is sufficient to show f is strictly increasing. Since

$$f'(t) > 0 \quad \Leftrightarrow \quad \frac{g'(e^{-\gamma(T-t)})e^{-\gamma(T-t)}}{g(e^{-\gamma(T-t)})} > \frac{g'(e^{-\gamma(T'-t)})e^{-\gamma(T'-t)}}{g(e^{-\gamma(T'-t)})},$$

and $e^{-\gamma(T-t)} > e^{-\gamma(T'-t)}$, the strict increasingness of the elasticity $\frac{g'(x)x}{g(x)}$ shows the result. ■

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