Memoryless Contract in Dynamic Moral Hazard with History Dependence*

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November, 2004

Abstract

This paper examines the role of history dependence in a two-period dynamic moral hazard model. If history dependence is relatively strong, then the signal in the first period becomes completely useless to the principal. Therefore, in the optimal long-term contract, payments to the agent are made dependent upon performance in the second period only. A sufficient condition for the offer of such *memoryless* contracts are found to be characterized by stochastic dominance. *Journal of Economic Literature* Classification Numbers: D82, J31, M52.

Keywords: repeated moral hazard; repeated principal-agent problem; history dependence; time lag.

1 Introduction

In long-term employment relationships, the question arises as to how the employer can provide proper incentives for employees to work harder. Since the employee's effort levels are sometimes either unobservable or unverifiable,

*The author is grateful to Michihiro Kandori and Hitoshi Matsushima for valuable comments and suggestions. The helpful advice of Eddie Dekel, Hideshi Itoh, Dan Sasaki and Satoru Takahashi, seminar participants at the University of Tokyo and attendees at the 2003 Annual Meeting of the Japanese Economic Association, is also acknowledged.

the employer must write payment schedules that depend, to some extent, on period-by-period performance that is partially related to the level of efforts. In firms, for instance, the employee occasionally receives a bonus that is linked to performance, and achievements in past periods are often reflected in renewal agreements.

In the field of contract theory, such economic situations have been studied in models of "repeated moral hazard" or "dynamic moral hazard."[1] One of the most important results in the literature is the role of memory in the optimal contract (Lambert (1983), Rogerson (1985), Malcomson and Spinnewyn (1988) and Chiappori et al. (1994)). It is shown that payments in the optimal long-term contract will generally have a *memory effect*. If it is optimal for the principal to make a first-period payment dependent on the performance of that period, then the principal will make the second-period payment dependent on the performance of the first period as well. Intertemporal smoothing and wealth effects on the agent's payoff provide motivations for the principal to offer contracts with such memory effects.

In most economic situations contaminated by dynamic moral hazard problems, however, we can easily find examples in which payments are not necessarily dependent on the realization of past performances as a whole. In firms, monthly wages do not always depend on the month-by-month performance that could provide detailed information about the worker's effort levels, but usually consist of fixed salaries in regular periods plus a bonus in a particular period. Other examples include an instructors' grading policy that mainly focuses on scores in the final exam, although week-by-week homework may reflect students' effort levels in detail. From this standpoint, it is difficult to conclude that studies of repeated moral hazard have accurately explained all practical long-term contracts in real economic environments.

In the existing repeated moral hazard literature, it is assumed that there are no exogenous links between one period and the next. Current performances are related only to current actions, and these probability distributions are assumed to be independent over time. Simplifications of this kind, seen not only in the study of repeated moral hazard but also in other economic literatures, are mainly for the sake of tractability. If the environment bears some sort of history dependence in reality, it is possible that payments in the optimal long-term contract may no longer be dependent on the entire history of past outcomes. When history dependence is relatively strong, complementarity arises between incentives, i.e., an incentive to effort in one period can provide incentives for efforts in other periods.

[1] The moral hazard model is often called the "agency problem" or the "principal-agent model" in the literature.

This paper considers a two-period dynamic moral hazard model where the probability distribution of outcomes in period 2 is dependent upon the action taken in period 1 as well as that taken in period 2. It is shown in Theorem 1 that, when the impact of a first-period shirk on the performance of period 2 is relatively large compared with that of a second-period shirk (as stated formally in Assumption 2), payments in the optimal long-term contract are not at all dependent on first-period performance.

Historical dependence of this sort is often seen in real economic environments. For example, if there is a time lag between the effort and its effect, then the probability of success in period 2 will be influenced by the effort level in period 1. If the production technology bears irreversibility, then the model becomes history dependent in a similar manner.[2]

The intuition behind the result is as follows. For the principal who is willing to induce the agent to work harder in period 2, it is necessary to make the second-period payment dependent on the second-period performance as this is the only source of incentive power available. However, such a payment schedule would induce the agent to work harder in period 1 since the performance in period 2 is also affected by the effort level in period 1. Moreover, this is sufficient for the agent to work hard in period 1: If the history dependence is relatively strong, shirking in period 1 would always make the agent worse off than shirking in period 2. Thus, the principal does not need to make the payment schedule dependent on first-period performance in any way.

A brief review of the related literature is as follows. Lambert (1983), Rogerson (1985), Malcomson and Spinnewyn (1988) and Chiappori et al. (1994) show that payments in the optimal long-term contract have memory effects in repeated moral hazard models without history dependence. The original statement of the memory effect in Rogerson (1985) is quite weak and applies to all optimal contracts in the model. However, if we look into a specific situation in which the principal should like to implement positive efforts in both periods and probability distributions satisfy the *monotone likelihood ratio condition* (MLRC), the role of memory in the optimal long-term contract can be strengthened. In such setups, optimal payments should always be dependent on the realization of entire past performances. In this paper, this strong consequence is referred to as the *strong memory* effect in contrast with the *weak memory* effect of the original statement.[3] The main result of this paper (Theorem 1) in the history-dependent model is in complete contrast to the strong memory effect in the history-independent

[2]These examples are examined in detail in Section 5.

[3]Proposition 2 rigorously states these two types of memory effect.

model.

Ma (1991) studies a two-period dynamic moral hazard model with history dependence, although the main focus is on renegotiation-proof contracts. In that paper, it is assumed that the agent takes action only in period 1 but the action has a persistent effect on outcomes over two periods. This setup is slightly different from that in the current paper where the agent takes action every period. In Ma's model there is no complementarity between incentives as discussed earlier and, therefore, the optimal (full commitment) long-term contract has a strong memory effect as in the history-independent model.

Also related are multitask incentive problems as discussed in Holmström and Milgrom (1991), and Laffont and Martimort (2002, Ch. 5). In principle, dynamic moral hazard models follow the lines of multitask incentive problems. However, the former stresses the sequentiality of actions and outcomes whereas the latter focuses on the technological interaction among tasks. In repeated moral hazard without history dependence, such an interaction among tasks (i.e., an interaction among periods) has no importance. However, in a model with history dependence, as in the present analysis, the multitask aspects of the repeated moral hazard are highlighted.

Finally, Fernandes and Phelan (2000) present a recursive method for dynamic moral hazard models with history dependence. Fernandes and Phelan study the infinite-period model and show that problems with history dependence can be solved recursively if one considers "threat keeping constraints" as well as the usual incentive compatibility and participation constraints. This paper in effect solves the model for the special case and presents an explicit solution, although the method is not recursive and the model is a finite-period one.

The remainder of the paper is organized as follows. Section 2 describes the model and formally defines the *history-dependent* model (Definition 1). Section 3 provides existing results in the history-independent model and states the two types of memory effect in the literature (Proposition 2). The main result is presented in Section 4. There, it is shown that the optimal long-term contract may not have the strong memory effect and a sufficient condition for the result is presented. Section 5 provides some examples of environments in which *memoryless* contracts are to be offered. Section 6 contains some concluding remarks.

2 Model

We study a simple dynamic moral hazard model with "history dependence." The relationship between a principal (she) and an agent (he) lasts for two periods ($t = 1, 2$).

In each period, the agent chooses his action a^t from the action space $A = \{\underline{a}, \bar{a}\}$. These actions are kept unobservable to the principal. We may find it convenient to interpret those actions as effort levels, and say that he works hard (respectively, shirks) when he chooses $\bar{a}$ (respectively, $\underline{a}$).

In period t, after the agent has chosen his action a^t, the outcome $x^t \in \{x_1, \cdots, x_N\}$ realizes according to probabilities that depend on the history of agent's actions; that is, the distribution of x^1 depends on a^1, whereas that of x^2 depends on the pair (a^1, a^2). These outcomes are immediately observed by both parties (and assumed to be verifiable to third parties, such as a court). We may regard these outcomes as performances, and identify each of them with the corresponding revenue to the principal.

We assume that x^1 and x^2 are independently distributed[4]; hereafter, we will write the distributions as follows:

$$\begin{aligned} p_i(a^1) &= \Pr\left[x^1 = x_i \mid a^1\right] && (i = 1, \cdots, N), \\ p_j(a^1, a^2) &= \Pr\left[x^2 = x_j \mid (a^1, a^2)\right] && (j = 1, \cdots, N). \end{aligned}$$

Throughout the paper, we assume that the distributions are of full supports:

$$\begin{aligned} p_i(a^1) &> 0 && \text{for all } (i, a^1) \in \{1, \cdots, N\} \times A, \\ p_j(a^1, a^2) &> 0 && \text{for all } (j, a^1, a^2) \in \{1, \cdots, N\} \times A^2. \end{aligned}$$

In later sections, we often refer to the model either as "history dependent" or as "history independent". In the existing literature in which history-independent models are mostly considered, it is assumed that the performance in period 2 depends only on the action chosen in period 2. In the history-dependent model on which we will focus in the paper, we deal with the influence of the action chosen in period 1 on the performance in period 2. Here, we define them more rigorously.

Definition 1. The model is *history dependent* if there exists $j \in \{1, \cdots, N\}$ such that $p_j(\bar{a}, \cdot) \neq p_j(\underline{a}, \cdot)$. It is *history independent* if, for all $j \in \{1, \cdots, N\}$, $p_j(\bar{a}, \cdot) = p_j(\underline{a}, \cdot)$.

At the beginning of the game (i.e., before $t = 1$), the principal and the agent sign a contract in the manner described in detail below.

[4] This assumption says that the realized value of x^1 does not influence the distribution of x^2, so that the former yields no information on the current likelihood of any particular production levels in period 2. "History dependence" discussed in this paper treats the case where x^2 is affected by a^1, but not by the realization of x^1.

First, the principal offers a long-term contract $\ell = (w^1, w^2)$, where $w^1 = (w_i)$ and $w^2 = (w_{ij})$ are payment schedules for periods 1 and 2, respectively, under outcome realizations $(x^1, x^2) = (x_i, x_j)$. Such a contract stipulates $N + N^2$ possible payments, depending on the realizations of outcomes. Next, the agent decides whether to accept or refuse the contract offered by the principal. If the agent refuses the offered contract, both parties receive their reservation utilities, and the game comes to an end. If the agent accepts the contract, the game enters into the two times moral hazard repetition discussed above.

We assume that the principal can commit to the long-term contract that she has offered before $t = 1$ and so, once the contract is accepted by the agent, the principal cannot change the payment schedule ℓ and must make the payment each period according to the history of outcome realizations up to the date. We also assume that the agent can commit to his participation to the game and so, once he accepts the contract, he cannot exit in the midst of the game and must participate in it until the end of period 2.

In each period, the agent attains a payoff of $u(w)-c(a)$, where u is strictly increasing and strictly concave (the agent is risk-averse) and $c(\underline{a}) < c(\bar{a})$ (harder work makes more cost). We normalize this as $c(\underline{a}) = 0$ and $c(\bar{a}) = C$.

Given a long-term contract ℓ, the agent's strategy consists of two parts: one is the action he takes in the first period, a^1, and the other is the action schedule for the second period $a^2 = (a_i^2)_{i=1}^N$, each of which specifies the action he will take in period 2 under the outcome realization of x_i in period 1.[5] Let $U_i(a^1, a_i^2)$ denote the expected utility in period 2 for the agent when he took a^1 and the outcome was x_i in the first period:

$$U_i(a^1, a_i^2) = \sum_{j=1}^{N} p_j(a^1, a_i^2)u(w_{ij}) - c(a_i^2).$$

Using this notation, the intertemporal expected utility for the agent $U(a^1; a^2)$ under the agent's strategy $(a^1; a^2)$ can be written as

$$U(a^1; a_1^2, \cdots, a_N^2) = \sum_{i=1}^{N} p_i(a^1)\left[u(w_i) + U_i(a^1, a_i^2, w^2)\right] - c(a^1).^6$$

[5] Accordingly, we allow the agent to change his action in period 2 after he observes the outcome realization in period 1, which is one of the standard assumptions in the literature. Once we cease this assumption and assume that the agent had to commit to a pair of actions (a^1, a^2) *ex ante*, then the model reduces to a one-shot multitask incentive problem. We shall take the sequentiality assumption to focus on the dynamics of the model, but note that the main result of the paper (Theorem 1) also applies to the one-shot multitask model.

[6] We assume that both the Principal and the Agent have the common discount factor

The optimization problem for the principal when she wishes to implement an action profile (a^1, a^2) can now be written as:

$$\min_{(w^1,w^2)} \sum_{i=1}^{N} p_i(a^1) \left[w_i + \sum_{j=1}^{N} p_j(a^1, a_i^2) w_{ij} \right], \tag{P}$$

subject to

$$U(a^1, a^2) \geq U(a', a''), \quad a' \neq a^1, \quad \forall a'' \in A^N, \tag{IC1}$$

$$U_i(a^1, a_i^2) \geq U_i(a^1, a'), \quad a' \neq a_i^2, \quad i = 1, \cdots, N, \tag{IC2}$$

$$U(a^1, a^2) \geq 2\bar{u}, \tag{PC}$$

where $\bar{u}$ denotes the reservation utility for the agent.

At this point, we should emphasize how the optimization problem (P) differs from the one for the history-independent model. When the model is history independent, the action taken in period 1, a^1, does not affect the probability distribution of outcomes in period 2 so that $U_i(a', a_i^2) = U_i(a'', a_i^2)$ for any $a' \neq a''$. This would reduce the incentive constraints for the first period (IC1) to

$$U(a^1, a^2) \geq U(a', a^2), \quad (a' \neq a^1), \tag{IC1$^{\text{ind}}$}$$

under which we must only take into account the deviation strategies from a^1 to the other a', with a^2 fixed. For the history-dependent model, on the other hand, this would not be sufficient: we must take into account all possibilities of deviation the agent might make during the two periods, as it is no longer assured that he will always take a^2 regardless of the action he takes in period 1, even if (IC2) is satisfied for the a^1.

3 Existing Results

This section gives two existing characterizations for the optimal long-term contract in repeated moral hazard models (Propositions 1 and 2).

Proposition 1 shows that the basic relationship in the literature between payments across two periods applies to the history-dependent model as well.

of 1. If the common discount factor were less than 1 (but positive) and the outcome space consists of three elements or more, we cannot attain plausible sufficient conditions as in Assumption 2, which can be described only with the nature of $(p_i(\cdot))$ and $(p_j(\cdot,\cdot))$. An independent related paper by Mukoyama and Sahin (2004) shows in case of $N = 2$ that an extension of Assumption 2 is a sufficient condition for w_i to be constant in a similar model in which both players have a common discount factor less than 1.

Proposition 1. *If an optimal long-term contract* (w^1, w^2) *implements the action profile* (a^1, a^2)*, then*

$$\frac{1}{u'(w_i)} = \sum_{j=1}^{N} \frac{p_j(a^1, a_i^2)}{u'(w_{ij})}, \quad i = 1, \cdots, N.$$

Proof. The same proof as in Rogerson (1985). □

Next, we focus on the history-independent model and state two kinds of memory effect in the optimal long-term contract. One (*weak memory*) is suggested by Rogerson (1985), and the other (*strong memory*) is a straightforward consequence of the former when (i) the model satisfies monotonicity condition (Assumption 1) and (ii) the contract is to induce the agent to make positive efforts in both periods.

Assumption 1. $p_i(a^1)$ satisfies the *monotone likelihood ratio condition* (MLRC); that is,

$$p_i(\bar{a})/p_i(\underline{a}) \text{ is increasing in } i.$$

Assumption 1 says that the switching from low effort $\underline{a}$ to high effort $\bar{a}$ increases the probability of getting a higher outcome at least as much as it increases the probability of getting a lower outcome.

Rogerson (1985) showed that, if the first-period performance plays any role in determining payments in period 1, then it also plays a role in determining payments in period 2. This statement of memory effect is quite weak and applies to all optimal contracts in the history-independent model. If we look into specific economic situations, however, the above statement of memory effect can be strengthened. In particular, if the probability distribution $p_i(\cdot)$ satisfies Assumption 1, then the first-period outcome always plays a role in determining payments both in period 1 and in period 2 when the principal wishes to implement positive efforts in both periods.[7] Proposition 2 states it formally.

Proposition 2. *Suppose that the model is history independent. Then the optimal long-term contract* ℓ *is such that*

(i) $w_i \neq w_k \Rightarrow [\exists j,\ w_{ij} \neq w_{kj}].$ (WM)

(ii) w_i *is increasing in* i*, if the model satisfies Assumption 1 and* ℓ *is to implement* $a^1 = \bar{a}$ *and* $a^2 = (\bar{a}, \cdots, \bar{a})$.

[7] We say that an action of the agent, a, is a positive effort if it satisfies $c(a) > \min_{a' \in A} c(a')$. In our model where the action set is of two elements, $\bar{a}$ is the only one positive effort action in A.

Proof. (i): Suppose, on the contrary, that $w_{ij} = w_{kj}$ for all j. Then, from (IC2), the agent's optimal action in the period 2 would be such that $a_i^2 = a_k^2$, which implies $u'(w_i) = u'(w_k)$ from Proposition 1: a contradiction.

(ii): Let the Lagrange multipliers with respect to (IC1$^{\text{ind}}$) be λ^{ind}, and that to (PC) be ν. Then, the first-order condition for w_i in the optimization problem (P) is

$$\frac{1}{u'(w_i)} = \lambda^{\text{ind}} \left[1 - \frac{p_i(\underline{a})}{p_i(\bar{a})}\right] + \nu. \tag{1}$$

Now, we can show that $\lambda^{\text{ind}} > 0$. Suppose, on the contrary, that $\lambda^{\text{ind}} = 0$. Then $w_i = w_k$ and $(w_{ij}) = (w_{kj})$ for all $i \neq k$. However, such (w^1, w^2) violates (IC1$^{\text{ind}}$) because $C > 0$: a contradiction.

From Assumption 1 and strict concavity of $u(\cdot)$, we conclude that w_i must be increasing in i. □

Corollary. *In the history-independent model that satisfies Assumption 1, the optimal long-term contract would be such that*

$$\begin{aligned} &w^1 \text{ depends on } x^1\text{, and} \\ &w^2 \text{ depends on both } x^1 \text{ and } x^2, \end{aligned} \tag{SM}$$

if it is to implement $a^1 = \bar{a}$ *and* $a^2 = (\bar{a}, \cdots, \bar{a})$.

In the following section, we say that a long-term contract has *weak memory* if it satisfies (WM) in Proposition 2 and that a long-term contract has *strong memory* if it satisfies (SM) in the Corollary.

4 Memoryless Contracts

In this section, we show that the optimal long-term contract does not have *strong memory* in the history-dependent model if the probability distributions of the second period outcomes satisfy certain conditions as presented below. The result (Theorem 1) lies in contrast to that in the history-independent model in that the optimal long-term contract would always have *strong memory* as described in Proposition 2.

If the model is history dependent, the payment schedule for a period can influence the agent's incentives in the preceding periods as well as that of the current period, as the actions taken in preceding periods may affect the probability distribution of current outcomes. Furthermore, if the probability distributions of the second period are more affected by the first-period action than by the second-period action, it is possible that the payment schedule in the second period alone would give enough incentive to take the indicated actions for both periods, without the need to incentivize by first period payments. If so, then the optimal long-term contract would become *memoryless*, making w^1 a constant payment and w^2 dependent only on x^2.

The following assumption gives the sufficient condition for memoryless contracts. We may regard this assumption as "strong history dependence" in the sense that the action chosen in period 1 has a stronger influence on the outcome in period 2 than the action chosen in period 2.

Assumption 2. $p_j(a^1, a^2)$ satisfies the following three conditions:

(i) $p_j(a^1, \bar{a})/p_j(a^1, \underline{a})$ is increasing in j for all a^1. (MLRC)

(ii) $\sum_{j=1}^{J} p_j(\underline{a}, \bar{a}) \geq \sum_{j=1}^{J} p_j(\bar{a}, \underline{a})$ for all $J \in \{1, \cdots, N\}$.

(iii) $\frac{1}{2} \sum_{j=1}^{J} (p_j(\underline{a}, \underline{a}) + p_j(\bar{a}, \bar{a})) \geq \sum_{j=1}^{J} p_j(\bar{a}, \underline{a})$ for all $J \in \{1, \cdots, N\}$.

In Assumption 2, (ii) says that the action profile $(\bar{a}, \underline{a})$ stochastically dominates the action profile $(\underline{a}, \bar{a})$ in the distribution of x^2, while (iii) says that $(\bar{a}, \underline{a})$ stochastically dominates the half-by-half randomization between $(\underline{a}, \underline{a})$ and $(\bar{a}, \bar{a})$. We should note that neither (ii) nor (iii) in Assumption 2 can be satisfied in the history-independent model.

Theorem 1. *Suppose that the model is history dependent and satisfies Assumptions 1 and 2. Suppose also that the principal should like to implement* $a^1 = \bar{a}$ *and* $a^2 = (\bar{a}, \cdots, \bar{a})$. *Then, the optimal long-term contract* ℓ *is such that*

(i) w_i *is a constant for all* i,

(ii) w_{ij} *is independent of* i, *and is increasing in* j.

Proof. This proof is as follows: 1. to show that the optimal long-term contract satisfies the properties (i) and (ii) when (IC1) is not binding, and 2. to verify that the derived contract automatically satisfies (IC1) under Assumptions 1 and 2.

1. Suppose (IC1) is not binding. Then, the first-order condition for w_i in the optimization problem (P) is

$$\frac{1}{u'(w_i)} = \nu,$$

where ν is the Lagrange multiplier with respect to (PC). Thus, w_i is a constant for all i.

The first-order condition for w_{ij} in the optimization problem (P) is

$$\frac{1}{u'(w_{ij})} = \frac{\mu_i}{p_i(\bar{a})}\left[1 - \frac{p_j(\bar{a},\underline{a})}{p_j(\bar{a},\bar{a})}\right] + \nu,$$

where μ_i is the Lagrange multiplier with respect to (IC2) for the corresponding i. First note that, since (IC1) is not binding, w_{ij} is independent of i (otherwise the principal could be strictly better off by offering the certainty equivalence $\tilde{w}'_j$ such that $u(\tilde{w}'_j) = \sum_i p_i(\bar{a})u(w_{ij})$, without affecting the remaining constraints (IC2) and (PC)). Hence, the ratio $\mu_i/p_i(\bar{a})$ is a constant for all i.

If $\mu_i = 0$, then w_{ij} would be a constant for all j, which violates (IC2) for i. Hence, $\mu_i > 0$ should be satisfied for all i, which means that (IC2) is binding in the optimum. Therefore, from Assumption 2 (i) and the concavity of $u(\cdot)$, w_{ij} must be increasing in j.

2. Firstly, we check that (IC1) is satisfied for two deviation strategies $(a^1;a^2) = (\underline{a};\bar{a},\cdots,\bar{a})$ and $(a^1;a^2) = (\underline{a};\underline{a},\cdots,\underline{a})$ under the optimal contract derived in 1. Here, we write $w_i = w$ and $w_{ij} = w'_j$ as the contract is not dependent on i.

As shown in 1., (IC2) is binding at the optimum; therefore,

$$C = \sum_{j=1}^{N} p_j(\bar{a},\bar{a})u(w'_j) - \sum_{j=1}^{N} p_j(\bar{a},\underline{a})u(w'_j) \tag{2}$$

(IC1) to hold against deviation strategy $(a^1;a^2) = (\underline{a};\bar{a},\cdots,\bar{a})$ is equivalent to

$$\sum_{j=1}^{N} p_j(\bar{a},\bar{a})u(w'_j) - 2C \geq \sum_{j=1}^{N} p_j(\underline{a},\bar{a})u(w'_j) - C,$$

which, by substituting (2), yields

$$\sum_{j=1}^{N} p_j(\bar{a},\underline{a})u(w'_j) \geq \sum_{j=1}^{N} p_j(\underline{a},\bar{a})u(w'_j).$$

Since $u(w'_j)$ in increasing in j, a sufficient condition for this inequality to hold is that $(\bar{a},\underline{a})$ stochastically dominates $(\underline{a},\bar{a})$ in the probability distribution of x^2: Assumption 2 (ii).

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$$\sum_{j=1}^{N} p_j(\bar{a},\bar{a})u(w'_j) - 2C \geq \sum_{j=1}^{N} p_j(\underline{a},\underline{a})u(w'_j),$$

which, by substituting (2), yields

$$2\sum_{j=1}^{N} p_j(\bar{a},\underline{a})u(w'_j) \geq \sum_{j=1}^{N} p_j(\bar{a},\bar{a})u(w'_j) + \sum_{j=1}^{N} p_j(\underline{a},\underline{a})u(w'_j).$$

Likewise a sufficient condition for this inequality to hold is that $(\bar{a},\underline{a})$ stochastically dominates the half-by-half randomization between $(\bar{a},\bar{a})$ and $(\underline{a},\underline{a})$: Assumption 2 (iii).

Finally we check that (IC1) is satisfied for any deviation strategies $(a^1;a^2) = (\underline{a};a_1^2,\cdots,a_N^2)$. Suppose the agent is to take $a_i^2 = \bar{a}$ if $i \in \bar{I} \subset \{1,\cdots,N\}$ and $a_i^2 = \underline{a}$ if $i \in \underline{I} = \{1,\cdots,N\} \setminus \bar{I}$. The intertemporal payoff to the agent following this deviation strategy would satisfy

$$\begin{aligned}
& u(w) + \sum_{i\in\bar{I}} p_i(\underline{a}) \left[\sum_{j=1}^{N} p_j(\underline{a},\bar{a})u(w'_j) - C\right] + \sum_{i\in\underline{I}} p_i(\underline{a}) \left[\sum_{j=1}^{N} p_j(\underline{a},\underline{a})u(w'_j)\right] \\
& \leq u(w) + \max\left\{\sum_{j=1}^{N} p_j(\underline{a},\bar{a})u(w'_j) - C,\ \sum_{j=1}^{N} p_j(\underline{a},\underline{a})u(w'_j)\right\} \\
& = \max\{U(\underline{a};\bar{a},\cdots,\bar{a}),\ U(\underline{a};\underline{a},\cdots,\underline{a})\} \\
& \leq U(\bar{a};\bar{a},\cdots,\bar{a}),
\end{aligned}$$

where the last inequality comes from the previous result that (IC1) is satisfied both for $(a^1;a^2) = (\underline{a};\bar{a},\cdots,\bar{a})$ and for $(a^1;a^2) = (\underline{a};\underline{a},\cdots,\underline{a})$. Hence, (IC1) is satisfied for any deviation strategy $(a^1;a^2) = (\underline{a};a_1^2,\cdots,a_N^2)$. □

The intuition behind the proof is as follows. For the principal who is willing to induce the agent to exert the positive effort $\bar{a}$ in period 2, it is necessary to make the second-period payment w_{ij} dependent on the second-period outcome j as this is the only source of incentive power available. However, such a payment schedule would induce the agent to work hard in period 1 since the distribution of second-period outcomes is affected not only by a^2 but also by a^1. Moreover, this gives the agent an incentive enough to work hard in period 1. Under Assumption 2, shirking in period 1 would always make the agent worse off than shirking in period 2, as a^1 has larger influence on the second-period outcome than a^2 does. Thus, the principal does not need to make the payment schedule dependent on the first-period outcome at all.

5 Examples

In this section, we give a few examples in which $p_j(\cdot)$ satisfies Assumption 2. These examples incorporate "strong history dependence" in the sense that the action chosen in period 1 has a stronger influence on the outcome of period 2 than the action chosen in period 2. Under such circumstances, the optimal long-term contract is *memoryless* by which we mean that the payment schedule would be dependent only upon the second-period outcome.

In the following examples, we suppose $N = 2$ ("success" and "failure") and let $\pi_t(\cdot)$ denote the probability of "success" in period t; that is, $\pi_t(\cdot) = p_2^t(\cdot)$ and $1 - \pi_t(\cdot) = p_1^t(\cdot)$.

Example 1 (Time lag)**.** There is a time lag between the effort and its effect.

If the agent works hard in period t, it not only increases the probability of success in the same period by α but also increases the probability of success in the following period by β. We assume $0 < \alpha < \beta$ in which we can regard β as a "full effect" of the effort and α as a "partial effect" of the effort. Let $\underline{\pi}$ denote the probability of success when the agent has never taken any positive efforts. Then, we can write $\pi_t(\cdot)$ as follows:

$$\begin{aligned} \pi_1(\underline{a}) &= \underline{\pi}, & \pi_1(\bar{a}) &= \underline{\pi} + \alpha, \\ \pi_2(\underline{a}, \underline{a}) &= \underline{\pi}, & \pi_2(\underline{a}, \bar{a}) &= \underline{\pi} + \alpha, \\ \pi_2(\bar{a}, \underline{a}) &= \underline{\pi} + \beta, & \pi_2(\bar{a}, \bar{a}) &= \underline{\pi} + \alpha + \beta. \end{aligned}$$

Assumption 2 (ii) is satisfied as $\pi_2(\bar{a}, \underline{a}) > \pi_2(\underline{a}, \bar{a})$; this is the "time-lag effect" since the positive effort $\bar{a}$ taken in period 1 has greater influence β than it has if taken in period 2 (α). Assumption 2 (iii) is also satisfied as $\pi_2(\bar{a}, \underline{a}) > \frac{1}{2}\left[\pi_2(\bar{a}, \bar{a}) + \pi_2(\underline{a}, \underline{a})\right]$.

Example 2 (Irreversibility)**.** The agent has to make a positive effort every period to maintain the highest probability of success $\bar{\pi}$. If he shirks, the probability of success declines by γ and this will never be recovered, even if the agent makes a positive effort in the following period:

$$\begin{aligned} \pi_1(\underline{a}) &= \bar{\pi} - \gamma, & \pi_1(\bar{a}) &= \bar{\pi}, \\ \pi_2(\underline{a}, \underline{a}) &= \bar{\pi} - 2\gamma, & \pi_2(\underline{a}, \bar{a}) &= \bar{\pi} - \gamma \\ \pi_2(\bar{a}, \underline{a}) &= \bar{\pi} - \gamma, & \pi_2(\bar{a}, \bar{a}) &= \bar{\pi}. \end{aligned}$$

It is clear that the distribution satisfies Assumption 2 (ii) and (iii) with equalities.

6 Concluding Remarks

This paper has examined the role of history dependence in a dynamic moral hazard model. It is shown that, under certain conditions on the probability distributions of outcomes, the optimal long-term contract is such that the payment schedules are not contingent upon the realization of past outcomes. This finding lies in striking contrast to the results in existing models without history dependence in that the optimal long-term contracts will generally have *memory* effects.

In real economic environments, we see a variety of examples where both the assumption that the outcome can be history dependent and the result that the payments do not fully reflect the realization of past outcomes. However, the assumption of full commitment may be too strong in some of these economic contexts. In the study of moral hazard problems, renegotiation-proof contracts have been investigated by Fudenberg and Tirole (1990), Ma (1991, 1994) and Matthews (1995). In this respect, the current paper calls for further research on renegotiation in models with history dependence.

This paper has favored the simplest models to focus upon the role of history dependence. In particular, we have assumed two actions and independent distributions over periods in the paper. Generalizations of this model also deserve further investigation.

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