National Adversity: Managing Insurance and Protection

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Abstract

This paper concerns self-insurance and self-protection that countries may implement at a national level in pursuit of their security. The distinctions self-insurance, self-protection, and market insurance were first made by Ehrlich and Becker (1972). Nevertheless, extension of their models to international security where market insurance for entire countries is usually unavailable is surprisingly sparse. We show that, in the absence of market insurance, self-insurance alone raises important new issues as to the definition of "fair pricing" and as to the relations between pricing, optimization, risk aversion, and inferiority that are significantly different from standard, conventional market analysis. We also discover a hitherto unrecognized tendency for misallocation between self-protection and self-insurance when both are available and considered together. Because of external effects running from self- protection to self-insurance, governments trying to find the right balance face incentives that encourage extreme, self-inflicted moral hazard, to the detriment of self-protection. Moreover, rather innocuous assumptions concerning countries' preferences lead to pervasive goods inferiority for both insurance and protection. Consequently, unlike the conventional wisdom in the economic theory of alliances with its neat interior Nash-solutions, we show when "defense" or "security" is disaggregated into more realistic categories such as protection and insurance that instabilities and corner solutions will be the conventional standard. The resulting perverse incentives that we discover point to conflicts and other difficulties among agents trying to cooperate in their management insurance and protection must overcome. Moreover the analysis implies that the paradigm Olson-Zeckhauser model (1966) model for alliance allocative behavior was fundamentally insufficient for the problem it was designed to address.

Keywords: self-insurance, self-protection, actuarially fair condition, inferior goods, alliances, public goods.

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1. Introduction

Among the first to examine combinations of instruments open to individuals to manage risks to their well-being were Ehrlich and Becker ("EB," 1972). EB identified several types of preparation available to expected utility maximizing individuals faced with what we will call "costs of emergency." These costs consist of any mix of (a) probability of loss and (b) magnitude of loss (hereafter together referred to as "risk profile"). Among such preparations were (a) "self insurance" to compensate for or reduce the magnitude of loss (b) "self-protection" to reduce the probabilities of loss --- neither of which involved market choice or market insurance --- and (c) insurance purchased from others in a market.

It would now be generally agreed that these ideas apply to entire societies attempting to cope with diverse conflicts. Governments need not passively accept risks that production and/or consumption decline in unwelcome situations. The EB distinctions apply with respect to nations also. For them diverse instruments exist to manage national adversity including self-insurance and self-protection. Most governments, at present, treat them as totally separate. Separation of security provision into self-protection and self-insurance is a bad idea. There needs to be much more coordination between these two seemingly disparate functions of government.

Moreover, security benefits may spill over becoming a public goods among countries (Olson and Zeckhauser, 1966) so that they interact in their allocations of national income to international (or regional) welfare and safety. For example, formation of international organizations, collective military preparedness, active international diplomacy, and foreign aid may reduce the probabilities of regional and international tension. And special trading agreements or collaborative stockpiling or common strategic defenses may provide mutual self-insurance to reduce or offset emergency losses. Nevertheless, utilization of EB to model collective improvements of the entire "risk profile" as an international public good is sparse, except for some work on terrorism such as Sandler (1992, 1997, and 2005). In particular, economists' voluntary public good (VPG) models have not been well extended to understand incentives and behaviors of sovereign agents desiring to manage risks along multiple channels analogous to EB's self-protection and self-insurance. The purpose of this paper is to contribute to that extension.

Analysis of multiple instruments of collective risk management must extend the standard market insurance paradigm in several respects. First, sovereign agents will face *increasing costs* for self-insurance coverage in contrast to the competitive linear prices relevant to individual market insurance. Second, when instruments both for self-insurance and for self-protection are available to a single decision maker, reducing risks through protection changes the price of self-insurance and thus the two allocations will interact in an unfamiliar manner. This essay will show that prominent among these is a type of self inflicted moral hazard. Third, if both self-insurance and self-protection are available to all members of a group, we must deal with problems with incentives caused by goods inferiority that can preclude determinate Nash equilibria. Lastly, since inter-agent incentives will be transmitted by income effects in the VPG model the issue of normality vs inferiority of protection and insurance is of special importance. Therefore, we must integrate into such non-market situations known results concerning inferiority of market insurance (Mossin 1968, Hoy and Robson 1981) and of protection (Ihori and McGuire, 2006, 2007).

When an agent provides security by spending on commercial market insurance, then a standard result states that with actuarially fair linear pricing, complete coverage (net of premiums) is purchased from resources available in the good contingency (EB, Mossin 1968, Hirshleifer and Riley 1975). We compare this benchmark result with *self-insurance* by an entire country, and use it also to examine the effect of such self-insurance on the choice of *self-protection*. This necessitates close attention to fair pricing in a context of diminishing returns, and we discover that here the definition of actuarial fairness itself is ambiguous.

First we show that if the country provides of self insurance as a national public good and if its spending on this self-insurance is subject to diminishing returns due to decreasing productivity of insurance premiums, then (contrary to the standard market result just mentioned) complete coverage (net of premiums) need not be purchased from resources available in the good contingency, irrespective of the ambiguity of fairness in pricing.

Next, we consider the incentives and conflicts facing a government that can both insure and protect at the same time. We identify three effects of security expenditure: (a) "insurance benefit" provided by the insurance branch, (b) "reduction of dispersion or variance of outcomes" by the protection branch, (c) "*insuranc*e cost savings benefits" provided by the protection branch. We show that this distinction is central to our paper. We show for comparison that if self-insurance can be provided at a market-like fair price, a country that fails to coordinate protection and insurance *will never self-protect at all*. Failure to recognize the insurance benefits generated by linear priced *self-protection* leads to its shutdown. Thus, we show that the difficulties of extreme moral hazard can be self inflicted. The incentive to take excessive risk is not due to a desire to pass costs on to a second party, but instead to information deficits and mal-coordination by the providers of protection. In fact, any agent, including entire governments, that is capable of both self-insurance and self-protection but who being a price-taker is also unaware of the *effects of protection on the price of insurance* will succumb to incentives similar to moral hazard.

We then apply the same idea when insurance is provided by a large agent, say the size of an entire nation, where such self insurance is usually characterized by the decreasing returns. Here, we argue, nations will have access to actuarially fair returns to their insurance outlays. And we show that diminishing productivity of insurance together with actuarial fairness in returns in no way resolves the self-inflicted moral hazard problem.

Moreover, when insurance is provided as a public good by/among nations in a group, as we demonstrate another difficulty is expected: goods-inferiority and the problems of equilibria and instability associated therewith. The necessary inferiority of *market* insurance is well established when risk aversion diminishes with wealth, resources are allocated optimally, and insurance is unfairly priced (Mossin 1968, EB). As we demonstrate *this effect is even stronger* when returns to insurance are diminishing since then unfair pricing is no longer necessary for inferiority. In addition, as Ihori-McGuire (2006, 2007) show, improvements in protection may easily be inferior under either decreasing or increasing risk aversion. Thus, when both types of security improvement are inferior, such incentives and conflicts should lead to specialization in provision of both public goods to Nash equilibria at corners for both insurance and protection.

This leads us to distinguish between specialization that is reciprocal or Ricardian, and specialization that is monopsonistic or centralized. We then have multiple Nash equlibria with multiple corner-solutions. In the case of centralized specialization where only one country provides both types of security, we show that the free-riding country always gains compared to isolation since it contributes neither to insurance nor protection, while the security-providing country cannot gain compared to isolation. But in the case of reciprocal specialization where each country provides one type of public good, each country free-rides with respect to either insurance or protection (while supplying all of the other good to the group) and both countries gain compared to isolation. However, even if those special assumptions concerning cardinality needed to permit welfare comparisons between allied countries are allowed still such comparisons are ambiguous.

Our arguments conclude that unless properly coordinated, self-insurance will push out self-protection completely, driving m_1 to zero in our terminology. This says that there is far too little consideration given to measures that will improve the odds against war, catastrophe etc (not particularly military measures, all measures). Our arguments lead to the implication that too much attention tends to be given to insurance type effort and too little to risk improving effort.

This paper consists of five sections. First, we formulate a basic analytical framework. Section 2 reviews characteristics of optimization under market insurance, self-insurance, and alternatively self-protection for a single country. Section 3 considers interaction between self-insurance and self-protection. Then, in section 4 extending the single country model to a two country world we comment on how the provisions of self-insurance and self-protection as public goods demonstrate an inherent potential for an "unstable conflict" leading to centralized or decentralized specialization in provision. Finally section 5 concludes.

2. Analytical Framework for a Single Agent

Let us begin with a narrative to illustrate the concept that our models will try to capture or summarize. Imagine there is an island nation that is subject to flooding. Whenever a flood happens there is a big loss. No matter how big the flood, the loss is the same, L. To protect itself against this loss the country can reduce the frequency of flooding by building flood-barriers, dikes, channels etc. If it builds no dikes a flood happens every other year. If it builds dikes that are 6 feet tall, the country will be flooded every 6 years. If this country builds dikes 15 feet tall it will be flooded every 11 years. The frequency of flooding depends on the height of its flood barriers. The relationship between cost of dikes and frequency of flood (1-p) will be known with certainty.[1]

To prepare for this loss, the country can also stockpile food and other necessities. Ignore time discounting and assume to start a linear relationship such that that if a flood happens every other year, each year without a flood the country can set aside x pounds of goods for the next year when there is a flood and have available in that year x pound of goods. If a flood happens every 6 years then to have x pounds available during the flood, the country only needs to give up x/5 lbs during each of the 5 dry years. If a flood happens every 11 years the country can provide x lbs during the flood by giving up only x/10 lb in each of the dry years. In other words, as an initial

[1] Let "d" be the number of dry years and "w" the number of wet years. Then p = d/(w+d) gives the frequency of dry years or of success, and (1-p) give the frequency of wet years or of failure.

assumption suppose the country can self-insure at an actuarially fair price, (1-p)/p, irrespective of the scale of provision, "x."

2.1 Risk Profiles and Emergency Cost

Congruent with the foregoing story we consider a single agent and two contingent states, a good state "1" and a bad state, "0". Ignoring all insurance and compensation possibilities (that is taking L as a fixed parameter) expected utility for this agent is given as:

$$\begin{aligned} W &= pU^1(Y) + (1-p)U^0(Y-L) \\ C^1 &= Y; \quad C^0 = Y - L \end{aligned} \tag{1}$$

or
$$W = W(Y, p) \tag{2}$$

where W is expected utility, C is consumption, L is loss in the bad state, and p is the chance of a good state. Our analysis will focus on the two canonical types of Ehrlich-Becker (EB) defense; (i) EB's "self-protection;" which raises p and reduces (1- p), (ii) EB's "self-insurance" which reduces L. Aside from our flood narrative, the variable "p" might be risk of trade interruption, disease outbreak, environmental calamity, or war. Later in section 4 we will assume these are shared indivisibly by the two coalition partners. Utility function U( ) is assumed to be the same whether luck is good or bad. U^1 denotes realized utility if the good event happens, and U^0 if the bad event happens, and $U_Y \equiv \partial U / \partial Y > 0, U_{YY} \equiv \partial^2 U / \partial Y^2 < 0$.

To establish the incentives for a single agent Eq. (3) shows the individual budget constraint: where Y is a fixed income and m_k denotes allocations (k = 1,2) to risk reduction, p(m_1), and or loss reduction -L(m_2) --- here

$$Y = C + m_k \tag{3}$$

considered a variable of choice. Therefore, if our concern is with insurance only --- with -L a variable but p taken to be a parameter --- Eq. (1) can be written

$$\tilde{W} = \tilde{W}(Y, m_2) \quad . \tag{4}$$

Eq (4) then shows it can be natural and helpful to consider m rather than L to be the national security public good.

2. 2 Market Insurance: Standard Result

We will emphasize presently how the structure and context of self-insurance for a large entity such as an entire nation is inherently quite different from market insurance. So it is for later comparative purposes useful to set out in brief summary the standard market insurance model, where again m_2 represents quantity of coverage in bad times.

A basic feature of the standard market insurance model (which distinguishes it from self-insurance) is that the cost of insurance coverage in bad times is linear. For a linear insurance recovery function instead of $-L(m_2)$ we write (with numeraire income being consumption in good times):

$$C^1 = Y - m_1 - \pi m_2 \tag{5}$$

and

$$C^0 = Y - m_1 - (L - m_2) \tag{6}$$

where πm_2 gives the <u>expenditure</u> on market insurance in good times (measured in units of C^1), and m_2 represents the <u>amount</u> of insurance coverage purchased at price π. Entered as a parameter, m_1 gives the allocation to risk improving self-protection. Then welfare becomes

$$W = p(m_1)U^1[Y - m_1 - \pi m_2] + (1 - p(m_1))U^0[Y - m_1 - (\bar{L} - m_2)] \tag{7}$$

If instead we took m_2/π to mean the <u>amount</u> of coverage and m_2 contingency-1 expenditures, we would write

$$W = p(m_1)U^1[Y - m_1 - m_2] + (1 - p(m_1))U^0[Y - m_1 - (\bar{L} - \frac{m_2}{\pi})] \tag{8}$$

Eqs. (7) and (8) are equivalent in the standard linear case, but we will favor (7) as more conventional. Then for optimal insurance, maximizing (7) or (8) with respect to m_2 yields necessary condition (9).

$$-p\pi U_Y^1 + (1-p)U_Y^0 = 0 \tag{9}$$

If the price of insurance happens to be actuarially fair then as in (10) the FOC would entail $U_Y^1 = U_Y^0$ whence

Actuarial fair price: $\pi = (1-p)/p$ (10)

$U^1 = U^0$ and, therefore, $C^1 = C^0$ or $Y - m_1 - \pi m_2 = Y - m_1 - (\bar{L} - m_2)$. And from this it follows that at the optimum, insurance coverage purchased is

$$m_2 = p\bar{L} \tag{11}$$

so that the total cost of such fairly priced insurance at the optimum becomes

$$\pi m_2 = (1-p)\bar{L} \tag{12}$$

This standard result states that with fair linear pricing, complete coverage (net of premiums) is purchased. We will use this *market insurance* summary as the benchmark for later to comparison with *self-insurance* by an entire country, and also in examination of the effect of insurance on the choice of protection.

Fair Pricing and Non-Inferiority of Market Insurance

Eq. (11) shows that at the fair insurance optimum, the amount of coverage purchased is independent of Y and income effect is zero. (But if $\bar{L}$ is an increasing function of Y, then the income effect becomes positive.) To summarize the conventional wisdom about linear market insurance, actuarial fairness in pricing, and goods inferiority, we introduce Figures 1-2. With p given, we consider just the utility function Eq. (7) at first and disregarding π for the moment, the indifference curves are as drawn in the diagram. Along a 45^O line the slope of indifference curves is (1-p)/p since $Y^0 = Y^1$ and therefore $U^0 = U^1$ and $U^0_Y = U^1_Y$. In other areas of the diagram we have to write $\frac{(1-p)U_Y^0}{pU_Y^1} = -\frac{dY^1}{dY^0}$ for the slope since in these areas $U_Y^0 \neq U_Y^1$. The slope of each member of a family of indifference curves will change systematically with changes in p. For example, considering decreasing values of p (probability of good contingency) with $p^A > p^B > p^C$, the slopes of the indifference curves along the 45^O diagonal or any ray through the origin change become steeper, when going from with p^A to p^B to p^C.

$$W=pU^1(Y^1 - m)+(1-p)U^0(Y^0 + (m/\pi))$$

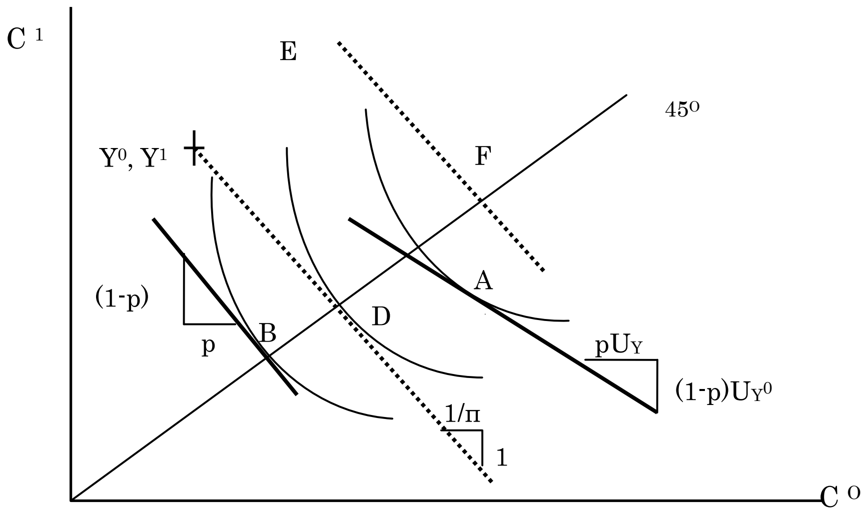

Figure 1

Now introduce opportunities to insure. If insurance is linear and fairly priced at π = (1-p)/p it will always be purchased to bring equilibrium to the 45^Odegree diagonal. <u>This solution is independent of risk aversion</u> --- whether it is constant, increasing, or decreasing --- and it eliminates possibility of inferiority since equilibrium is along the diagonal. However, if insurance is not fairly priced then the irrelevance of risk aversion changes. Now whether insurance is inferior, normal, or borderline depends on two factors (1) risk aversion and (2) whether price of insurance is greater than the "fair" price or less expensive.

Begin with the more ordinary case where prices are less favorable than actuarially fair, i.e. $\pi > (1-p)/p$. Then in Figure 2 the endowment point is given at the cross (Y^0, Y^1). With $\pi = \pi^H$ (H for "high"), higher than the value of π in Figure 1 the consumer will trade to point G, which is not on the 45^Odegree diagonal.

$$W=pU^1(Y^1 - m)+(1-p)U^0(Y^0 + (m/\pi))$$

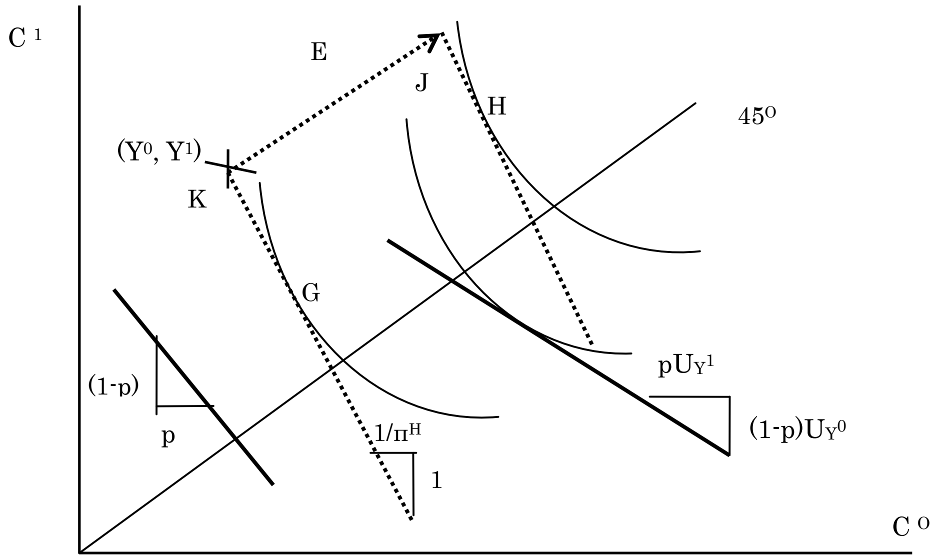

Figure 2[2]

[2] To depict the income effect let the endowment move to J so that with price unchanged at π^H the consumer trades to point H. We know that if risk aversion is decreasing the consumer will purchase less insurance going from J to H than going from K to G, which is to say that when risk aversion is decreasing and insurance is unfairly priced it is an inferior good. If the line connecting points G and H is steeper than 45^Odegree then less insurance is purchased with an increase in income and insurance therefore is inferior. If the indifference curves were such that the line from G to H were less steep than 45^Odegrees --- as would be true if risk aversion were increasing --- then the increase in income would yield an increase in insurance purchase and insurance would be normal.

Suppose now that insurance is priced below actuarial fairness --- "super fairly" priced. Then optima will occur below the 45^{O} degree line, and allocations below this diagonal amount to gambles. On inspection of the figure --- a symmetric relationship holds if insurance is "super fair." [3]

2.3 Self Insurance

Self-insurance provided to itself by a large entity such as a nation differs in two important respects from standard market insurance. To show this, we alter notation slightly. Rather than -L(m) where L(0) was a threatened loss if nothing is spent on insurance, we write

$$-L(m) = -[\bar{L} - \mathbf{L}\ (m)] \tag{13}$$

Now the entire, total, insurance <u>benefit</u> is shown by $\mathbf{L}$, and L(0) is given by $\bar{L}$.

2.3.1 *Diminishing Returns*

First of all, self-insurance differs from standard market insurance in that self-insurance function $\mathbf{L}$ should show diminishing returns or increasing costs. $\mathbf{L}' > 0$, $\mathbf{L}'' < 0$. EB make this assumption also, and refer glancingly to the role of human capital in providing for self-insurance as a source of diminishing returns. We believe (i) that scale considerations appropriate for an entire country along an extensive margin as well as (ii) other cooperating factors of production, as in EB, argue that "self-insurance" has such declining marginal

[3] Then the tangency points would be say G' and H' (not shown), now below the 45^{O}degree line. Now if the line G'H' were less steep than 45^{O}degrees --- as it would be if risk aversion is decreasing ---then greater income would lead to more gambling being purchased. But if risk aversion were to increase with income, then the line G'H' (below the 45^{O}degree line) would have a slope greater than 45^{O}degrees and increases in income would lead to reductions in gambling purchases

productivity. National self-insurance may often involve actions like stockpiling or standby production maintenance and these surely will show diminishing returns[4]. If m_2 is very productive, -L may even conceivably be negative for high values of m_2 (recognized also by EB as "negative insurance" or as termed here, “gambling.”), so that over some region $-L(m_2) = -[\bar{L} - \mathbf{L}\ (m_2)] > 0$. However, we ignore this case as it implies a reversal between good and bad contingencies. Declining "productivity" of "m_2" thus is the first source of a distinction between sovereign self-insurance vs. lesser scale decentralized market insurance.

Such diminishing returns also will introduce issues in the formulation of inter-contingency pricing of self-insurance that are absent from market insurance --- a fact not recognized in the literature as far as we can to allow for further interactions between determine.[5] Quite arguably, the self-insurance function should be written more generally as $\mathbf{L}\ (m_2, p, \pi)$ to allow for more complicated interactions between insurance, risk, and price. This would cause the definitions and formulation of "actuarial fairness" become ambiguous. But we relegate these complications to the appendix and concentrate here on the most salient formulation of pricing when insurance is non-linear with inter-contingency price π as shown in (14):

$$W = pU^1[Y - \pi m_2] + (1-p)U^0[Y - \{\bar{L} - \mathbf{L}\ (m_2)\}] : m_1 \ \textit{not shown} \tag{14}$$

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[4] In our story of flood protection and insurance, as greater quantities of consumables are set aside during dry years, their costs of preservation and delivery during good years might increase more than proportionately. For example, as an extreme case, if p = ½ setting aside m_2 =1 provides 1 unit in bad times, but saving m_2 =8 yields only 4 units in bad times, etc. Here $\mathbf{L} = (m_2)^{2/3}$.

[5] EB state in passing that self-insurance is independent of risk, but this is surely a mistake. Stockpiling for a seven year recurring famine is surely more expensive than for one that comes every 25 years.

where π shows the actuarial price per unit in good times necessary to yield m_2 units of resources in adversity.

2.3.2 *Salience of Fair Pricing*

Complications like writing $\mathbf{L} = \mathbf{L}(m_2, p, \pi)$ aside, a second major difference between self-insurance as provided by an entire country and ordinary market insurance is that when a whole nation provides insurance to itself, fair pricing would seem to be the standard case and not an outlier just referenced for comparison. Of course nation's can make mistakes, have imperfect information etc. But countries in this position are "bargaining with themselves" as to how much insurance and at what price to provide it. They should not in principle have to worry about adverse selection or moral hazard. So they should not give themselves deductibles, "load" prices nor impose arbitrary insurance limits to control fraud[6]. Moreover, it is plausible to assume that the nation as a price maker, not as a price taker, incorporates this actuarially fair condition at its optimization.

2.3.3 *The Insurance Optimization Problem*

Now similar to EB's derivation, expected utility (14) is maximized with respect to m_2. This gives (15) as the first order condition. Eq. (15) shows the marginal cost of providing $\mathbf{L}$,

$$\text{FOC:} \quad -p\pi U_Y^1 + (1-p)U_Y^0 \mathbf{L}'(m_2) = 0 \tag{15}$$

i.e. $[p\pi U_Y^1]$,equal to the marginal benefit of providing $\mathbf{L}$,i.e. $(1-p)U_Y^0\mathbf{L}'$, evaluated at the solution value of m_2 If this necessary condition is rewritten as in (16) then its actuarial meaning becomes clear. The RHS there gives the probability weighted marginal <u>insurance/benefit receipt</u> under adversity for the last, probability-weighted

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[6] Of course countries have corruption, rent seeking, and numerous misalignments of incentives to concern them.

dollar of premium paid in good times

$$U_Y^1 / U_Y^0 = [(1-p)/p\pi]\mathbf{L}\ '(m_2) \tag{16}$$

2.3.4 *Definitions of "Actuarial Fairness:"*

If self-insurance is actuarially fair (henceforth simply "fair") as we believe should be the paradigm for an entire country then the concept must be defined. An obvious parallel to the fairness under linear market insurance is resource allocation fairness as in Eq. (10) which is the same as (17).

<u>Resource allocation fairness</u>: $\pi = (1-p)/p$. (17).

An alternative definition of actuarial fairness would incorporate the marginal productivity of resources as applied to the bad contingency. This we label "marginal productivity fairness," as defined by Eq. (18):

<u>Marginal productivity fairness</u>: $\pi = \mathbf{L}\ '(1-p)/p$ (18)

2.3.4.b Optimal Solution Allocations

If fairness under marginal productivity ---Eq. (18) --- obtains then at the optimum $U_Y^0 / U_Y^1 = 1$ and thus $U_Y^1 = U_Y^0; U^1 = U^0; Y^1 = Y^0$ so that the optimum occurs along the 45^O and the analysis of inferiority proceeds just as in the linear case. Although this definition of actuarial fairness leads to a nice symmetry it assumes that the self-insuring agent somehow knows its optimal purchase of insurance "in advance," so as to have knowledge of $\mathbf{L}\ '$ before actually making its allocations. But this seems implausible. Moreover, under Eq. (18) one could not tell whether insurance was fairly priced until the optimum was actually chosen. This supports the first, more conventional, and we believe preferable, definition of actuarial fairness. Under fairness by that first definition Eq. (17) then the optimum simplifies to

$$U_Y^1 / U_Y^0 = \mathbf{L}'(m_2) \tag{19}$$

Eq. (19) is simply a familiar equality of MRS and Marginal Rate of Transformation, which obtains irrespective of risks (1-p) so long as the price of insurance is fair. But note, in contrast to the linear market insurance case no equivalence is implied between U_Y^1 and U_Y^0, with significant implications, we shall see for the inferiority vs. normality of insurance.

2.3.6 *Effects of Risk Aversion on Self-Insurance*

Economists have long known of a systematic inter-dependence between risk aversion and provision of market "linear" insurance. Established in the theory of market insurance is that insurance will be an inferior good if (a) risk aversion is diminishing with income, (b) insurance is *not* fairly priced, and (c) its purchase is optimized as in Eqs. (14-19) (Mossin 1968, Hoy and Robson 1981). To explore when this continues to hold for self-insurance we must investigate the sign of the income effect on insurance spending.

Consider Eq. (14) as the maximand, and the FOC given by Eq. (15). To analyze this case we follow the same procedure as we did for market insurance. Indifference curves are the same as in Figure 1 with slopes $(1-p)U_Y^0 / pU_Y^1 = -[dY^1 / dY^0]$ which simplify to $(1-p)/p = -[dY^1 / dY^0]$ only along the diagonal where $U_Y^0 = U_Y^1$. Considering $\mathbf{L}(m_2)$ to be a cost or transformation function the optimum occurs at tangency between the dashed MRT and the MRS. If insurance is "resource allocation fair," that is $\pi = (1-p)/p$ then the FOC simplify to Eq. (19). If tangency occurs at A in Fig. 3 then it must be the case that at the optimum $\mathbf{L}' = 1$. But if the tangency is at a point such as B then $\mathbf{L}\,' > 1$. At A, irrespective of the risk aversion properties of the utility function, insurance is normal, but at B declining risk aversion will generate insurance as an inferior good, *even though it is fairly priced*, that is, "resource allocation fairly" priced.

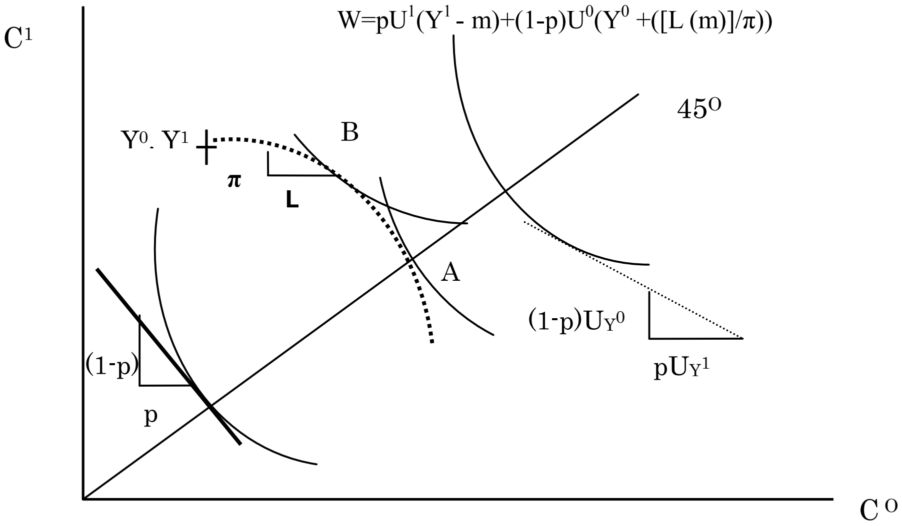

Figure 3

Another way to state this conclusion is to repeat the optimum

$$\pi\, p/(1 - p)\mathbf{L}\,'(m) = U_Y^0 / U_Y^1 \tag{20}$$

This suggests two cases:

Case I: optimum below the 45^O $\qquad \pi\, p > (1 - p)\mathbf{L}\,' \Rightarrow U_Y^0 > U_Y^1 \Rightarrow C^1 > C^0 \qquad (21)$

and

Case II: optimum above the 45^O $\qquad \pi\, p < (1 - p)\mathbf{L}\,' \Rightarrow U_Y^0 < U_Y^1 \Rightarrow C^1 < C^0 \qquad (22)$

In support of the foregoing illustration, therefore, we want to derive when m_2 is an inferior and when a normal "good". Total differentiation of FOC (15) gives:

$$\partial m_2 / \partial Y \equiv M_Y = [p\pi U^1_{YY} - (1-p)U^0_{YY}\mathbf{L}\,']/D \tag{23}$$

where D represents the second order condition.

$$\text{SOC:}\quad \text{D}= p\pi U^1_{YY} - (1-p)\mathbf{L}\,' U^0_{YY} + (1-p)U^0_{Y}\mathbf{L}\,'' < 0 \tag{24}$$

From the SOC we know therefore

$$p\pi U^1_{YY} - (1-p)U^0_{YY}\mathbf{L}\,' < -(1-p)U^0_{Y}\mathbf{L}\,'' > 0 \tag{25}$$

Hence the sign of the numerator in Eq. (23) is ambiguous [7] and m_2 as self insurance *may* be inferior depending on the pricing of insurance.

Marginal Productivity Fairness

"**Marginal** actuarial productivity" meaning $\pi / \mathbf{L}\,'(m)$ may be actuarially fair following Eq (18). Then $\pi / \mathbf{L}\,'(m) = (1-p)/p$ and we could say that when "actuarial marginal productivity" is fairly priced, at the optimum insurance cannot be inferior since under such assumptions $U^1_Y = U^0_Y$, therefore, $Y^1 = Y^0$ and $U^1_{YY} = U^0_{YY}$. Under this condition then the first two terms in Eq. (24) vanish and thus D < 0, while the numerator of Eq. (23) = 0. This corresponds to a tangency such as point A in Figure 3.

Resource Allocation Fairness

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[7] The numerator of (24) is actually the <u>difference</u> between the effects of greater income on marginal cost $\Delta MC = MC_{MY} = p\pi U^1_{YY} < 0$ and marginal benefit $\Delta MB = MB_{MY} = (1-p)U^0_{YY}L' < 0$. Note for later use that whereas MC_{MY} has elements in the good state of the world, MB_{MY} refers exclusively to the bad state.

We now consider the case of resource allocation fairness, which seems more plausible. Then, the sign of (23) is ambiguous. Specifically, if the numerator is positive, given the SOC, the sign of Eq. (23) is negative, and m_2 becomes inferior[8]. When m_2 is inferior, greater income lowers marginal cost $\Delta MC = MC_{MY} = p\pi U^1_{YY} < 0$ by more than it reduces marginal benefit $-\Delta MB = -MB_{MY} = -(1-p)U^0_{YY}\mathbf{L}\,' > 0$. This required relationship between MC_{MY} and MB_{MY} can be derived from the risk aversion properties of the utility function, with absolute risk aversion R defined as

$$R = -U_{YY}/U_Y \qquad \text{or} \qquad -U_{YY} = RU_Y \tag{26}$$

As shown in note 8, first if R is constant, then irrespective of fairness or unfairness of insurance pricing, from the FOC at an optimum the numerator of Eq. 23 vanishes and insurance is a borderline normal good. However if risk aversion is increasing ($R_1 > R_0$), H < 0 then sign of (23) is positive, and negative if risk aversion is decreasing[9];

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[8] Note if self-insurance is fair and therefore (1-p) = - pπ, then the numerator of (24) becomes $p(U^1_{YY} - U^0_{YY})$.

[9] This being another approach to Mossin's (1968) result, the numerator of (22) can be rewritten as:

$$H = -(p\pi R_1 U^1_Y - (1-p)R_0 U^0_Y \mathbf{L}\,') \tag{27a}$$

Considering the FOC, we obtain

$$H = p\pi U^1_Y (R_0 - R_1) = (1-p)U^0_Y (R_0 - R_1)\mathbf{L}\,' : \mathbf{L}\,' > 0 \tag{27b}$$

With the numerator of Eq. (23) written as $MC_{MY} - MB_{MY}$ it is understood that both MC_{MY} and MB_{MY} are negative. If $|MC_{MY}| < |MB_{MY}|$ then $M_Y < 0$ and m_2 spent on self-insurance is inferior. Using this notation to write (27a) gives

$$H = [MC_{MY} - MB_{MY}]$$

And

$$\begin{aligned} MC_{MY} &= -R_1 p\pi U^1_Y \\ -MB_{MY} &= +R_0 p\pi U^1_Y \end{aligned} \tag{27c}$$

In MC_{MY} the terms p and R interact just as in the case of m_1 spent for self protection considered below. However, for self insurance when m_2 and, therefore, -L are optimized this interaction is washed out of the sum of MC_{MY} and $-MB_{MY}$.

that is, if risk aversion is increasing (decreasing), m_2 is normal (inferior).

It follows then that the case for *inferiority of insurance is even stronger for self-insurance* than for market insurance since for diminishing return self-insurance fair pricing does not rule inferiority out. Generally, we should expect absolute risk aversion to decrease with wealth, so that for optimal/tangency-outcomes in the region of inferiority above the 45^O line the amount of insurance purchased will decline with wealth. Market insurance produces outcomes in this region of inferiority only if it is unfairly priced. But self-insurance leads to the region of inferiority even when it is fairly priced just provided the productivity of self-insurance is not so excessively great that $L' < 1$. This conclusion goes beyond the standard case of market insurance. Inferiority in a market insurance setting requires unfair insurance pricing to avoid the line of equalized wealth; but self insurance will prove inferior even if it is fairly priced and optimally provided since even optimal and fair self-insurance because of diminishing returns will not in general equalize incomes or marginal utilities across contingencies. Thus, (as will be the focus of Section 4), the dilemmas implicit in goods-inferiority as they apply to group allocation become more probable and serious when a group is composed of "self-insurers".

2. 4. Self Protection

Now to return to our flood protection anecdote we suppose a country can reduce the frequency of flooding

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The change in marginal benefit of more insurance when Y is increased depends only on its impact in one contingency, i.e. on. $(1-p)R_0U^0{}_Y L'$ When m_2 for insurance is optimized, as shown in Eq. (19) $U_Y{}^0$ and $U_Y{}^1$ are balanced as per eq. 19 so that the independent effect of $U_Y{}^0$ and (1-p) are all absorbed in $U_Y{}^1$ and p or vice versa. When similar analysis is performed on self protection, the FOCs do not cancel out the interdependence between R and p, so that our conclusions for the two cases differ.

by building flood-barriers, dikes, channels etc. (See Cornes, 1993). The frequency of flooding depends on the height of the flood barriers, and the relationship between cost of dikes and frequency of flood is known with certainty i.e. (1-p) in our model. Thus the second risk management instrument to consider is self-protection with m_1 spent to reduce the chance of a bad event, 1-p, i.e. to decrease what we call "baseline risk[10] of [1-p(0)]." Ihori and McGuire (2007) demonstrated (with insurance fixed parametrically) that for self-protection the issue of normality-inferiority is substantially more involved than it has proven to be for self-insurance as analyzed here. We now desire to extend the Ihori-McGuire analysis of self-protection developed for the special case of fixed uninsured loss to the more general case of (1) variable loss and (2) self-insurance where (3) insurance benefit is non-linear, and (4) actuarial fairness interacts with diminishing returns. Whatever the risk-reduction/self-protection function, $p(m_1)$, $p' > 0$, and $p'' < 0$ are assumed throughout .[11].

To begin, we repeat (14) now including both variables m_1 and m_2. Inserting the condition for resource actuarial fairness, $\pi = [(1-p)/p]$ directly gives:

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[10] Our "baseline risk" corresponds to what is sometimes referred to as "background risk" in economics of insurance analyses. Background risk distinguishes "independent" background risk where p(0) is not influenced by the value of L(0) as in our model here, versus "non-independent" background risk where p(0) and L(0) are interdependent, and asks how the choice of protection or insurance varies with the independence property (See Schlesinger, 2000).

[11] Where useful we can re-write Eq. (2) as (28) where L, m_2, and L are now taken to be parameters. Differences in functions $\tilde{W}_i(\cdot)$ *and* $\widehat{W}_i(\cdot)$ are implicit in each model, so the notational distinction will be omitted henceforth

$$\widehat{W} = \widehat{W}(C, m_1) \tag{28}$$

.

$$W = p(m_1)U^1[Y - m_1 - \frac{1-p(m_1)}{p(m_1)} m_2] + (1-p(m_1))U^0[Y - m_1 - \{\bar{L} - \mathbf{L}\ (m_2)\}] \qquad \text{(14 repeated)}$$

The FOC for determining optimal expenditure on self-protection becomes:

$$[p'(U^1 - U^0)] - [pU_Y^1 + (1-p)U_Y^0] + [(p'/p)\, m_2 U_Y^1] = 0\,. \qquad (29)$$

We can characterize this optimality condition on the provision of self protection saying that there are "direct" marginal benefits in the form of the gain in utility $p'(U^1 - U^0)$, "direct" marginal costs $[pU^1_Y + (1\text{-}p)\, U^0_Y]$ and "indirect" benefits, $p' m_2 U_Y^1 / p$,comprised of, an unambiguous gain from the decrease in insurance premiums paid for *the same* m_2 coverage received stemming from the lower price implied by lower risk (1-p). Note that since the sole variable of choice here is m_1 any possible implications of the change in $p(m_1)$ on subsequent choices of m_2 and, therefore, of insurance purchased (by the agent choosing insurance) are irrelevant.

$$\text{SOC} \qquad E = \begin{array}{l} p''(U^1 - U^0) - 2p'(U_Y^1 - U_Y^0) + [pU_{YY}^1 + (1-p)U_{YY}^0] \\ -U_{YY}^1 \dfrac{p' m_2}{p}(-1 + \dfrac{m_2 p'}{p^2}) + U_Y^1 \dfrac{m_2}{p^2}(pp'' - p'p') < 0 \end{array} \qquad (30)$$

Without further specification these SOCs need not always hold for mutual self protection, but we assume they are satisfied; then taking total differentiation of FOC (30) gives:

$$\frac{\partial m_1}{\partial Y} = - \frac{p'(U_Y^1 - U_Y^0) - [pU_{YY}^1 + (1-p)U_{YY}^0] + U_{YY}^1 p' \dfrac{m_2}{p}}{E} \qquad (31)$$

Condition (30), assuming the SOC actually obtains, determines the denominator in (32) as negative at an optimum. But again the sign of the numerator is ambiguous, and the normality or inferiority of m_1 depends on this numerator, just as in the self insurance model. Now, however, Ihori-McGuire demonstrate that there is an interaction between risk aversion R and baseline probability, so the normality/inferiority of m_1 is more involved than that of m_2 in the case of self-insurance. Here if absolute risk aversion is increasing/decreasing and (1-p) is initially low/high, m_1 will be normal/inferior, while if absolute risk aversion is decreasing/increasing and (1-p) is low/high, m_1 becomes inferior/normal[12]

3. Interaction between Self-Protection and Self-Insurance

How do a country's security programs or provisions effected by protection vs. by insurance influence one another? Do they complement or, substitute for each other, compete or mutually reinforce? How does the availability and utilization of information among programs for protection vs. insurance influence these relationships? These questions seem to have received little attention.

Before addressing the larger question, however, we analyze a special case to illustrate the crucial importance of information availability and accuracy. This is the case where the information and decision processes of the self-insuring/self-protecting agent are independent and isolated from each other. We demonstrate that under such restrictions, *especially when insurance is actuarially fair,* moral hazard behavior will drive provision of self-

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[12] Ihori and McGuire (2007) demonstrate that there is a critical-crossover probability p* such that if risk aversion is increasing M switches from normal to inferior while if risk aversion is decreasing M switches from inferior to normal as this value p* is crossed. In a generalized analysis of risk taking and insurance, Eeckhoudt and Gollier (2000) comment on the fact that a decline in first order stochastic risk may increase an optimizing agent's optimal exposure to risk (p. 122) considering it to be a"puzzle." This relation between RA and p accounts for one source of such a "puzzle."

One objection to this analysis might be our assumption that loss L(M) is independent of wealth. One might argue that the richer an agent/country the greater its loss from adversity. We could include this effect by replacing the loss, L(M), with a proportional reduction of income.

$$W = p(m)U^1(Y-m) + (1-p(m))U^0(\alpha Y - m) : 0 < \alpha < 1 \qquad (32)$$

This formulation implies a single crossover probability --- not constant p* but rather p = p(α) --- such that income effects change sign when probability changes and are neutral at p(α).

protection to zero. First we show this to be the case when returns to self-insurance are constant and linear, as would be the case with perfect and competitive market insurance.

3.1 A Special Example: Linear Fair Market Insurance (LFI)

As a benchmark case then, consider linear, constant unit cost, insurance as in a market where a small agent takes the price of insurance as fixed. As before, for expected welfare write:

$$W = p(m_1)U^1[Y - m_1 - \pi m_2] + (1 - p(m_1))U^0[Y - m_1 - (\bar{L} - m_2)] \qquad \text{(7 repeated)}$$

where m_2 and m_1 and π are as previously defined.

Complete Information Utilized by the Protection Branch

Here we assume that the agent who provides insurance does not take the price of insurance π as fixed, but rather incorporates the actuarial fair price condition (10) into his optimization. The standard result for a single coordinated and fully informed agent takes welfare the same as Eq. (7). Fair pricing, optimal self-insurance, and maximization with respect to m_2 as in Eq. (11) gives the optimum m_2 coverage purchased as $m_2 = p\bar{L}$. Thus complete coverage is purchased net of premium cost. Next, assume self-protection $p(m_1)$ can also be improved, so fair insurance price declines since risk declines with the increase in p. How much will be spent on protection?

The answer in this standard case depends, on whether a *self-protection provider* is aware that protection improves the price of insurance. Assume full information --- an alternative to the compartmentalized and unshared assumption of the next sub-section --- for an entity such as a nation that "bargains with itself" over the price of insurance. The protection branch knows that an increases in m_1 will lower the price of insurance, π, by raising p and also knows this should affect the optimizing behavior of the insurance branch. In a sense, the protection branch is above the insurance branch in a hierarchy. We could suppose that the protection branch moves first, anticipating that its provision of lower risk saves insurance costs but having no knowledge whether the lower premium price it causes will stimulate or curtail the quantity of insurance. Then, the insurance branch moves second[13]. We begin with the necessary condition, differentiating (7) with respect m_1.

$$p'(U^1 - U^0) - [pU_Y^1 + (1-p)U_Y^0] + [U_Y^1 m_2 (p'/p)] - [\{\pi p U_Y^0 - (1-p)U_Y^0\}\frac{dm_2}{dm_1}] = 0 \qquad (33)$$

Then using the fact that pricing is fair after including $U_Y^1 = U_Y^0 = U_Y$ Eq. (33) simplifies to

$$p'[U^1 - U^0] - [pU_Y + (1-p)U_Y] + m_2(p'/p)U_Y - [\{\pi p U_Y - (1-p)U_Y\}(dm_1/m_2)] = 0 \qquad (34)$$

When price is fair the final bracketed term vanishes at the optimum for insurance; therefore, the "protection branch" can ignore this effect (i.e. ignore dm_2 / dm_1) without undermining the overall optimum, since the effect vanishes when the insurance branch is doing its job properly. But Eq. (34) also includes $m_2(p'/p)U_Y^1$ to indicate that because of the cost savings that m_1 generates for insurance, the optimal value of m_1 depends on the choice of m_2 even at that stationary optimum for m_2. So the decision of how much to protect, to be optimal, must include *a part of* the effect of protection on the price of insurance, namely the "insurance cost savings" effect. The "insurance branch," controlling only m_2 will not recognize this effect attributable to m_1; the protection branch must

[13] For individual protection and insurance, a hierarchal separation between protection and insurance decisions sounds implausible. The smoker anticipates lower life insurance rates when he quits smoking. He may think of buying more insurance at the lower rates as well. But to impute this sort of foresight to a government however is not at all obvious. Thus our "complete information" case might be regarded as ideal, even utopian.

recognize it. In this <u>extended sense</u>, the protection branch is the "leader," and the insurance branch is the "follower." If the protection branch fails to recognize it then as shown in the next sub-section the FOC-implementing choices of insurance and protection agents will become incompatible, and m_1 will be driven to zero. But if this effect of m_1 is recognized by the protection branch then from $m_2 = p\bar{L}$ and $U^1 = U^0$, at the overall optimum the optimal values of m_1 and m_2 imply

$$p' = 1/\bar{L} \tag{35}$$

<u>So one of the benefits of spending to raise p when risk is already completely covered by insurance (and will continue to be so covered after p and therefore π improves) derives from the decrease in the cost of the optimal amount of insurance coverage when the implied actuarially fair premium declines</u>[14]. In this case both m_1 and m_2 are independent of income; the income effect is zero.

Effect of Imperfect Information: No Information Sharing Between Insurance and Protection Providers

Now consider the case where the protection branch of the country is a price taker as the benchmark case for a small open world. We investigate the outcome of non-cooperative Nash equilibrium in a small open world by exploring the incentives of the protection agent if it ignores the cost savings it generates for providers of

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14 We could have reached the same conclusion more directly by observing that since optimal insurance when fairly priced equalizes income as between contingencies, the optimal value of m_1 must be that which maximizes this insured income, or minimizes $\{-m_1 - (1 - p(m_1))\bar{L}\}$. This must be the value of m_1 such that $p' = 1/\bar{L}$.

insurance. Here, although it may recognize that p determines π, nevertheless the protection branch takes π as a fixed parameter for its optimization problem and focuses solely on its role in improving the weights on good and bad outcomes in the expected utility balance. From the insurance provider's choice of m_2 at his optimum, maximization of (7) with respect to m_2 yields as

$$\frac{\partial W}{\partial m_2} \equiv T(m_2) = -p\pi U_Y^1 + (1-p)U_Y^0 . \tag{36}$$

Crucially, assume that although m_1 determines $p(m_1)$, and p determines π, and π determines the cost of insurance coverage m_2, nevertheless, the agent choosing m_1 is ignorant of these relationships and overlooks the benefit that m_1 creates by lowering the cost of existing insurance. That is, assuming that the agent who provides m_1 regards the price of insurance π as fixed (despite the fact that because of that agent's decisions this price continually adjusts to satisfy the assumption of actuarial fairness). Then for this case --- which might describe behavior among government agencies in a small open world that is myopic and uncoordinated even though they share the same over all objective function --- the protection provider's welfare maximization is given as:

$$\frac{dW}{dm_1} \equiv T(m_1) = p'(U^1 - U^2) - (pU_Y^1 + (1-p)U_Y^0) \tag{37}$$

Now, go back to the provider of insurance and suppose $T(m_2) = 0$. Then, from (36) plus an actuarially fair condition,[(1-p)/p] = π we know $U_Y^0 = U_Y^1$; hence $U^0 = U^1$. Substituting this into (37), gives $T(m_1) = -U_Y^1 < 0$. Thus, when (36) and the actuarially fair condition (10) both obtain, it follows $T(m_1) = -U_Y^1 < 0$. Next considering the provider of self-protection again let $T(m_1) = 0$. Then from (37) we know $p'(U^1 - U^0) = [pU_Y^1 + (1-p)U_Y^0] > 0$ and hence $U^1 > U^0$. And this implies $T(m_2) = -pU_Y^1 + pU_Y^0 > 0$.

To interpret this define curve $T(m_1)=0$ as the locus of (m_1, m_2) which satisfies $T(m_1)=p'(U^1-U^0)-[pU_Y^1+(1-p)U_Y^0]=0$; and define curve $T(m_2)=0$ as the locus of (m_1, m_2) which satisfies $T(m_2)=-pU_Y^1+pU_Y^0=0$. Then to summarize:

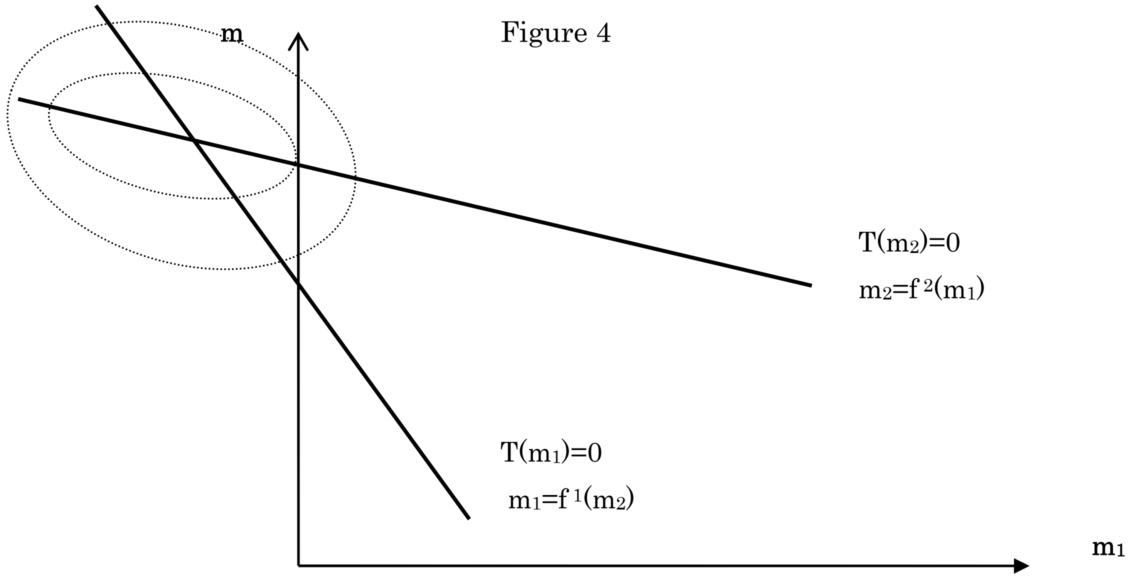
$$\text{If } T(m_2)=0\text{, then } T(m_1)<0 \tag{38}$$

$$\text{If } T(m_1)=0\text{, then } T(m_2)>0 \tag{39}$$

We can never find values along Eq. (36) =0 for which Eq. (37) =0, since whenever Eq. (36) = 0, then Eq. (37) < 0. Therefore, at every point along the optimal insurance curve Eq. (36) = 0, the agent providing protection wants to *reduce* m_1 and thus increase risk. If he could increase risk so much that p < 0 and (1-p)>1, and be compensated for this increase by negative m_1, the agent will want to do this. *This is moral hazard in the extreme.*

On the other hand, everywhere along the curve $T(m_1)=0$, we see that $dW/dm_2 > 0$, meaning that an insurance agent would want to insure more at each value of m_1. Thus it seems --- in stark contrast to the incentives for a unitary government agent who coordinates provision of (m_1, m_2)--- that a decentralized security agencies would not provide both types of security spending at the same time. Rather they would normally provide $m_2>0$ only. The two reaction curves $T(m_1)$ and $T(m_2)$ in space m_1-m_2 with a Nash solution at $m_1=0, m_2>0$ are shown in Figure 4, where the closed contours are loci of constant “security.”

Figure 4



Need for Inter-Program Information Sharing

Bureaucratic compartmentalization like this seems to us entirely plausible, and not diminished by the assumption that fair pricing is applicable and available. The demands for inter bureau information exchange to achieve a first best optimum are significant, since country wide benefits from m_1 must be calculated to include cost savings in insurance “premiums” that increases in m_1 entail. That is the provider of protection must include insurance cost-savings benefits in his calculations to select the correct value of m_1. This exercise emphasizes the daunting information requirements necessary for the standard result.

3.2 General case: Self Insurance and Self protection

According to the story in our introductory narrative the island nation subject to flooding, can reduce its frequency by building flood-barriers, dikes, channels etc. and can reduce the magnitude of loss by stockpiles, say of food and other necessities, self-insuring at an actuarially fair price, $\pi = (1\text{-}p)/p$. Here we dwell more generally on how these two instruments interact with each other. In particular we show how the effectiveness of insurance plus protection varies crucially with the quality of information shared within the government and of cooperation among insurance and protection measures, programs, or bureaus.

Since agents individually or in a group may spend on both insurance and protection, we are interested in how any one agent's incentives interact with respect to the two instruments when both are available. Specifically, taking fair insurance to be the norm, we focus on how anticipation that expenditure on p will affect π influences the optimal choice of m_1 and therefore of $p(m_1)$. First, suppose the government knows all these relationships. It knows the effect of dike height and cost on frequency, and knows it can self-insure at fair prices.

To begin we repeat (14) now including both variables m_1 and m_2, inserting the condition for actuarial fairness, $\pi = [(1\text{-}p)/p]$ directly, and writing W in the form where m_2 indicates quantity of insurance coverage purchased while πm_2 represents total expenditure on insurance coverage :

$$W = p(m_1)U^1[Y - m_1 - \frac{m_2\{1-p(m_1)\}}{p(m_1)}] + (1-p(m_1))U^0[Y - m_1 - \{\bar{L} - \mathbf{L}(m_2)\}] \qquad \text{(14 repeated)}$$

As derived in above, --- assuming that the insurance branch accepts m_1 as a parameter and thus disregards any anticipated change in $p(m_1)$ and therefore in π --- the FOC with respect to m_2 gives

$$S = -U_Y^1[Y - m_1 - \frac{m_2\{1-p(m_1)\}}{p(m_1)}] + \mathbf{L}'U_Y^0[Y - m_1 - \{\bar{L} - \mathbf{L}(m_2)\}] = 0 \qquad (40)$$

The FOC for determining optimal expenditure on self-protection m_1 becomes:

$$[p'(U^1 - U^0)] - [pU_Y^1 + (1-p)U_Y^0] + [U_Y^1 m_2 p'/p] - [p\pi(U_Y^1 - (1-p)U_Y^0\mathbf{L}')\frac{dm_2}{dm_1}] = 0 \,. \qquad (41)$$

As in the case of linear insurance, at the optimum value m_2, the bracketed term which includes dm_2/dm_1 vanishes, such that in a properly decentralized organization of (m_1, m_2) the "protection branch" can ignore it. However we cannot ignore the effects of protection on the price of insurance, so that the third bracketed term in Eq. (41) must be incorporated in the protection branch calculus [15].

Organization And Information Sharing
Between Protection and Insurance Branches

We can consolidate our argument for information sharing among government branches if we think of implementation as solving these two equations:

$$\partial W / \partial m_1 = \phi^1(m_1, m_2) = 0 :\Rightarrow m_1 = f^1(m_2) \tag{42a}$$

and

$$\partial W / \partial m_2 = \phi^2(m_1, m_2) = 0 :\Rightarrow, m_2 = f^2(m_1) \tag{42b}$$

To illustrate the solution draw the "reaction" curves $f^2(m_1) = 0$ and $f^1(m_2) = 0$ over contours of "constant security" as in Figure 5. At their intersection we would have the true optimal values of m_1* and m_2*.

---

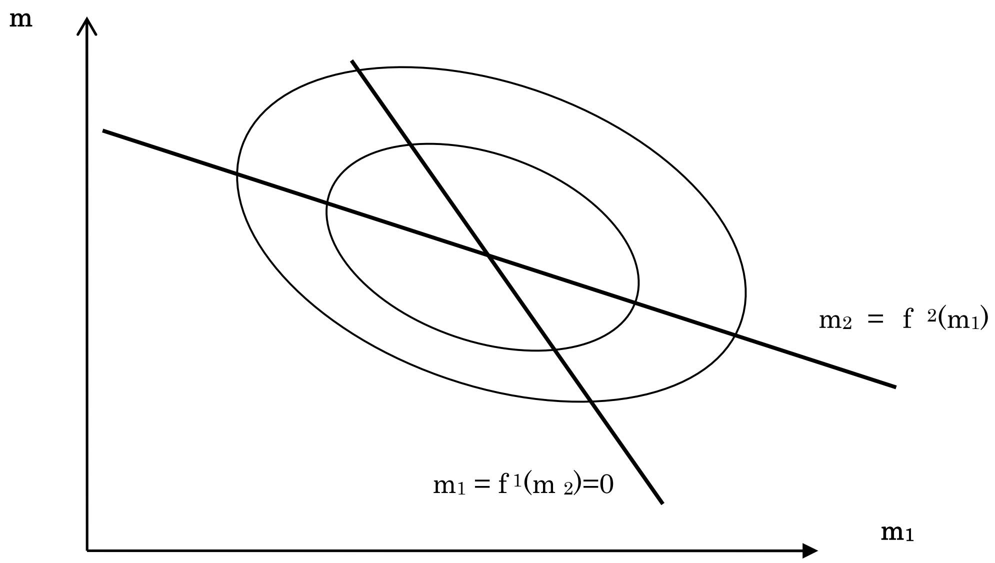
15 From Eq. (40) : $$\frac{dm_2}{dm_1} = -\frac{\partial S / \partial m_1}{\partial S / \partial m_2} = -\left[\frac{-U^1_{YY}[-1 + m_2 p'/ p^2] - U^0_{YY}\mathbf{L}'}{U^1_{YY}\pi + U^0_Y \mathbf{L}'' + U^0_{YY}(\mathbf{L}')^2}\right] \gtreqless 0 \tag{43}$$

By SOCs for Eq (40) the denominator of (43) is negative. As a special case, if $\mathbf{L}'$ = 1 at the insurance optimum of Eq. (40) so that $U^1 = U^0;\ U^1_Y = U^0_Y;\ U^1_{YY} = U^0_{YY}$ then the sign of (43) is > 0 so that just as in the linear insurance case, quantity of coverage m_2 and of protection are gross complements. We can also simplify (43) using the definition R = -U_{YY}/U_Y, where R is the coefficient of risk aversion. Then

$$d\mathrm{m}_2/\mathrm{dm}_1 = -[R_1 U^1_Y + R_0 U^0_Y \mathbf{L}' - U^1_{YY} p' \mathrm{m}_2 \{1/(p)^2\}]/[(-)] \tag{44}$$

Then, under constant risk aversion, from the FOC the first two terms of the numerator cancel so that <u>increases in expenditures on protection raise optimized quantity of self insurance</u> coverage that is $dm_2/dm_1 > 0$ and we know in this case m_2 is a borderline normal good. Moreover, even if risk aversion is declining so that $R_1 < R_0$ still m_2 may be gross complements since only a very great value of R_0 will offset the positive weight of the final term in the numerator.

Figure 5



To see the effects of compartmentalization, suppose, rather than a unitary decision maker, an insurance branch within the government and a protection branch are each charged with providing m_2 and m_1 respectively. If we assume full information sharing then that government would understand its welfare to be as shown in (45a,b) and would strive to implement m_1* and m_2*. How could it do that? Here are some approaches.

Bureaucratic Coordination

A. Black box approach: Just assume that the government makes all the calculations, derives the answer and instructs the insurance branch to spend m_2* and the protection branch to spend m_1*. This approach avoids the governance problem of externalities as between protection and insurance branches by assuming it away.

B. Central Control Plus Decentralized Execution: The government informs each branch of its correct reaction function $m_1 = f^1(m_2) \quad and \quad m_2 = f^2(m_1)$. (Note that the first of these incorporates the effect of m_1 on π

and the insurance-cost-savings attributable to protection outlays). From any point in Figure 5, each branch moves toward its correct reaction function in Cournot fashion, until the intersection optimum, of m_1* and m_2* is reached.

C. Hierarchical Execution. The government entrusts one branch to make a final "all at once decision." This avoids the zig-zag iteration implied by B. Instead, the government gives the function $m_2 = f^2(m_1)$ to the protection branch or the function $m_1 = f^1(m_2)$ to the insurance branch. The "Leader-branch" that receives the reaction function then implements it and solves for the overall optimum and provides to the "Follower branch" its solution value of m_L ("L" for leader) after which the "Follower-branch" just follows its reaction function, providing m_F as anticipated by the leader.

To sum up, if a large agent the size of a nation, may spend on both insurance and protection, we expect that both m_1 and m_2 should (if information is shared and coordinated) be provided at an interior solution. But if information is not correctly distributed and recognized---specifically, if the insurance cost savings from improvements in self protection are not allowed to influence those self-protection decisions --- then in aggregate decisions will be suboptimal with losses from extreme moral hazard and corner solutions where only m_2 is provided, just like the benchmark case of market insurance in a small open world.

4. Mutual Provision of Public Goods: A Presumption of Specialization and Corner Solutions

We now turn to the case where countries can provide self-insurance and self-protection mutually, as public goods for each other. We have shown that for both decreasing returns and for linear technology --- i.e. for self-insurance and market insurance --- incomplete information sharing and benefit recognition may create extreme moral hazard leading to zero provision of protection. To avoid the implied inefficient bias against risk reduction in favor of insurance we will assume that these intra-bureaucratic externalities are recognized. This heroic assumption however will not paper over conflicts and instabilities that countries desiring to cooperate in mutual security must face due to their own incentives.

The Shadow of Security As An Inferior Good

When each country could provide both types of public good[16], as shown above each type of security can easily become inferior if absolute risk aversion to declines with wealth. And as recognized, for example, in Kerschbamer and Puppe (1998) or in Ihori and McGuire (2006), when a public good is inferior, the Nash equilibrium solution for two countries becomes unstable implying a corner solution where only one country provides the public good. The intuition is as follows. When one country (home) provides security it creates a positive externality for its partner country (foreign), and hence the partner's effective income rises. But when public goods are inferior, the partner will react to an increase in its income by decreasing its provision of the public good. This generates a negative externality for home which then will react by raising its provision further. Thus, at the Nash equilibrium only the home country provides the public goods.

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[16] Our simple solution to the problem of diminishing returns and distribution of infra marginal costs/gains in a public good spillover environment will be to assume a "summation *finance* aggregator," $M = \sum m$, in the provision of public good L, even though L(M), p(M), represents a "non-summation *consumption* aggregator" (e.g. $p(M) \neq \sum p_i(m_i)$) Then, importing an idea from contest theory we take primitive preferences as being over *contributions* to insurance or to risk reduction, rather than insurance coverage or risk reduction itself.

The Shadow of Cornes-Itaya

If we consider multi-types of pure public goods, a second source of instability and imbalance follows from incentives discovered by Cornes and Itaya (2004). They demonstrate that when two agents in a partnership or alliance both could provide two different (pure) public goods, then at a Nash equilibrium, it is impossible (unless both agents have identical preference functions) for both agents to share in the provision of both goods. Curiously, then our pessimistic conclusion that both protection and insurance may easily (though not necessarily) be inferior goods is less damaging than one might otherwise think. Much of the damage has been done already --- by Cornes-Itaya. However, they do not exclude nice textbook interior solutions if both countries are identical. So our analysis completes the demolition of interior Nash solutions, at least when the public goods are mutual self-insurance and self-protection. That is we have shown that interior solutions *are excluded* if public goods are inferior even in the case of identical preferences. To see this consider more closely those configurations and welfare outcomes which are not excluded by Cornes-Itaya.

Merely for illustration and simplicity assume two identical countries then, A and B, and suppose that both types of security spending are inferior. Since there are two public goods and each type is provided by one country only, then for the case of inferior goods, we have two possibilities.

Centralized Specialization

One country, say, A provides both public goods, while the other country, B, provides none at all. If A provides both m_1 and m_2, while B provides nothing, Country B completely free-rides off the provision by

A. Then, *ceteris paribus* B's welfare in the two-country alliance is higher than it was in isolation and (if interpersonal cardinality could be demonstrated) also higher than A's welfare was in isolation. On the other hand, the alliance creates no benefit for A; A's welfare is the same as in isolation. Since we assume that both countries are identical, we might conjecture that in isolation where both countries provided positive levels of both m_1, and m_2 the welfare of A and B would be equal (if interpersonal cardinality obtained).

Next suppose the two countries become allies and alliance-specialization is "centralized." Now Country A provides the same amount of m_1, m_2 as it did in isolation. Since B provides nothing there is no positive spillover from country B's provision of m_1, m_2. Hence, welfare of country A remains the same as it was in isolation. On the other hand, welfare of country B increases because B now enjoys the positive spillover effect of A's provision of public goods.

Decentralized Specialization

But if the specialized provision of public goods is decentralized between alliance members, then Country A provides one of the public goods, while B provides the other. To be specific, suppose A provides m_1, and B provides m_2. Then Country B partially free-rides off provision m_1 by A, and similarly A partially free-rides off provision m_2 by B. Under Nash-Cournot behavior then, both countries gain compared to isolation. But is A indifferent between this decentralized provision where it provides only m_1 and B provides only m_2 and the symmetric alternative where it provides only m_2 and B provides only m_1? Or will A (and B) definitely prefer one configuration over the other?

One approach to this question will assume that interpersonal cardinal welfare comparisons are meaningful. Then since we assume that both countries are identical the welfare of countries A and B considered separately will be equal. Now again in isolation each country provides a positive level of both m_1, and m_2 for itself; whereas once the two form an alliance decentralized specialization occurs. As above let A's provision of $m_1>0$ and of $m_2=0$ and let B's the provision of $m_1=0$ and of $m_2>0$. Since each country receives positive spillovers from the provision by the other country, both gain from the alliance. However, welfare comparisons even between these two identical countries after forming an alliance is generally ambiguous. Since both countries have the same preferences and incomes, and two public goods m_1 and m_2 are pure, relative welfare can be inferred by comparing the amounts of private consumption after provision of public goods. Specifically, if $m_1 > m_2$, then C of country A is less than C of country B, and hence insofar as interpersonal welfare comparisons are meaningful the welfare of A is less than the welfare of B[17] (and vice versa). Thus surprisingly, under Nash-Cournot behavior --- even identical countries should have a definite preference for one and the same decentralized configuration over the other --- precisely which one depending on specifics. In other words when "common defense" among alliance partners consists of the two distinct activities, collective-protection and collective-insurance, with provision governed by Nash-Cournot behavior, there is a built-in incentive for conflict, disagreement, and gamesmanship between allies.

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17 Alternatively, using the same logic but without cardinal interpersonal comparisons, we could infer that the welfare of A is higher in the decentralized configuration where its expenditure on public good m_1 or m_2, is less--- and symmetrically for B.

But will this perverse incentive persist under Stackelberg Leader-Follower relationships (which are probably more realistic) or will it dissolve? A Stackelberg leader always prefers not to contribute to the public good at all and to free ride on the provision of followers. Therefore, if specialization is centralized, and A is leader for both types of public goods, then A provides nothing at all at the Stackelberg solution. But if specialization is decentralized with A the follower in the provision of protection and B the follower in the provision of insurance, we obtain an outcome mirror-symmetric to decentralized Cournot provision, with similar perverse preferences, incentives, and conflicts over choice of good to provide.

This case of decentralized seems more interesting than that of centralized specialization because then both countries gain by joining an alliance partnership. In the real world we may observe this situation. For example, in the US-Japan alliance, the US provides mainly protection --- military spending to reduce the risk of attack from enemies; but, Japan provides insurance --- through military spending to defend itself --- which will reduce costs when/if war occurs.

To sum up, in a world where mutual collective security is feasible a natural aspiration might be that both components insurance and protection be provided by allies cooperatively and in an efficient first best amount. However, our analysis has shown that for the plausible case of decreasing risk aversion when both kinds of security are expected to be inferior goods, then at a non-cooperative Nash outcome each of those components of security will be provided by one country alone, while other countries ride free. The prevalence of free riding means that the total of security provision is too small. And the incentive configuration that gave inferiority and corner solutions makes it is difficult for both countries to cooperate for their mutual security.

5. Conclusion

This paper has investigated two types of preparation available to expected utility maximizing agents faced with "costs of emergency", namely self-insurance and self-protection. Self-insurance provided to itself by a large entity such as a nation differs in two important respects from standard market insurance. First of all, the self-insurance function should show diminishing returns or increasing costs whereas market insurance typically has linear or piecewise linear pricing. Second, self-insuring countries far more readily than individuals should have access to actuarially fair prices, which they can then incorporate into their optimization.

If a small price taking agent provides insurance at a market price, then the standard result holds that with pricing fair and linear, complete coverage (net of premiums) is purchased. We have used this standard result as a benchmark to compare with self-insurance by an entire country, and also in our examination of the effect of insurance on the choice of protection.

First, we show that the effect of decreasing productivity (ordinary diminishing returns) when a nation provides self insurance is for complete coverage (net of premiums) not to be purchased except by unlikely chance.

Second we describe the incentive structure when both types of protection are available. In doing this we have focused on the mis-incentives that follow when providers of protection and of insurance (within the same government) ignore the benefits and cost they confer on each other. More specifically, we show that without recognition of such external benefits and costs, the security providers of a country are likely to drive themselves into a corner, in effect, of extreme moral hazard where insurance is over-provided and protection abandoned. Thus, an agent trying to provide both self-insurance and self-protection may find that both will never be provided at the same time because of ignorance of the insurance cost savings attributable to protection outlays. The resolution of this mis-incentive requires that the providers of risk-reducing protection recognize in their calculus the cost-savings they provide to insurers. Since improvements in protection lower the actuarially fair price available to self-insuring agents, the decision of how much protection to provide must incorporate this benefit. Once such benefit is recognized and included in the calculus of protection, the moral hazard corner will be avoided.

Our model of individual self-insurance and self-protection readily extends to groups of two countries where such insurance and protection are public goods. Here we demonstrate an inherent potential for destabilizing incentives to generate corner outcomes with complete perfect specialization in the provision of these public goods. The source of the difficulty is that both protection and insurance are rather likely to be economically inferior and economic inferiority leads to corner solutions. When absolute risk aversion declines with wealth, we expect both types of security to be inferior, and hence Nash equlibria to be given by corner solutions as between countries. In such a case we observe specialization by country in the provision of the public goods. Then for "centralized specialization" the free-riding country always gains from membership in a group compared to isolation. But under "decentralized specialization" where both countries free-ride off of each other, welfare comparison between the two countries becomes ambiguous but both countries gain compared to isolation. In either case, the total level of security spending is too little. Moreover, because of free riding incentives, and their inherent potential for generating conflict among allies, our analysis indicates that cooperation with respect to the public provision against adversity should be more problematic than in an ordinary voluntary provision group. Furthermore, we demonstrate that postulating Stackelberg Leader-Follower behavior provides no resolution of the mis-incentives.

Thus, we have shown that separation of security decision-making into self-protection and self-insurance is a bad idea. In fact most governments, at present, treat them as essentially separate. There needs to be much more coordination between these two seemingly disparate functions of government. Also, our arguments conclude that unless properly coordinated, self-insurance will push out self-protection completely, driving m_1 to zero in our terminology. This says that there is far too little consideration given to measures that will improve the odds against war, catastrophe etc (not particularly military measures, all measures). Our arguments lead to the implication that too much attention tends to be given to insurance provision and too little to risk improvement.

APPENDIX

Alternatives in the Structure and Price of Insurance

The foregoing text assumes one simple relationship between premiums paid, πm_2, during good times and $\mathbf{L}(m_2)$ benefits received during bad times, where the variable of choice is units of coverage m_2. We could cast the problem in terms of units of expenditure (where we define $x_2 = \pi m_2$, "x" for "expenses," and where $m_2 = x_2/\pi$) such that the variable of choice is x. Here $1/\pi$ would indicate the efficiency of resource transfer across contingencies. Thus in place of Eq. 14

$$W = pU^1[Y - \pi m_2] + (1-p)U^0[Y - \{\bar{L} - \mathbf{L}\ (m_2)\}] : m_1 \ \textit{not shown} \qquad \text{(14 repeated)}$$

We could write:

$$W = pU^1[Y - x_2] + (1-p)U^0[Y - \{\bar{L} - \mathbf{L}\ (x_2/\pi)\}] : m_1 \ \textit{not shown} \qquad \text{(14a)}$$

Optimization of (14a) gives the same results as derived above in the text but measured in terms of expenditure rather than quantity of coverage. However, when transfer of resources across contingencies is written as in (14a) it becomes obvious that every unit of x_2 may *not* transfer into just exactly x_2/π units to the bad contingency. Once this is recognized then the idea of "fair" pricing becomes not so clear. For example, in place of (14a) we could write (14b)

$$W = pU^1[Y - x_2] \ + \ (1-p)U^0[Y - \{\bar{L} - \frac{1}{\pi}\mathbf{L}\ (x_2)\}] : m_1 \ \textit{not shown} \qquad \text{(14b)}$$

Eq. (14b) represents another structure of diminishing returns. There $1/\pi$ indicates not the productivity or accumulation of x_2-resources but the accumulation in bad times of $\mathbf{L}$ units of benefit. Now, in a sense, the self-insurance function operates during good times to create transfers available only under adversity; so that for the total $\mathbf{L}\ /\pi$ received in adversity, x_2 was set aside in the good contingency[18].

When insurance is supplied in the market and its marginal costs and benefit are linear, this distinction

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[18] One way to think about this is to assume there is a steady state. We can then frame self-insurance in terms of changes in, or alternative parameters for this steady state. Normalizing the notation of note 1, let w = 1, d = n., Here is the steady state. Every n+1 years there is a crisis, requiring that the self-insurance accumulation be utilized. Each year from 1 to n, \$$m_2$ is set aside in anticipation of year n+1. That is "self-insurance" is fair. At year n+2 the process starts over again, and on and on. Therefore p = n/[n+1], and (1-p) = 1/[n+1]. $\pi = 1/n$ so that $m_2/\pi = nm_2$. Changes in the risk of adversity then are given by changes in n. If n is large adversity happens only rarely so that (1-p) is small, and if n is small, the risk of adversity (1-p) is high. We ignore discounting, and uncertainty.

Case I: Here the total insurance consumed in year n+1 is $n\mathbf{L}\ (m)$. Here diminishing returns apply to each year's insurance savings individually so that every new year the country begins a new decreasing returns process.

Case II: Here the total insurance coverage consumed in year n+1 is $\mathbf{L}\ (nm)$. Here m_2-savings set aside each year generate less marginal return than the same m savings of the year before.

In both I and II, greater m_2 produces more diminishing returns, although in different manner and to different degree. Note however, that with case II, diminishing returns, in addition to being greater for larger values of m_2, are more severe the rarer the emergency (higher value of n).

between (14a) and (14b) does not arise or it doesn't matter; but when $\mathbf{L}$ displays diminishing returns how to describe the cost of insurance seems to be ambiguous. Do diminishing returns apply both to quantities reserved and to the lapse of time between adversities, or only to the former? As for comparing (14a) vs. (14b), the difference seems to lie in "when" the resources transferred across contingencies become productive --- before (as in Eq. (14b)) or after (as in Eq. (14a)) the transfer. The novelty of this distinction, or the fact that it seems to have been overlooked, may be because when the insurance function is linear, as in market insurance, no such difference arises.

For either of these options (14a) or (14b) the idea of actuarial fairness is still applicable, but its implications with respect to outcomes vary. For example, if marginal utilities just happened to be equalized at the optimum such that $\mathbf{L}'$ = 1 then consumption would also be equalized, and this would imply $\mathbf{L}(m_2) = (1-p)\overline{L}$, in the one case or $\mathbf{L}(x_2/\pi) = (1-p)\overline{L}$ in the other. Although these formulas have a nice symmetry to market insurance neither of these conditions is required to obtain even though coverage is fairly priced --- not required because there is no necessity that $\mathbf{L}'$ = 1. In either case FOCs inform us of optimal insurance protection and allow us to compare configurations (14a) and (14b), assuming self-insurance to be fairly priced.[19] More generally, if the benefit in the bad contingency depends in a more complicated manner on resources set aside in good times then we should write $\mathbf{L} = \mathbf{L}(\pi, p, x_2)$ and the effects of fair insurance and even its definition may become ambiguous.

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19 First, note that for π = [(1-p)/p] = 1, $\mathbf{L}'(x_2/\pi) = \mathbf{L}'(m_2)$, while for π > 1, $\mathbf{L}'(x_2/\pi) > \mathbf{L}'(m_2)$ and for π < 1, $\mathbf{L}'(x_2/\pi) < \mathbf{L}'(m_2)$. So if self-insurance has the form of Eq. (14a) i.e., $\mathbf{L}(m_2)$, there is an interaction between insurance and protection not present in the case of market insurance. For π < 1 (and hence risk (1-p) low) the solution value of "m_2" will be less than it is when odds are equal. On the other hand, for π > 1 (and risk (1-p) high) optimal m_2 will be greater than the best choice of "m_2" when odds are equal (maintaining throughout these comparisons, the fair insurance equality π = (1-p)/p).

But if self-insurance has the form of Eq. (16a), i.e. $\mathbf{L}(x_2/\pi)$, then there is an additional interaction between insurance and protection. There, when π = (1-p)/p) varies as a parameter, both the MRS between contingencies shifts and MRT shifts as well so that the overall effect of π on the optimal m_2 cannot be deduced *a priori*.

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