

# Overlapping Tax Revenue, Local Debt Control and Soft-Budget Constraint

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## Abstract

This paper investigates how the soft budget constraint with grants from the central government to local governments tends to internalize the vertical externality by stimulating insufficient local expenditure when both the central and local governments impose taxes on the same economic activities from public investment. The theoretical model incorporates local governments' rent-seeking activities in a multi-government setting with and without central controls on local borrowing. Two channels through debt issuance and public investment cause the soft budget outcome. In the unrestricted scheme of local debt issuance we have the positive effect on public investment and debt issuance although it would also stimulate wasteful rent seeking activities. In the restricted scheme of local debt issuance the soft budget case may not stimulate public investment since its effect through debt issuance is absent. In either case the soft budget constraint is welfare improving if the marginal valuation of central public goods is relatively small and/or the tax share of local government is relatively small.

JEL classification: E6, H5, H6

Keyword: Soft budget constraint, local investment, rent-seeking activities, local debt control

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## 1. Introduction

This paper investigates how local governments increase insufficient local public expenditures by highlighting the soft-budget constraint of grants from the central (or federal) government to subnational governments (hereafter local governments) based on the rent-seeking activities by the local governments. Namely, this paper will analyze theoretically welfare implications of the soft-budget constraint of intergovernmental financing on local expenditures by developing a simple game between the two governments with the overlapping tax bases. Incorporating the rent-seeking activities of local politicians or governments, we explore both positive and negative effects of soft budget using two cases with and without central controls on local borrowing.

It is well recognized that if local governments face soft budget constraints, they will have an incentive to over-spend, over-borrow, and/or pay insufficient attention to the quality of the investments that their borrowing finances. Such over-spending/borrowing can occur through the common pool mechanism. See, for example, Wildasin (1997, 2004), Goodspeed (2002), Akai and Sato (2005), and Boadway and Tremblay (2005) among others. That is, the natural conjecture is that if the central government imposes soft budget constraints, inefficient too much investment should arise. On the other hand, recently Besfamille and Lockwood (2004) show that hard budget constraints can be too hard and discourage investment that is socially efficient. Namely, they point out the possibility that the hard budget constraint over-incentives the soft budget constraint to provide effort by penalizing it too much for project failure, thus leading ultimately to the possibility that socially efficient projects may not be undertaken.

In this paper without incorporating any uncertainty or imperfect information of effort with respect to public investment and other government activities but with incorporating rent-seeking activities by local politicians and vertical tax externality between the local and central governments, we show that two channels through debt issuance and public investment cause the soft budget outcome. While the soft budget constraint does not necessarily realize the first best solution and it would stimulate wasteful rent seeking activities, the soft budget constraint may be better off than the hard budget constraint case by stimulating local public works and hence by internalizing the vertical externality to some extent.

We pay attention to the vertical externality of shared tax bases between the central and local governments. Multileveled government normally means some commonality of tax base between central and local governments. As a result the tax base may overlap and shared tax bases create another type of common pool problem. It is now well recognized in the tax competition literature that such vertical externalities are likely to leave local taxes too high. This is because each local

government unduly discounts the pressure on central government's spending it creates by raising its own tax rate. See Keen and Kotsogiannis (2002), Keen (1998), and Wilson (1999) among others. In this paper we do not consider such vertical/horizontal tax competition between central and multi local governments and would simply assume that tax rates are given both for central and multi local governments. Rather, we would like to focus on another externality of local expenditures due to overlapping tax bases.

By assuming that the tax share is exogenously given, local public investment may have a positive vertical externality effect. Namely, if an increase in local expenditure on infrastructure stimulates macroeconomic activities, it may enlarge the overlapping tax base, which would then increase taxes for the central government at the given share of tax base between two governments. This is a positive spillover of vertical externality. In this sense, the non-cooperative Nash equilibrium level of local public investment is too low.

In a two period model the benevolent central government may have an incentive to issue more debt and stimulate local expenditure in period 2 by means of additional grants *ex post*. However, such additional grants could produce soft budget problems by increasing grants when the rent-seeking local government spends and borrows more in period 1. Two channels through debt issuance and public investment cause the soft budget outcome. First, local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy, as stressed by Goodspeed (2002). Second, public investment more means an increase in the revenue of central government and hence the central government may subsidize the local government more in period 2, which is a new channel explored in this paper since this channel comes from the vertical externality.

In this game, the behavior of local government and hence the soft budget outcome are also dependent on the scheme of local debt finance. The central government may have some power to control the issuance of local debt. We consider two cases with and without *ex ante* central controls on local borrowing and compare the hard budget and soft budget constraints in each case of local debt scheme. The soft-budget constraint may not be worse since it would stimulate local public investment due to the above two channels, which is beneficial. On the other hand, it would also stimulate the rent-seeking activity and additional grants, which is not beneficial. We shall show that if the marginal valuation of central public goods is relatively small and/or the tax share of local government is relatively small, the former benevolent effect is more powerful. In the restricted scheme the soft budget due to debt issuance disappears, and it may not always stimulate public investment, while it would not stimulate additional grants much. Its welfare effect is qualitatively the same as in the unrestricted scheme of local debt issuance.

This paper consists of five sections. In Section 2 we develop a theoretical model of the central and local governments, and then consider the outcome of the first best as the reference point. In section 3 we investigate the game between the benevolent central government and the rent-seeking local government with and without controls of local debt issuance. In section 4 we introduce the ex post transfer from the central government to the local government to explore the soft-budget problem with and without central controls on local borrowing. Finally, we present some concluding remarks in Section 5.

## 2. Analytical Model of Central and Local Governments

### 2.1 Analytical Framework

We develop a two-period intergovernmental financing model of two governments, the central government (or CG), the lower-level local government (or LG) in a small open economy, in order to explain how local public investment and wasteful spending may be stimulated under the soft-budget constraint. For simplicity, we consider the representative local government, and do not consider the free-riding and/or spillover effects within multi local governments. This is just an assumption for simplicity. There are many papers to explore the horizontal and vertical externalities due to non-cooperative competition among multi local governments. See Wilson (1999) among others. As shown in Appendix 1, the analytical results would be qualitatively the same even if we consider non-cooperative behavior of multi-local governments. Moreover, this is in particular a good approximation of Japan's case where many local governments behave cooperatively against the central government and their behavior may be summarized by the representative local government (the Ministry of Home Affairs).

In the conventional literature the soft-budget outcome occurs only if there are many local governments, who behave non-cooperatively. One contribution of this paper is to show that the soft-budget outcome could occur even in the case of the representative local government. This is a new result since the conventional literature on the soft budget always assumes multi-local governments.

The representative local government (LG) provides useful local public goods  $g_t$ , and the central government (CG) provides useful nation-wide public goods  $G_t$  in each period. Each public good is beneficial and its utility is given by a twice-continuously differentiable and strictly quasi-concave function. Moreover, we assume that all goods are normal ones. The relative price of each good is set to be unity for simplicity. Thus, the social welfare  $W$ , which reflects the representative agent's preferences over public goods, is given by

$$W = u(G_1) + v(g_1) + \delta\{u(G_2) + v(g_2)\} \quad (1)$$

where  $0 < \delta < 1$  is a discount factor. Since S is wasteful, it does not appear in (1). For simplicity, private consumption is assumed to be fixed and hence we only consider

the utility from public goods. This formulation may be justified since we assume that the tax rate on consumers is fixed and labor supply is exogenously given. Even if we consider private consumption explicitly in the social welfare, the analytical results are qualitatively the same. See section 5.6.

The local government conducts public investment  $k$  in period 1, which has a productive effect of raising tax revenue in period 2. Let  $Y_t$  represent total tax revenue of the two governments in period  $t$  ( $t = 1, 2$ ). We assume that  $Y_1$  is exogenously given but  $Y_2$  is dependent on public works conducted by the local government in period 1.  $Y_2 = Y_1 + f(k)$ .  $k$  denotes local public investment in period 1, which would increase total tax revenue of period 2. Investment product function  $f(\cdot)$  satisfies the standard Inada condition:  $f'(\cdot) > 0, f''(\cdot) < 0$ . For simplicity we do not consider public investment by the central government. In a multi-local government setting local public investment may have spillover effects over regions. However, in this paper since we consider the representative local government, we do not incorporate such spillovers. Still public investment has the vertical externality effect on the central government's tax revenue. See section 5.5.

Both central and local governments levy taxes on overlapping economic activities. Since the tax base is overlapping, the tax revenue is shared by the two governments. We set  $\beta$  as local government's portion of total tax revenue,  $0 < \beta < 1$ . The central government gains a portion of the total tax revenue. Thus  $1 - \beta$  means share of the central government to total tax revenue. The share parameter  $\beta$  is assumed to be exogenously given and constant over time. Or we could regard  $\beta Y$  as the subsidy from CG to LG. See also section 5.2.

We consider pork barrel spending by the local government. As shown in DelRossi and Inman (1999), pork barrel projects are too high due to subsidies from the central government caused by local governments' political demand. In the tradition of Leviathan models of government (see Brennan and Buchanan (1980) among others), the local politicians prefer "wasteful" public spending ( $S$ ), which provides them with rent-seeking opportunities but does not voters or consumers.

Next, we specify each government's budget constraint. The period-by-period budget constraints of CG are given as follows,

$$B = G_1 - (1 - \beta)Y_1 \quad (2-1)$$

$$G_2 + (1 + r)B = (1 - \beta)Y_2 \quad (2-2)$$

where  $B$  is the central government debt.  $r > 0$  is the exogenously given world interest rate.

The period-by-period budget constraints of LG are given as follows,

$$D = g_1 + k - \beta Y_1 + S \quad (3-1)$$

$$g_2 + (1 + r)D = \beta Y_2 \quad (3-2)$$

where  $D$  is the local government debt. Without loss of generality  $S$  appears only in

period 1.

From (2-1,2) and (3-1,2) we can rewrite the intertemporal budget constraints of the central and local government, respectively, as follows.

$$G_1 + \frac{G_2}{1+r} = (1-\beta)Y_1 + \frac{(1-\beta)Y_2}{1+r} \quad (2-3)$$

$$g_1 + \frac{g_2}{1+r} + k + S = \beta Y_1 + \frac{\beta Y_2}{1+r} \quad (3-3)$$

## 2.2 Pareto Efficient Solution

First of all, we investigate the Pareto efficient first best allocation in this model as a benchmark. Since we do not incorporate any uncertainty or imperfect information with respect to public investment and other government activities in the unitary system, unitary benevolent government, consolidating CG and LG, could attain the first best by allocating optimally the total tax revenues among nation-wide public goods and local public goods in each period. Namely, the unitary government, who implements the optimal allocation  $\{G_t, g_t, k\}$ , maximizes social welfare (1) subject to the following overall feasibility constraint

$$Y_1 + \frac{Y_2}{1+r} = G_1 + \frac{G_2}{1+r} + g_1 + \frac{g_2}{1+r} + k + S \quad (4)$$

which is obtained from (2-3) and (3-3) by eliminating  $\beta$ .

First order conditions of this optimization problem are as follows,

$$u_{G1} - \mu = 0$$

$$\delta u_{G2} - \frac{\mu}{1+r} = 0 \quad \text{where } u_{Gt} \equiv \frac{\partial u(G_t)}{\partial G_t}$$

$$v_{g1} - \mu = 0$$

$$\delta v_{g2} - \frac{\mu}{1+r} = 0 \quad \text{where } v_{gt} \equiv \frac{\partial v(g_t)}{\partial g_t}$$

$$\mu \left\{ \frac{f'(k)}{1+r} - 1 \right\} = 0$$

$$S=0$$

$\mu$  is the Lagrangian multiplier of equation (4). From these conditions we have

$$v_{g1} = u_{G1} \quad (5-1)$$

$$u_{G2} = v_{g2} \quad (5-2)$$

$$\frac{u_{G1}}{u_{G2}} = \frac{v_{g1}}{v_{g2}} = (1+r)\delta \quad (5-3)$$

$$f'(k) = 1+r \quad (5-4)$$

$$S=0 \tag{5-5}$$

The above optimality conditions (5-1,,5) and the feasibility condition (4) determine the Pareto efficient allocation as the benchmark case. Conditions (5-1)(5-2) mean that the marginal benefit of public goods is equalized between CG and LG. Condition (5-3) governs the standard (intertemporal) optimal allocation of public spending between two periods. Condition (5-4) is the standard first-best criterion of public investment. Finally, condition (5-5) is obviously the efficiency condition. Here exist any rent-seeking activities:  $S=0$ .

### 3. Outcome in a Decentralized System

#### 3.1 Benchmark Case of Decentralization: Game I

##### Unrestricted Scheme of Local Debt Issuance

We now investigate outcomes in a decentralized system of a multi-government non-cooperative world where benevolent central and rent-seeking local governments decide their policy variables non-cooperatively. CG is the leader and LG is the follower. The game is done at the beginning of period 1. We consider two cases with respect to the issuance of local public debt.

First of all we investigate the fully (or isolated) decentralized Nash equilibrium at the exogenously given  $\beta > 0$ , where there is no restriction of issuing LG' debt imposed by CG. Namely, in this game (Game I) at the first stage CG determines public goods  $G_1, G_2$ , and then at the second stage LG determines its expenditures,  $g_1, g_2, k, D$ , and  $S$ . Here we have the unrestricted scheme of local debt issuance, and hence LG may choose any amount of  $D$ .

CG maximizes (1) subject to (2-3) by choosing nation wide public goods. On the other hand, LG, who represents the interest of rent-seeking local politicians, maximizes wasteful public spending or rent,  $S$  by choosing local public goods and investment, subject to her budget constraint (3-3) and the following survival constraint.

$$v(g_1) + \delta v(g_2) = \bar{U} \tag{6}$$

where  $\bar{U}$  means the reservation utility which represents the preferences of voters. If (6) is not satisfied, voters do not re-elect them and local politicians cannot stay at the office of local government. It is plausible to assume that

$$\bar{U} < U^F \equiv v(g_1^F) + \delta v(g_2^F)$$

where  $g_1^F, g_2^F$  are the first best levels of  $g_1, g_2$ , respectively.

#### Second Stage

Let us investigate the outcome of this game, Game I. LG's problem is

$$\text{Max } S = \beta(Y_1 + \frac{Y_2}{1+r}) - (g_1 + \frac{g_2}{1+r} + k) \quad \text{st. (6)}$$

Then, first order conditions are as follows,

$$\begin{aligned} -1 - \psi v_{g_1} &= 0 \\ -\frac{1}{1+r} - \psi \delta v_{g_2} &= 0 \\ \frac{f'(k)\beta}{1+r} - 1 &= 0 \end{aligned} \quad (7-1)$$

where  $\psi$  is the Lagrangian multiplier of (6). Thus, we have

$$\frac{v_{g_1}}{v_{g_2}} = (1+r)\delta \quad (7-2)$$

From these conditions (6), (7-1)(7-2),  $g_1, g_2, k, D$ , and  $S$  are determined. Condition (7-2) means that the total expenditure on local public goods,  $g_1 + \frac{1}{1+r}g_2$  is minimized under the survival condition. (6) and (7-2) determine  $g_1, g_2$  in this game,  $g_1^*, g_2^*$ . (7-1) determines  $k$  in this game,  $k^*$ , and (3-2) determines  $S$ . Note that the optimal levels of  $g_1, g_2, k, D$ , and  $S$  are not dependent on CG's choice variables of  $G_1, G_2$ . Hence, the subgame perfect solution is the same as the one-shot Nash solution.

### First Stage

CG maximizes (1) subject to (2-3) by choosing nation wide public goods at given levels of  $g_1, g_2, k, D$ , and  $S$ , which are determined by LG from the second stage of the game. Then, we have

$$\begin{aligned} u_{G_1} - \Psi &= 0 \\ \delta u_{G_2} - \frac{\Psi}{1+r} &= 0 \end{aligned}$$

where  $\Psi$  is the Lagrangian multipliers of equations (2-3). Thus, we have

$$\frac{u_{G_1}}{u_{G_2}} = (1+r)\delta \quad (8)$$

Condition (8) means that social welfare  $u(G_1) + \delta u(G_2)$  is maximized under the given level of total expenditure,  $G_1 + \frac{1}{1+r}G_2$ , associated with  $k^*$ .

### Outcome

The subgame perfect outcome of this game is given by

$$f'(k) = \frac{1+r}{\beta} > 1+r \quad (7-1)$$

$$\frac{v_{g1}}{v_{g2}} = (1+r)\delta \quad (7-2)$$

$$\frac{u_{G1}}{u_{G2}} = (1+r)\delta \quad (8)$$

Conditions (7-2)(8), which are the same as (5-3), imply that relative (intertemporal) allocation between  $g_1$  and  $g_2$  as well as relative (intertemporal) allocation between  $G_1$  and  $G_2$  is efficient. But the levels of these public goods and local investment are not necessarily provided optimally. In other words, conditions (5-1)(5-2) do not necessarily hold since the total levels of public goods,  $G_1 + \frac{G_2}{1+r}$  and  $g_1 + \frac{g_2}{1+r}$ , are arbitrarily set, depending on the exogenous parameter,  $\beta$ , the rent-seeking behavior of LG, and the survival condition (6).

Moreover, considering  $\beta < 1$ , (7-1) means that  $k$  is under-provided due to the vertical externality of the overlapping tax base:  $k^* < k^F$ , where  $k^*$  is the solution of  $k$  in this game and  $k^F$  is the solution of  $k$  at the first best solution. Condition (5-4) does not hold. Since the local government does not take into account the positive spillover effect of increasing the overlapping tax base on public goods provided by the central government, local public investment provided by the local government is not sufficient and total tax revenue shared by both governments in period 2 is inefficiently low. Condition (5-5) does not hold either due to the rent-seeking behavior of LG.

To sum up, there are three sources of inefficiency in the decentralized system. First,  $\beta$  is not necessarily set at the optimal level and hence the allocation of public spending between CG and LG is not determined optimally. Second, there is a vertical externality of public investment due to the overlapping tax base, and hence  $k$  is too little. Finally, due to the rent-seeking activities of LG ( $\bar{U} < U^F$ ), local public goods  $g_1, g_2$  are too little and wasteful public spending  $S$  becomes positive.

### 3.2 Restriction of D: Game II

Suppose CG cannot control the rent seeking behavior of LG, but can control local debt issuance. We now consider the restricted scheme of local debt case where CG can determine the amount of  $D$  at the first stage of this game as  $\bar{D}$  in Game II. Namely, CG can set any level of  $\bar{D}$  in order to maximize the social welfare. In the second stage LG determines  $g_1, g_2, k$  at given level of  $\bar{D}$ . In this game LG faces a

sort of liquidity constraint. The subgame perfect equilibrium is not necessarily the same as the one-shot Nash equilibrium.

### Second Stage

LG maximizes

$$S = \bar{D} + \beta Y_1 - g_1 - k$$

by choosing  $g_1$  and  $k$  subject to the following survival constraint

$$v(g_1) + \delta v(\beta Y_2 - (1+r)\bar{D}) = \bar{U}. \quad (6)'$$

Then from the first order conditions we have

$$\delta \beta f' = \frac{v_{g1}}{v_{g2}} \quad (9)$$

Since local debt is not used for intertemporal transfer by LG, the effective relative price between  $g_1$  and  $g_2$  is now given as  $\delta \beta f'$ , which is the (discounted) effective marginal product of local investment.

From this condition (9) and the survival condition (6)' we may derive the response functions of LS as

$$g_1 = \tilde{g}(\bar{D}) \quad (10-1)$$

$$k = \tilde{k}(\bar{D}) \quad (10-2)$$

In Figure 1 the SS curve shows a locus of  $(g_1, k)$  which satisfies condition (6)'. This curve is downward sloping. The FOC curve shows a locus of  $(g_1, k)$  which satisfies condition (9). This curve is upward sloping. An increase in  $\bar{D}$  would shift both curves to the right. Hence, it would stimulate  $k$ , and its effect on  $g_1$  is also positive.  $\tilde{k}'(\bar{D}) > 0$  and  $\tilde{g}'(\bar{D}) > 0$ . Intuition is as follows. When  $\bar{D}$  rises at a given level of  $k$ ,  $g_2$  declines and  $v_{g2}$  rises, which raises the right hand side of (9), the marginal benefit of transfer from period 1 to period 2. Hence, it would stimulate  $k$ .  $g_1$  has to rise to maintain (6)'.

Let us discuss the economic implication of (9). When both (7-1) and (7-2) hold, we get (9). In other words, if  $\bar{D}$  happens to be equal to the solution of  $D$  in Game I,  $D^*$ , then Game II produces the same outcome as Game I. On the contrary, if  $\frac{v_{g1}}{v_{g2}} < (1+r)\delta$ , then (9) implies  $\beta f' < 1+r$  (and vice versa). In such a case  $k$  is greater than in Game I. Since  $\tilde{k}'(\bar{D}) > 0$ , this case corresponds to  $\bar{D} > D^*$ .

We can show that  $S$  is always smaller than in Game I. In Game II the total

spending on local public goods  $g_1 + \frac{1}{1+r}g_2$  is larger than in Game I. And, so long as  $\bar{k}^*$ , the solution of  $k$ , is not equal to  $k^*$ ,  $\frac{\beta Y_2}{1+r} - k$  is smaller than in Game I. Hence,  $S$  becomes smaller at Game II than at Game I.

### First Stage

At the first stage of game, CG maximizes the national welfare (1) subject to its budget constraints and the response functions of local governments by choosing  $\bar{D}$  as well as nation-wide public goods  $G_1, G_2$  and public debt  $B$ .

$$\text{Max } u(G_1) + v(\tilde{g}(\bar{D})) + \delta\{u(G_2) + v[\beta f(\tilde{k}(\bar{D})) - (1+r)\bar{D}]\}$$

subject to

$$G_1 + \frac{1}{1+r}G_2 = (1-\beta)[Y_1 + \frac{1}{1+r}f(\tilde{k}(\bar{D}))]$$

Then, the first order conditions are

$$u_{G_1} + \lambda = 0$$

$$\delta u_{G_2} + \lambda \frac{1}{1+r} = 0$$

$$v_{g_1}\tilde{g}' + \delta v_{g_2}[\beta f'\tilde{k}' - (1+r)] - \lambda \frac{(1-\beta)f'\tilde{k}'}{1+r} = 0$$

where  $\lambda$  is the Lagrange multiplier of the above budget constraint of CG.

From these equations we have (8) and

$$\frac{v_{g_1}\tilde{g}'}{u_{G_1}} + \frac{v_{g_2}\tilde{k}'\beta f'}{u_{G_2}(1+r)} + \frac{(1-\beta)f'\tilde{k}'}{1+r} = \frac{v_{g_2}}{u_{G_2}} \quad (11)$$

The left hand side of (11) means a marginal benefit of an increase in  $\bar{D}$ , while the right hand side means a marginal cost of an increase in  $\bar{D}$ . Namely, an increase in  $\bar{D}$  stimulates  $k$  and hence tax revenues of CG, resulting in an increase in  $G_1, G_2$ , which is the merit. On the other hand, it directly reduces  $g_2$  by raising interest payments, which is the cost. Condition (11) determines the optimal level of  $\bar{D}$ ,  $\bar{D}^*$ , where the marginal benefit is equal to the marginal cost. The subgame perfect solution of  $\bar{D}$ ,  $\bar{D}^*$ , is not necessarily equal to the equilibrium value in Game I,  $D^*$ . When  $D^* = \bar{D}^*$ , then  $k^* = \bar{k}^*$  and hence the solution of Game II is identical to that of Game I.

### Remark

Under the constraint (6) or (6)' the ex post social welfare may be rewritten

as

$$W = u(G_1) + \delta u(G_2) + \bar{U} \quad (1)'$$

Since  $\bar{U}$  is exogenously given, ex post  $W$  is only dependent on CG's provision of public goods. In both Games I and II CG faces the same budget constraint (2-3) and chooses the same optimality condition with respect to nation-wide public goods (8). It follows that the total utility from provision of  $G_1, G_2$  is maximized at given level of  $k$ . Thus, if  $k$  is higher, then the right hand side of (2-3) is larger, and the resulting social welfare is also higher.

When CG takes the survival condition into her optimizing behavior, then it would be optimal to stimulate  $k$  as much as possible by raising  $\bar{D}$ . However, the optimizing behavior in this section does not assume this possibility. Namely, we assume that even if CG knows LG's response functions (10-1)(10-2) well, CG does not consider the survival condition (6). In such a case (11) gives the optimal level of  $\bar{D}$ ,  $\bar{D}^*$ , which is not necessarily larger than  $D^*$ ,  $D$  in Game I. In other words, if  $\bar{D}^* > D^*$ , then Game II produces higher welfare than Game I (and vice versa).

On the other hand, if CG takes the survival condition into her optimizing behavior, then CG intends to increase  $\bar{D}$  as much as possible. In reality it is possible to restrict the amount of  $D$  to the level less than  $D^*$ , but it is not easy to set  $D$  to the level more than  $D^*$ . One plausible assumption is that CG could control  $D$  only under the condition of  $\bar{D} < D^*$ . If this condition is imposed, then the optimal level of  $\bar{D}$ ,  $\bar{D}^*$ , is equal to  $D^*$ . In other words, if CG takes the survival condition into her optimizing behavior, then it is optimal for CG not to restrict the issuance of local debt finance.

## 4 Additional Transfers

### 4.1 Game I

#### CG's Ex Post Transfer: Third Stage

In Game I, when period 2 comes, CG may not want to commit to the initial level of  $\beta$ . CG may effectively raise  $\beta$  by creating grants to LG ex post. This is a time inconsistency problem. Thus, LG faces to the soft-budget constraint. We first investigate the optimizing behavior of CG at the beginning of the second period as the third stage of this game. After LG determines local expenditures,  $g_1$ ,  $S$  and  $k$ , in period 1, CG may effectively choose its public spending  $G_2$  and  $g_2$  subject to the budget conditions (2-2) and (3-2) by creating an additional grant,  $A$ , appropriately in period 2.

The budget constraint of the central government in period 2 is rewritten as

$$G_2 + (1+r)B = (1-\beta)Y_2 - A \quad (2-2')$$

Similarly, the budget constraint of the local government in period 2 is rewritten as

$$g_2 + (1+r)D = \beta Y_2 + A \quad (3-2)'$$

From (2-2') and (3-2') eliminating  $A$  gives the relevant overall budget constraint in period 2 as

$$G_2 + g_2 + (1+r)(B+D) = Y_2 \quad (12)$$

By choosing  $A$  ex post in period 2, the central government may in fact choose the allocation of  $G_2$  and  $g_2$  under the above overall constraint (12) to maximize the social welfare in period 2;  $u(G_2) + v(g_2)$ . The rent seeking activity has already done in period 1. Here at ex post the survival condition (6) is no longer binding. Thus, the first-order condition at the third stage of this game is given by

$$u_{G_2} = v_{g_2} \quad (13)$$

From the above optimality condition (13) and the ex post budget constraints (2-2'), (3-2'), at given levels of local expenditures  $D$  and  $k$ , which are chosen in period 1, we may derive the optimal response of  $A$ ,  $g_2$  (and hence  $G_2$ ) of the central government as functions of  $D$  and  $k$ , respectively.

$$A = J(D, k) \quad (14-1)$$

$$g_2 = P(D, k) \quad (14-2)$$

By totally differentiating the budget conditions (2-2') and (12) and the optimality condition (5-2), we have

$$dG_2 + dg_2 + (1+r)dD = f'(k)dk$$

$$(1-\eta)dG_2 = \eta dg_2$$

$$dG_2 = (1-\beta)f'(k)dk - dA$$

where  $\eta \equiv \frac{v_{g_2}}{[|u_{G_2}| + |v_{g_2}|]}$  means the relative evaluation of  $G_2$  compared with  $g_2$ . It is assumed for simplicity that  $0 < \eta < 1$  is constant. Then, considering (2-2'), we have as the property of response functions

$$J_D = \frac{\partial A}{\partial D} = -\frac{\partial G_2}{\partial D} = \eta(1+r) > 0 \quad (15-1)$$

$$J_k = \frac{\partial A}{\partial k} = -\frac{\partial G_2}{\partial k} + (1-\beta)f'(k) = (1-\beta)f'(k) - \eta f'(k) \quad (15-2)$$

$$P_D = \frac{\partial g_2}{\partial D} = -(1-\eta)(1+r) < 0 \quad (15-3)$$

$$P_k = \frac{\partial g_2}{\partial k} = (1-\eta)f'(k) \quad (15-4)$$

(15-1) shows the standard outcome of the soft budget constraint due to local debt (See Goodspeed (2002)). An increase in  $D$  results in a decrease in  $g_2$  at the given level of  $G_2$ , leading to more grants  $A$  from the central government.  $J_D > 0$ .

Intuition is as follows. When more debt  $D$  is issued,  $g_2$  falls from (3-2) while  $G_2$  rises from (2-2). This outcome is not good for the central government since it would like to realize the optimality condition (5-2) to raise social welfare. Thus, the central government has an incentive to make additional subsidies to the local government in period 2 to raise the ex post level of  $g_2$  and reduce the ex post level of  $G_2$ .

Moreover, we have another outcome of the soft-budget result due to public investment,  $J_k > 0$ , which is a new channel due to the vertical externality. As shown in (15-2), the sign of  $J_k$  is generally ambiguous. If  $1-\beta > \eta$ , then  $J_k > 0$  (and vice versa). That is, if the marginal valuation of  $G_2$  is relatively small and  $1-\beta$  is too high,  $g_2$  is too low compared with  $G_2$ , and hence the central government would react to increase  $A$  in order to maximize the ex post social welfare. Intuition is as follows. An increase in  $k$  results in an increase in the tax revenue of CG by the amount of  $(1-\beta)f'$  and an increase in  $G_2$  by the amount of  $\eta f'$ . If  $1-\beta > \eta$ , CG would react to give more grants  $A$  to LG.

A key part of the model is the interaction between the central government and the local government. The central government intends to allocate revenues to equalize marginal gains of public goods between the central and local governments. The central government's benevolent incentives result in a soft budget constraint by creating additional grants in period 2 when the local government borrows more and invest more in period 1. Two channels through debt issuance and public investment cause the soft budget outcome. First, local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy. Second, local investment more in period 1 means an increase in the tax revenue of the central government in period 2 and hence raises the amount of central public goods. Then, the central government intends to maximize the social welfare in period 2 by making additional grants in period 2 in response to local borrowing more and public investment more in period 1.

### **LG's Behavior: Second Stage**

We now investigate the optimizing behavior of the local government at the beginning of period 1 in the soft-budget version of Game I. The local government's survival constraint (6) is effectively binding here only under the condition that the central government changes  $A$  in response to local expenditures of period 1, as summarized by equation (14-1,2). Namely, the survival condition and the effective budget constraint for the local government are given by

$$v(g_1) + \delta v(P(D, k)) = \bar{U} \quad (6)'$$

$$g_1 + \frac{P(D,k)}{1+r} + k + S = \beta Y_1 + \frac{\beta[Y_1 + f(k)]}{1+r} + \frac{J(D,k)}{1+r} \quad (16)$$

The local government maximizes the objective S subject to (6)' (16) at given levels of tax share parameter  $\beta$ . Note that at this stage the survival condition (6)' is binding.

Therefore, the first order conditions with respect to its policy variables,  $g_1$ ,  $D$ , and  $k$ , are respectively given as follows,

$$-1 - \omega v_{g_1} = 0 \quad (17-1)$$

$$-\frac{P_D - J_D}{1+r} - \omega \delta v_{g_2} P_D = 0 \quad (17-2)$$

$$-(1 + \frac{P_k}{1+r} - \frac{\beta}{1+r} f'(k) - \frac{J_k}{1+r}) - \omega v_{g_2} P_k = 0 \quad (17-3)$$

where  $\omega (>0)$  is the Lagrange multiplier of constraint (6)'. Equations (17-1,2) govern the allocation of  $g_1$  and  $g_2$  at a given level of tax share parameter  $\beta$ , and  $\bar{U}$ .

Substituting (15-1,3) into (17-1)(17-2), we have

$$v_{g_1} = \delta v_{g_2} (1 - \eta)(1 + r) \quad (18-1)$$

Thus, the optimality condition between  $g_1$  and  $g_2$  given by (6) is not realized here at the subgame perfect solution. If CG did not make additional grants  $A$ , the optimizing behavior of LG could have attained condition (5-3) with respect to the relative allocation of  $g_1$  and  $g_2$ . When LG takes into account the response functions of CG, (14-1,2), it would effectively reduce the marginal cost of raising  $g_1$ , stimulating  $g_1$  in period 1. (18-1) means that  $g_1$  is too high, compared with  $g_2$  and  $G_2$ . Moreover, the total expenditure on local public goods,  $g_1 + \frac{1}{1+r} g_2$ , is larger than in section 3.1. The soft budget constrain would result in an increase in  $A$ , which has a positive effect on  $g_1$ .

Next, substituting (15-2,4) into (17-2), we have

$$v_{g_1} = \delta(1 - \eta) f' v_{g_2} \quad (18-2)$$

Considering (18-1) and (18-2), we finally get

$$1 + r = f' \quad (5-4)$$

It follows that at the subgame perfect solution  $k$  is larger than in section 3.1. This is a plausible result of the soft budget constraint. When  $k$  rises, LG may expect additional grants  $A$  from CG resulting from an increase in  $(1 - \beta)Y_2$  in addition to its own tax revenue  $\beta Y_2$ , so that the effective marginal benefit of an increase in  $k$  becomes  $f'$ , not  $\beta f'$ . Note that the first stage of this game is the same as in section 3.1. CG determines  $G_1, G_2$  to attain (8).

### Welfare Comparison

We have shown that the soft budget constraint stimulates public investment. However, at the same time  $A$  is generally positive, which hurts the social welfare. We now investigate the overall welfare effect of soft-budget constraint. When  $A$  is explicitly included, the ex post budget constraint of CG is given as

$$G_1 + \frac{1}{1+r} G_2 = (1-\beta)(Y_1 + \frac{1}{1+r} Y_2) - \frac{1}{1+r} A \quad (2-3)'$$

An increase in  $k$  may raise  $A$  directly and indirectly, as shown in (15-2) and (15-1). On the other hand, when  $k$  is raised, it can enlarge the tax revenue of CG. The overall impact of an increase in  $k$  on CG' net tax revenue excluding  $A$ , the right hand side of (2-3)', is

$$R \equiv (1-\beta) f' - (J_k + J_D \frac{dD}{dk}) \quad (19)$$

where considering (3-2)', we have

$$\frac{dD}{dk} = \frac{\beta f'}{1+r} + \frac{1}{1+r} (P_k + P_D \frac{dD}{dk}) - \frac{1}{1+r} (J_k + J_D \frac{dD}{dk})$$

Or we finally obtain

$$\frac{dD}{dk} = \frac{\beta f'}{(1-\eta)(1+r)} \quad (20)$$

Then, substituting (20) into (19), we have

$$R = \frac{(1-\beta-\eta)\eta}{1-\eta} f' \quad (19)'$$

If  $1-\beta-\eta > 0$ , then the left hand side of inequality (19),  $R$ , is positive and an increase in  $k$  raises the right hand of (2-3)', which is beneficial. In such a case, the soft-budget version of Game I actually raises the social welfare. On the other hand, if  $1-\beta-\eta < 0$ , then  $R$  is negative. This undesirable outcome could occur since  $A$  may be too much and the total spending on local public goods to maintain the reservation utility  $\bar{U}$  also increase much. In such a case, an increase in  $k$  does not enlarge the overall tax revenues available to CG, so that spending on central public goods would decrease.

## 4.2 Game II

### CG's Transfer: Third Stage

We then investigate Game II. Here again CG faces the same time inconsistency problem. Namely, after LG determines local expenditures,  $g_1$ ,  $S$  and  $k$ , in period 1, CG may effectively choose its public spending  $G_2$  and  $g_2$  subject to the budget conditions (2-2) and (3-2) by creating an additional grant,  $A$ , appropriately in

period 2. We consider the optimizing behavior of CG at the beginning of the second period in Game II as the third stage of this game. The analytical results are almost the same as in Game I.

Namely, in place of (14-1,2) we have

$$A = J(\bar{D}, k) \quad (21-1)$$

$$g_2 = P(\bar{D}, k) \quad (21-2)$$

Since  $\bar{D}$  is already given, LG's choice would affect CG's behavior only through  $k$ . The soft budget does not have the channel through debt issuance. It only comes from the channel through local investment.

By totally differentiating the survival condition (6), the optimality condition (5-2) and (13), we have

$$dg_2 = \beta f' dk + dA$$

$$(1 - \eta)dG_2 = \eta dg_2$$

$$dG_2 = (1 - \beta)f'(k)dk - dA$$

Then, considering (2-2'), we have as the property of response functions

$$J_k = \frac{\partial A}{\partial k} = (1 - \beta - \eta)f'$$

$$P_k = \frac{\partial g_2}{\partial k} = f'(1 - \eta) > 0$$

which are the same as (15-2) and (15-4).

### LG's Behavior: Second Stage

We now investigate the optimizing behavior of the local government at the beginning of period 1. As in section 4.1, the local government's survival constraint (6) is effectively binding here only under the condition that the central government changes  $A$  in response to local expenditures of period 1, as summarized by equation (20-1,2).

Hence, LG maximizes

$$S = \bar{D} + \beta Y_1 - g_1 - k$$

subject to the survival constraint

$$v(g_1) + \delta v(P(k)) = \bar{U}.$$

Then from the first order conditions we have

$$\delta(1 - \eta)f' = \frac{v_{g_1}}{v_{g_2}} \quad (18-2)$$

If (18-1) holds here, then as in Game I we have (5-4) in the soft-budget case. However, (18-1) does not necessarily hold. Let us compare the soft budget version of Game II with the hard budget version of game II. In the hard budget version of

Game II we have (9). Since  $v_{g1}/v_{g2}$  is not necessarily the same value in both cases, we do not know that  $k$  is really higher in the soft budget case than in the hard budget case. When CG controls the local debt issuance, the soft budget constraint does not necessarily stimulate public investment. If  $1-\beta = \eta$ , both games have the same solution. If  $1-\beta > \eta$ , in the soft budget version  $k$  is larger than in the hard budget version, and vice versa.

We could analyze the first stage of this game. Since CG again intends to attain (8), the property of first game is almost the same as in section 3.2.

### **Welfare Comparison**

Let us investigate the impact of an increase in  $k$  on CG's tax revenue. Since  $D$  is fixed, we have now

$$R = (1-\beta)f' - J_k = \eta f' > 0 \quad (19)$$

Unlike Game I this value is always positive. In other words, an increase in  $k$  always raises the right hand side of (2-3)', which is beneficial. If  $1-\beta > \eta$  and hence  $k$  is promoted, the resulting welfare in the soft-budget version of Game II is always greater than that in the hard-budget version of Game II. On the contrary, if  $1-\beta < \eta$  and hence  $k$  is depressed, then the soft budget constraint hurts the social welfare. In this sense, the welfare implication of soft budget constraint is qualitatively the same as in Game I.

## **5. Comments**

### **5.1 Welfare Implication of Soft Budget**

Several comments are useful. We call the committed versions of Games I and II the hard-budget games, while uncommitted versions of Games I and II the soft-budget games, respectively. When the central government commits to a predetermined value of  $\beta$  as the leader of intergovernmental game between central and local governments, the local government is subject to the hard budget constraint. However, in this case the outcome is not the first best due to several factors such as the predetermined value of  $\beta$ , the rent seeking activity of LG, and the vertical externality of  $\beta$ . Although the central government may not attain the first best by simply choosing  $\bar{D}$  appropriately, CG could raise the social welfare by increasing  $\bar{D}$ .

We have also shown that ex post the central government does not have an incentive to commit to a predetermined value of  $\beta$  in period 2 and may add new grants  $A$  after the local government determines its expenditures. Then, the local government is subject to the soft budget constraint. Namely, when the local government raises local expenditures and borrowing in period 1, the central government has an incentive to support such larger local expenditures and

borrowing by creating additional subsidies to the local government after the rent seeking activity was done. It follows that in such a game the local government has a strong incentive to increase the local expenditures and borrowing in period 1. The central government may respond to such demand in period 2 by overthrowing the commitment if the marginal valuation of  $G_2$  is relatively small and/or a predetermined level of tax share parameter  $\beta$  is too low at the subgame perfect solution ( $1 - \beta > \eta$ ).

Two channels through debt issuance and public investment cause the soft budget outcome. First, local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy, as stressed by Goodspeed (2002). Second, public investment more means an increase in the revenue of central government and hence the central government may subsidize the local government more in period 2, which is a new channel explored in this paper.

We have shown that in the unrestricted scheme of local debt issuance the two channels work together. The soft budget would always stimulate  $k$ , which is beneficial, while it would also stimulate  $A$ , which is not beneficial. If  $1 - \beta - \eta > 0$ , its positive welfare effect dominates the negative welfare effect of additional grants. We have also explored the ambiguous effect of soft budget in Game II on public investment in the restricted scheme of local debt issuance because the channel through debt issuance does not work here. If  $1 - \beta < \eta$ , the soft budget does not stimulate efficient public investment and hence it does not raise the social welfare. However, in this case an increase in  $k$  is always desirable. Hence, the welfare implication of the soft budget is qualitatively the same between two games, depending on the sign of  $1 - \beta - \eta$ .

## 5.2 Choosing $\beta$

### First Stage

We now consider the case where CG may choose the optimal level of  $\beta$ . In Game I LG's optimizing behavior at the second stage is the same as in section 3. Thus, we consider the optimizing behavior of CG at the first stage below.

At the first stage of Game I, the central government maximizes the national welfare (1) subject to its budget constraints (2-3), and the response functions of local government by choosing parameter,  $\beta$  as well as nation-wide public spending  $G_1$ ,  $G_2$  and public debt  $B$ . Then we have

$$\text{Max } u(G_1) + v(g_1) + \delta\{u(G_2) + v(g_2)\}$$

subject to

$$G_1 + \frac{1}{1+r}G_2 = (1-\beta)[Y_1 + \frac{1}{1+r}f(\widehat{k}(\beta))]$$

where  $\widehat{k}(\beta)$  is LG's response function with respect to  $\beta$ . It is easy to show than an increase in  $\beta$  stimulates  $k$  but its effects on  $g_1, g_2$  are null.

$$\widehat{k}_\beta \equiv \frac{dk}{d\beta} = -\frac{f'}{f''\beta} > 0.$$

Then, the first order conditions are (8) and

$$\frac{f - (1-\beta)f' \widehat{k}_\beta}{1+r} = 0 \quad (23)$$

The optimal level of  $\beta$  is given by

$$\frac{1-\beta}{\beta} = -\frac{f''f}{f'f'} \quad (24)$$

Let us investigate the welfare implication. In the hard budget version of Game I the ex post welfare is maximized when  $(1-\beta)Y_2$  is maximized. The impact of an increase in  $\beta$  on  $(1-\beta)Y_2$  is

$$-f + (1-\beta)f' \widehat{k}_\beta = -f - \frac{(1-\beta)f'f'}{\beta f''} \quad (25)$$

which reduces to zero since (23) holds. Thus, the ex post welfare is maximized by choosing  $\beta$  although it does not necessarily attain the first best solution.

### 5.3 Benevolent LG

Suppose that LG is also benevolent. Then, LG maximizes (1) subject to (3-3) at  $S=0$  by choosing local public goods and investment, while assuming nation-wide public goods fixed. Instead of the survival constraint (6), the following budget constraint is now binding.

$$g_1 + \frac{g_2}{1+r} + k = \beta Y_1 + \frac{\beta Y_2}{1+r} \quad (26)$$

The first order conditions of LG in Game I or II are the same as in section 2. For example at Game II LG now maximizes

$$v(\bar{D} + \beta Y_1 - k) + \delta v(\beta Y_2 - (1+r)\bar{D})$$

by choosing  $k$ . The optimality condition reduces to (9). The analytical results in section 3 would also be qualitatively the same.

As to the welfare implication, there are some differences between rent-seeking LG and benevolent LG, depending on the size of  $S$ . Since  $S=0$  in the case of benevolent LG, welfare is higher there. However, the welfare implication of soft-budget constraint may be qualitatively the same. In the case of benevolent LG,

an increase in  $A$  would always raise the ex post welfare from  $g_1, g_2$ . Thus, if the welfare from  $G_1, G_2$  can be raised by putting additional transfer  $A$  to LG and hence by stimulating  $k$ , the total social welfare is also raised.

As shown in 4.1 and 4.2, the welfare effect from  $G_1, G_2$  is ambiguous, depending on the sign of  $1 - \beta - \eta$ . Since we now have the positive welfare effect due to raising  $g_1, g_2$ , the overall effect may well be beneficial.

#### 5.4 Debt Financed Public Investment

We consider the restriction that local debt is only used for financing public investment.

$$D = k$$

Then, the optimizing problem of LG at the second stage of Game I is to maximize

$$S = \beta Y_1 - g_1$$

subject to

$$v(g_1) + \delta v(\beta Y_2 - (1+r)k) = \bar{U} \quad (27)$$

The optimality conditions are (27) and

$$\beta f'(k) = 1+r \quad (7-1)$$

The level of  $k$  is the same as in Game I. However, we do not necessarily obtain (7-2). Thus, the total expenditure on local public goods,  $g_1 + \frac{1}{1+r}g_2$ , is greater than in Game I, resulting in lower social welfare than in Game I. The main analytical results concerning the soft budget constraint would qualitatively be the same.

#### 5.5 Spillover Effect of Local Public Investment

The vertical externality is the key factor to obtain the result that in the hard budget constraint local public investment is too low. If we incorporate spillover effect of local public investment we could get the similar result as well. Namely, local public investment becomes too low in a world of multi-local governments where each local government determines its public investment non-cooperatively. In this sense the assumption of a given share of tax revenue between the local and central governments is not essential. We could obtain the similar analytical results if we incorporate the spillover effect of local public investment in a world of multi-local governments.

#### 5.6 Inclusion of Private Consumption

If we explicitly incorporate private consumption into the model, consumers' budget constraint may be given as

$$c_1 + \frac{1}{1+r}c_2 = (1-t)y$$

where  $c_1, c_2$  are private consumption in period 1 and period 2,  $t$  is a tax rate, and  $y$  is income. Then we may define the tax revenue as  $ty = Y$ . Since  $Y$  is an increasing function of  $k$ ,  $y$  is also an increasing function of  $k$  at an exogenously given tax rate. Thus, the welfare from private consumption is increasing with  $k$  as well. In section 4 we have shown that if  $k$  is raised due to the soft budget constraint, it may be better. Such an analytical implication holds even if we consider the welfare from private consumption. In other words, the main analytical results hold when we explicitly incorporate private consumption.

## 6. Concluding Remarks

In this paper, we have investigated theoretically the soft-budget constraint with grants from the central government to the local government by clarifying the vertical externality of local expenditures due to overlapping tax bases between two governments using a two-period model. We have also explicitly incorporated political rent seeking activities by local politicians to explore both the benefit and cost of soft budget constraint.

The central government's benevolent incentive results in creating a soft budget constraint by increasing grants. Two channels through debt issuance and public investment cause the soft budget outcome. First, local borrowing more in period 1 means a decline in local public goods in period 2 and hence upsets the central government's optimal allocation strategy, as stressed by Goodspeed (2002). Second, public investment more means an increase in the revenue of central government and hence the central government may subsidize the local government more in period 2, which is a new channel explored in this paper.

It is interesting to note that public investment becomes too little due to the vertical externality of tax revenues. Hence the soft budget constraint may well improve social welfare although it induces rent-seeking activities. We have shown that in the unrestricted scheme of local debt issuance the soft budget constraint is effective to stimulate both efficient local expenditures and additional grants because the two channels work together. If the marginal valuation of  $G_2$  is relatively small and/or a predetermined level of tax share parameter  $\beta$  is too low at the subgame perfect solution ( $1 - \beta > \eta$ ), then the soft budget case is welfare improving. We have also shown that in the restricted scheme of local debt issuance the channel through debt issuance does not work. Thus, the soft budget constraint may not stimulate public investment and hence may not be better than in the hard budget constraint. Here again the welfare implication depends on the sign of  $1 - \beta - \eta$ .

We could regard the soft budget constraint with local debt control the most

interdependent case of intergovernmental financing and the hard budget constraint without local debt control the most independent case of intergovernmental financing. We may say that the degree of decentralization is very high in the latter case, while the degree of decentralization is very low in the former case. When we consider some positive externalities of local public investment such as the vertical externality, these extreme cases may not perform very well. Namely, the relatively independent case of the soft budget constraint without debt control may perform better than the hard budget constraint without debt control if  $1 - \beta - \eta > 0$ . The welfare implication of soft budget constraint with debt control is generally ambiguous, and it could be better than the hard budget constraint without debt control if  $1 - \beta - \eta > 0$ . Our analysis have shown that the marginal valuation of  $G_2$  and the level of tax share parameter  $\beta$  are crucial to evaluate the relative performance of soft budget constraint and local debt control by the central government.

## Appendix 1: Multiple Local Governments

Suppose there are  $n$  ( $\geq 2$ ) local governments. If we define the total amount of local public goods as  $g_1, g_2$  and each local government's supply of public goods as  $g_1^i, g_2^i$ , then we have

$$g_1 = \sum_{i=1}^n g_1^i, \quad g_2 = \sum_{i=1}^n g_2^i \quad (\text{A1})$$

The social welfare (1) is now rewritten as

$$W^i = u(G_1) + v(g_1^i) + \delta \{u(G_2) + v(g_2^i)\} \quad (\text{A2})$$

where  $W^i$  is the social welfare in the representative agent in region  $i$ .

$$W = \sum_{i=1}^n W^i$$

We may define other variables of local governments as in (A1). Then the budget constraints of CG and LG are the same as in the text. For simplicity suppose all local governments are identical. It follows that in the section of 2.2 the first best conditions are given by

$$v_{g_1} = nu_{G_1} \quad (\text{A3-1})$$

$$u_{G_2} = nv_{g_2} \quad (\text{A3-2})$$

and (5-3)(5-4). (A3-1) and (A3-2) correspond to the well-known Samuelson condition of the pure public good,  $G$ . We have analytically the same results as in sections 3 and 4.

Regarding the game between CG and LG, we may assume that each LG behaves non-cooperatively and regards other LG's choice variables given. Then, the analytical results are the same as in the text. For example, (12) may be rewritten as

$$G_2 + g_2^i + \sum_{j \neq i} g_2^j + (1+r)(B + D^i + \sum_{j \neq i} D^j) = Y_2 \quad (\text{A4})$$

Then, central government's response functions are in place of (14-1)(14-2) given as

$$A^i = J^i(D^i, k^i) \quad (\text{A5-1})$$

$$g_2^i = P^i(D^i, k^i) \quad (\text{A6-2})$$

Similarly, we have

$$dG_2 + dg_2^i + (1+r)dD^i = f'(k^i)dk^i$$

$$(1-\eta)dG_2 = \eta dg_2^i$$

$$dG_2 = (1-\beta)f'(k^i)dk^i - dA^i$$

Hence, we have (15-1,2,3,4) as in the text.

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Table 1

	Game I		Game II	
	Hard budget	Soft budget	Hard budget	Soft budget
$v_{g1}/v_{g2}$	$(1+r)\delta$	$(1+r)(1-\eta)\delta$	$\beta f'\delta$	$(1-\eta)f'\delta$
$u_{G1}/u_{G2}$	$(1+r)\delta$	$(1+r)\delta$	$(1+r)\delta$	$(1+r)\delta$
$k$	$1+r = \beta f'$	$1+r = f'$	$k(\text{SD}) > k(\text{HD})$ if $1-\beta > \eta$	
$W$	$W(\text{HD}) < W(\text{SD})$		$W(\text{SD}) > W(\text{HD})$ if $1-\beta > \eta$	

Figure 1

