

The Effects of the Loss of Skill on Unemployment Fluctuations.

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Abstract

This paper studies the effects of the loss of skill on the persistence of unemployment and other macroeconomic variables. It combines a Real Business Cycle model with a search and matching labor market to explain how the loss of skill of workers and the subsequent decrease in their probability of finding new jobs creates more persistent business cycles. The paper proves that the introduction of this mechanism improves the performance of the model and is able to replicate cross country differences in unemployment and output persistence.

Keywords: Business Cycles, Search and Matching, Loss of Skill

JEL Classification: E32, J41, J31

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1 Introduction

Unemployment fluctuations over the business cycle are a serious concern for policy makers. One important aspect of unemployment is its persistence over the cycle. Disentangling the reasons behind this persistence and the differences across countries would help understand how to decrease high unemployment levels.

Macroeconomic variables, especially unemployment, show high persistence in their cyclical components. Analyzing quarterly data from 1980 to 2002, one observes that for the US economy, the serial correlation between unemployment rates one quarter apart is 90 percent and 23 percent with one year difference. In Europe, these numbers are bigger and an economy like the Spanish shows even more persistence, with 95 percent autocorrelation for one quarter difference and 58 percent when the distance is one year. These facts show the magnitude of the problem and raise the question of the determinants of such high persistence and its differences across countries.

This paper claims that a mechanism which affects the persistence of the business cycle, in particular the persistence of unemployment, is the loss of skill of workers which have been out of work for an extended period of time. It has been well documented that the probability of finding a job decreases with the duration of unemployment and that the longer is the spell of unemployment the higher is the possibility of deterioration of the human capital of the worker. Bover, Arellano & Bentolila (2002) show that for the Spanish economy the probability of finding a job decreases steadily with the duration of unemployment and is reduced for individuals who have been out of work for over a year. Jackman & Layard (1991) find that the exit rate from unemployment decreases when there is a higher proportion of long-term unemployed. They also note the relationship between the spell of unemployment and the possibility of destruction of the skills of the worker.

The loss of skill of unemployed workers has been previously used in the literature to explain the differences between unemployment levels in Europe and the US. Ljungqvist & Sargent (1997) and den Haan, Haefke & Ramey (2001) introduce this mechanism to analyze the rise of unemployment in Europe in the eighties. They show that the differences in institutions and labor market characteristics, like unemployment benefits, help explain how an increase in the rate of loss of skill of workers raises unemployment in an economy like the European, not having such a big impact in an economy with the features of the US. However, they do not study the cyclical behavior of unemployment and other macroeconomic variables.

Pissarides (1992) shows, within a search and matching model, that the temporary effects of employment shocks last longer when unemployed workers lose their skill over time. His analysis is purely qualitative and the mechanism which creates the more persistent dynamics is the fact when workers lose their skill they become less attractive to potential employers and few firms enter the market, making the labor market thinner and lowering the probability of finding work for every type of worker. Here we try to prove that this more persistent effect of shocks can be extended to the other macroeconomic variables, but to do so it is necessary to work with a more complete model, one which includes production, consumption and the other elements which the business cycle models take into consideration.

Mertz (1995) and Andolfatto (1996) studied these fluctuations within a framework which combines a business cycle model with a search and matching labor market. However, these two papers,

although performing well in other aspects, are not able to replicate the persistence in unemployment and output observed in the data. Mertz (1995) can only generate half the persistence shown in the data, and Andolfatto (1996) needs to assume a 40 percent unemployment rate for the US economy to obtain reasonable results. The claim of this paper is that the previous literature misses this additional mechanism of the loss of skill to generate enough persistence in the labor market and in the other macroeconomic variables.

The model presented in this paper is based on the framework proposed by the previous two authors and it incorporates a search and matching labor market into a Real Business Cycle (RBC) model. It assumes infinitely lived risk-averse agents who decide how to best allocate their wealth between consumption and savings. They also supply labor to firms which produce a homogenous good. It uses a simple version of a Real Business Cycle model which abstracts from capital accumulation or any type of rigidity or inefficiency except for the search and matching friction in the labor market¹. This market is modeled in the style of Mortensen & Pissarides (1994). I assume a single type of firm, which hires either low or high skilled workers. Low skilled workers must undergo training on their first period of work, whereas high skilled workers do not. The cost of training makes low skilled workers less attractive to firms since it reduces their profits. This in turn implies that low skilled workers find jobs with a lower probability than the high skilled. This probability is endogenous in the model and it depends on the cost of training. The loss of skill occurs over time and is modeled as a random process, transferring workers from the high to the low skilled unemployment pool. The longer a worker is unemployed the more probable it is that he will lose the skill.

This paper finds that introducing the loss of skill mechanism in this framework increases the persistence of unemployment with respect to previous studies. The serial correlation of unemployment and output rises in comparison with models with no loss of skill and the difference is close to 10 percent for the fourth and fifth order autocorrelations. This model can also closely replicate the differences in persistence in unemployment and output across countries. This is proven by taking the extreme cases of Spain and the US.

The intuition for the increase in persistence is as follows. When the economy suffers a negative shock, employment drops and unemployment increases. Given the matching friction in the economy, unemployed workers cannot return to work instantaneously. Both the reduction in vacancies, due to the reduction in profits, and the increase in unemployment, decrease the probability of unemployed individuals finding jobs, lengthening their unemployment spell. The increase in the duration of unemployment raises the chances of workers losing their skill, increasing low skilled unemployment. Both the increase in unemployment and the increase in the proportion of the low skilled, who have even lower probability of finding jobs, raises the average duration of unemployment in the economy and the persistence of unemployment and the other macroeconomic variables. This intuition also helps explain the ability of the model to replicate how countries which have a higher unemployment rate and higher proportion of long-term unemployment show higher persistence in unemployment and output.

The remainder of the paper is organized as follows. Section 2 presents the model, explaining

¹ The theoretical structure of the model is kept to the minimum to isolate the effects of the loss of skill from other mechanisms which affect unemployment and output fluctuations.

the problem of the household and the problem of the firm and worker. Section 3 explains the calibration of the parameters. Section 4 presents the results of the paper and Section 5 summarizes the main conclusions. The derivations and alternative calibrations of the model are left for the appendix.

2 The Model

2.1 Environment

The economy is composed by a continuum of infinitely lived agents. Time is discrete and agents decide optimally how much to consume, save and, if employed, how much labor to supply.

Workers and firms engage in employment relationships, which are composed of one firm and one worker.

Unemployed workers are assumed to be either high or low skilled. Workers who have just lost their job retain their skill for some time. The loss of skill, which only happens while being out of work, will occur with probability λ for the high skilled unemployed workers who have not matched with a firm.

Firms can hire either type of unemployed worker. Both types are equally productive, although the low skilled has to be trained during the first period of employment. Training has a cost t , which is shared between the worker and the firm. After the period of training the worker becomes high skilled.

Vacant firms and unemployed workers meet randomly according to a matching function $m(u_t, v_t)$, where $u_t = u_{lt} + u_{ht}$, u_t is total unemployment, u_{lt} and u_{ht} are low and high skilled unemployment respectively, and v_t is the number of vacancies. The matching function is assumed to be constant returns to scale, which implies

$$m(u_t, v_t) = m\left(1, \frac{v_t}{u_t}\right) u_t = m(\theta_t) u_t$$

where $\theta_t = \frac{v_t}{u_t}$ is the market tightness of the labor market.

Hence, the arrival rate of low and high skilled workers for firms is respectively $\frac{m(u_t, v_t)}{v_t} \frac{u_{lt}}{u_t} = \frac{m(\theta_t)}{\theta_t} \frac{u_{lt}}{u_t}$ and $\frac{m(\theta_t)}{\theta_t} \frac{u_{ht}}{u_t}$. The arrival rate of firms for workers is $\frac{m(u_t, v_t)}{u_t} = m(\theta_t)$.

If the search process is successful, firms produce output according the production function $Y_t = A_t L_t^v$ where A_t is the level of technology of the economy and L_t the number of hours worked by the employee. The aggregate technology level is $A_t = Ae^{z_t}$, where z_t is an aggregate technology shock which follows a first order autoregressive process $z_t = \rho_\epsilon z_{t-1} + \epsilon_t^z$. The costs of production for the firm are the wages, the cost of training if the worker is low skilled and a fixed cost η_t . This fixed cost, which can be interpreted as the cost of intermediate inputs other than labor, is idiosyncratic to the firm and independent and identically distributed across firms and time, with distribution function $F : [0, \infty] \rightarrow [0, 1]$. A new cost is drawn every period by the firm, and if the cost is high

enough it may be beneficial for the firm and the worker to discontinue the employment relationship. The value of η_t which dissolves the match is denoted by $\bar{\eta}_t$ and in principle is different for firms which hire high or low skilled workers, since the latter have to pay the extra cost of training. Therefore the probability of job destruction is $1 - F(\bar{\eta}_{ht})$ and $1 - F(\bar{\eta}_{lt})$ for high and low skilled workers respectively.

Wages are determined as a Nash bargaining process over the surplus of the match, where β_w is the share of the surplus for the worker. The cost of training, which reduces the surplus of the match with low skilled workers, is shared in the same manner.

The timing of the model is as follows. At the beginning of every period the level of technology of the economy is revealed and every matched firm draws an intermediate input cost. These two variables determine the number of productive and unproductive matches for the period. After destruction takes place, the levels of employment, high and low skilled unemployment are determined. At that point agents decide their level of consumption, savings and number of hours to work. Once these decisions are made production starts at firms and vacancies and unemployed workers try to meet. High skilled workers who do not match with firms can suffer the loss of skill, after which they become low skilled and will have to wait until the following period to search again.

2.2 The problem of the firm and the worker

Firms post vacancies in the labor market and, when matched with a worker, implement optimal production plans in order to maximize their profits. Posting vacancies has a flow cost of k for the firm. A vacant firm will match with a worker of type $i \in \{l, h\}$ with probability $q_{it} = \frac{m(\theta_t) u_{it}}{\theta_t u_t}$. If the firm is matched, and the idiosyncratic shock is low enough, the following period the firm will obtain the value of being filled by worker of type i , otherwise it will remain as a vacancy. Denote by V_t and $J_{it}(\eta_t)$ the values, measured in terms of consumption, of having a vacancy opened and of a match for a firm which hires worker of type i . Hence the value of a vacancy is

$$V_t = -k + \beta E_t \left[q_{lt} \int_{\eta_{\min}}^{\bar{\eta}_{lt+1}} J_{lt+1}(\eta_{t+1}) dF(\eta_{t+1}) + \right. \\ \left. + q_{ht} \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} J_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) + (1 - q_{lt}F(\bar{\eta}_{lt+1}) - q_{ht}F(\bar{\eta}_{ht+1})) V_{t+1} \right] \quad (1)$$

Free entry of firms is assumed in equilibrium, which implies that the value of a vacancy must be zero. Therefore:

$$0 = -k + \beta E_t \left[q_{ht} \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} J_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) + q_{lt} \int_{\eta_{\min}}^{\bar{\eta}_{lt+1}} J_{lt+1}(\eta_{t+1}) dF(\eta_{t+1}) \right] \quad (2)$$

The value for the firm with a high skilled worker is

$$J_{ht}(\eta_t) = A_t L_t^v - \eta_t - w_{ht}(\eta_t) L_t + \beta E_t \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} J_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) \quad (3)$$

The interpretation of the previous equation is as follows. During the current period, given the firm's idiosyncratic cost of intermediate inputs, η_t , it produces output and pays wages and the cost of these inputs. The following period, if the idiosyncratic intermediate input cost is below the threshold, the match will still be productive, with a value of $J_{ht+1}(\eta_{t+1})$, otherwise the match will be destroyed and it will become a vacancy, which has value zero.

Similar present value has a firm which hires a low skilled worker. It only differs in the fact that it has to pay its share of the cost of training, t_t^f , which also implies a different wage. Note that the continuation value is the same as the one for the high skilled firm, since the worker becomes high skilled after the first period.

$$J_{lt}(\eta_t) = A_t L_t^v - \eta_t - t_t^f - w_{lt}(\eta_t) L_t + \beta E_t \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} J_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) \quad (4)$$

Consider now the side of the worker. Denote by U_{it} and $N_{it}(\eta_t)$ the value, in terms of consumption, of being unemployed and being matched with a firm for a worker of type $i \in \{l, h\}$.

A high skilled unemployed worker obtains b_t utility from home production. If it matches with a firm, which happens with probability p_t , and the intermediate input cost for the firm is below the threshold, $\bar{\eta}_{ht+1}$, he will become a productive worker the following period. If the search process is not successful, he may lose the skill, event which occurs with probability λ , and become low skilled unemployed. If he does not enter into an employment relationship with a firm and does not lose the skill, he will remain a high skilled unemployed. Hence, the value of being high skilled unemployed at period t is:

$$U_{ht} = b_t + \beta E_t \left[p_t \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} N_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) + (1 - p_t F(\bar{\eta}_{ht+1})) \lambda U_{lt+1} + (1 - p_t F(\bar{\eta}_{ht+1})) (1 - \lambda) U_{ht+1} \right] \quad (5)$$

Similarly, a low skilled unemployed worker receives utility from the home production. He can meet a firm and if the idiosyncratic shock to the firm is favorable will start producing the following period. Otherwise he will remain a low skilled unemployed.²

$$U_{lt} = b_t + \beta E_t \left[p_t \left(\int_{\eta_{\min}}^{\bar{\eta}_{lt+1}} N_{lt+1}(\eta_{t+1}) dF(\eta_{t+1}) \right) + (1 - p_t F(\bar{\eta}_{lt+1})) U_{lt+1} \right] \quad (6)$$

²Note that the threshold for the intermediate input cost which makes the match unproductive is different from the one for the high skilled worker. During the first period of the match, the low skilled unemployed needs to be trained and its cost will lower the acceptable intermediate input cost, that is, the cost which will make the surplus equal to zero.

As in the case of the firm, the value of a match for a worker is a function of the idiosyncratic shock η_t . It also depends on the skill of the worker. The value of employment for a high skilled worker is composed by the high skilled wage, the disutility in terms of consumption from supplying labor and the continuation value, which is the value of being employed if the match is not destroyed or the value of being high skilled unemployed, if the intermediate input cost is too high.

$$N_{ht}(\eta_t) = w_{ht}(\eta_t)L_t - \frac{a_n}{1+\gamma_n}L_t^{1+\gamma_n}\frac{1}{u'(C_t)} + \quad (7)$$

$$+\beta E_t \left[\int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} N_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) + (1 - F(\bar{\eta}_{ht+1})) U_{ht+1} \right]$$

If the worker came from the low skilled unemployment pool, the value is very similar and only differentiated by the wages and the cost of training.

$$N_{lt}(\eta_t) = w_{lt}(\eta_t)L_t - t_t^w - \frac{a_n}{1+\gamma_n}L_t^{1+\gamma_n}\frac{1}{u'(C_t)} + \quad (8)$$

$$+\beta E_t \left[\int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} N_{ht+1}(\eta_{t+1}) dF(\eta_{t+1}) + (1 - F(\bar{\eta}_{ht+1})) U_{ht+1} \right]$$

When an employment relationship takes place it creates a surplus which is shared between the firm and the worker. The surplus of the match is defined as the sum of the values of a filled job for a firm and a worker minus their outside options, which are the value of a vacancy and the value of unemployment respectively. Since there is free entry of firms, the expression for the surplus is $S_{it}(\eta_t) = J_{it}(\eta_t) + N_{it}(\eta_t) - U_{it}$. The sharing rule for the surplus is obtained optimally as a Nash bargaining process which results in a constant fraction for both parties. If β_w is the bargaining power of the worker, then $N_{it}(\eta_t) - U_{it} = \beta_w S_{it}(\eta_t)$ and $J_{it}(\eta_t) = (1 - \beta_w) S_{it}(\eta_t)$. Combining these two expressions with equations (3) to (8) the surplus in terms of units of consumption for a high and low skilled match can be expressed as:

$$S_{ht}(\eta_t) = A_t L_t^v - \eta_t - \frac{a_n}{1+\gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_t + \quad (9)$$

$$+\beta E_t (1 - p_t \beta_w) \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} S_{ht+1} dF(\eta_{t+1}) - \beta (1 - p_t F(\bar{\eta}_{ht+1})) \lambda E_t (U_{lt+1} - U_{ht+1})$$

$$S_{lt}(\eta_t) = A_t L_t^v - \eta_t - t_t - \frac{a_n}{1+\gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_t - \beta E_t (U_{lt+1} - U_{ht+1}) + \quad (10)$$

$$+\beta E_t \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} S_{ht+1} dF(\eta_{t+1}) - \beta p_t E_t \beta_w \int_{\eta_{\min}}^{\bar{\eta}_{lt+1}} S_{lt+1} dF(\eta_{t+1})$$

The division of the surplus between firm and worker yields the wage paid to the employee. The expressions for the wages paid to a high and low skilled worker are respectively:

$$w_{ht}(\eta_t)L_t = \beta_w \left[A_t L_t^v - \eta_t + \beta p_t E_t \int_{\eta_{\min}}^{\bar{\eta}_{ht+1}} J_{ht+1} dF(\eta_{t+1}) \right] \quad (11)$$

$$+ (1 - \beta_w) \left[\frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} + b_t - \beta (1 - p_t F(\bar{\eta}_{ht+1})) \lambda E_t (U_{ht+1} - U_{lt+1}) \right]$$

$$w_{lt}(\eta_t)L_t = \beta_w \left[A_t L_t^v - \eta_t + \beta p_t E_t \int_{\eta_{\min}}^{\bar{\eta}_{lt+1}} J_{lt+1} dF(\eta_{t+1}) \right] \quad (12)$$

$$+ (1 - \beta_w) \left[\frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} + b_t - \beta E_t (U_{ht+1} - U_{lt+1}) \right]$$

The worker is compensated for a proportion β_w of the production of the firm net of intermediate input cost, and for a measure of the saved cost of searching for new matches. He is also compensated for a fraction $(1 - \beta_w)$ of the disutility from supplying labor, and the forgone home production. The last term of the expression reflects the fact that by being hired in a firm, a high skilled worker avoids the risk of becoming low skilled and a low skilled increases his value of unemployment once the match is destroyed.

An employment relationship is terminated when the idiosyncratic intermediate input cost to the firm is so high that it drives the surplus to zero. This determines the threshold cost above which both worker and firm will agree to dissolve the match and search for better options. Using equation (9) and (10) and equating them to zero we obtain the expressions for the low and high skill thresholds.³

Finally, given this timing explained earlier, the flows in and out of the different states for the workers are:

$$u_{ht} = (1 - p_{t-1} F(\bar{\eta}_{ht})) (1 - \lambda) u_{ht-1} + (1 - F(\bar{\eta}_{ht})) (n_{lt-1} + n_{ht-1}) \quad (13)$$

$$u_{lt} = (1 - p_{t-1} F(\bar{\eta}_{lt})) u_{lt-1} + (1 - p_{t-1} F(\bar{\eta}_{ht})) \lambda u_{ht-1} \quad (14)$$

$$n_{ht} = F(\bar{\eta}_{ht}) (n_{lt-1} + n_{ht-1}) + p_{t-1} F(\bar{\eta}_{ht}) u_{ht-1} \quad (15)$$

$$n_{lt} = p_{t-1} F(\bar{\eta}_{lt}) u_{lt-1} \quad (16)$$

$$1 = u_{lt} + u_{ht} + n_{lt} + n_{ht} \quad (17)$$

2.3 The Problem of the Household

The economy is composed of a big family where earnings are pooled together as an insurance mechanism. Firms are own by the consumers and all its profits are rebated to them. The family

³See Appendix for the expressions and derivations.

decides the level of consumption, savings and the numbers of hours to be worked by the agents employed in order to maximize its life time utility.

Therefore, the household will choose $\{C_{t+i}, B_{t+i}, L_{t+i}\}_{i=0}^{\infty}$ to max

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\frac{1}{1-\gamma_c} C_{t+i}^{1-\gamma_c} - n_{t+i} \frac{1}{1+\gamma_n} L_{t+i}^{1+\gamma_n} + (1-n_{t+i}) b_{t+i} \right] \right\} \quad (18)$$

subject to

$$\begin{aligned} C_{t+i} &= Y_{t+i} - \hat{\eta}_{t+i} - kv_{t+i} - t_{t+i} u_{t+i-1} p_{t+i-1} F(\bar{\eta}_{t+i}) - \frac{1}{1+r_{t+i}} B_{t+i} + B_{t+i-1} \\ Y_{t+i} &= n_{t+i} A_{t+i} L_{t+i}^v \\ \text{for } i &= \{0, \dots, \infty\} \end{aligned}$$

where $\beta \leq 1$ is the discount rate of the economy; $\gamma_c < 1$ and $\gamma_n > 0$ are the coefficients of risk aversion to fluctuations in consumption and hours worked; n_t is the number of employed workers, $n_t = n_{lt} + n_{ht}$; Y_t is the production of a single firm; $\hat{\eta}_t$ is the sum of costs of intermediate inputs for the productive firms; v_t is the number of vacancies and k the flow cost of being opened; $t_t u_{t-1} p_{t-1} F(\bar{\eta}_t)$ the cost of training the low skilled workers matched; b is the home production of those individuals who are not working; B_t is the number of real bonds purchased, and r_t the real interest rate in the economy.

This problem yields the following optimal conditions:

Consumption Euler equation, which shows how in equilibrium the individual is indifferent between saving or consuming one more unit.

$$C_t^{-\gamma_c} = \beta E_t \left\{ (1+r_t) C_{t+1}^{-\gamma_c} \right\} \quad (19)$$

Optimal labor supply, which depends positively on the level of technology, the labor share in the production function, v , and the marginal utility of consumption; and negatively on coefficient of risk aversion.

$$L_t = \left(v A_t C_t^{-\gamma_c} \right)^{\frac{1}{1+\gamma_n-v}} \quad (20)$$

In equilibrium the demand of bonds is equal to zero, and hence the last relationships to be satisfied are the economy wide resource constraint

$$Y_t = C_t + kv_t + t_t u_{t-1} p_{t-1} F(\bar{\eta}_t) + \hat{\eta}_t \quad (21)$$

and the aggregate production

$$Y_t = n_t A_t L_t^v \quad (22)$$

2.4 Equilibrium

An equilibrium in this economy is a recursive general equilibrium composed by the set of variables:

- unemployment rates, employment rates and vacancies u_{lt} , u_{ht} , n_{lt} , n_{ht} , v_t ,
- values of a match for a firm, $J_{ht}(\eta_t)$, $J_{lt}(\eta_t)$, for a worker, $N_{ht}(\eta_t)$, $N_{lt}(\eta_t)$; and value of unemployment, U_{ht} , U_{lt} ,
- wages, $w_{ht}(\eta_t)$, $w_{lt}(\eta_t)$,
- intermediate input cost thresholds, $\bar{\eta}_{ht}$, $\bar{\eta}_{lt}$,
- output, Y_t , consumption, C_t , number of hours of work supplied, L_t , and the real interest rate, r_t ,

which satisfy the following conditions:

- flows in the labor market, (13) to (17),
- value functions in the problem of the firm and the worker, (3) to (8),
- nash bargaining over the surplus to determine wages, (11) and (12),
- zero surplus conditions,
- optimal equations in the problem of the household, Euler equation, (19), optimal labor supply, (20), economy wide resource constraint, (21) and the aggregate production, (22).

3 Calibration

In this section we explain the parametrization of the model.

The parameters are chosen to match the empirical evidence on the long run values of the variables in the model, which would correspond to the steady state of the model. The benchmark calibration is done for the Spanish economy, since the model is more relevant for economies like that one, where the unemployment rate is high and so is the proportion of long-term unemployment. However, a short description of the parameters used for the simulations for the US economy can be found in Table A1 in the Appendix.

The length of a period is one quarter. The discount factor of the economy is $\beta = 0.98$, which implies a quarterly real interest rate of 2 percent.

The production function is assumed to be constant returns to scale, so $v = 1$. It seems reasonable to think that increases in the amount of hours worked will produce proportional increases in output. The results of the simulations are robust to changes in this parameter. Setting it equal to 66 percent, as in a standard production function with capital, does not change the main results. The steady state labor supply is assumed to be $L = 1/3$, which implies that on average 8 hours per day are devoted to work. The last parameter in the production function is the level of technology, A , which is calibrated using the optimal labor supply (equation (20)) and the steady state value of consumption implied by the equations in the model.

Home production is assumed to be 20 percent of the firm's production, and its steady state value is $b = 0.0045$. This parameter alters the volatility of unemployment, but not its persistence.

Following Mortensen & Pissarides (1994), the bargaining power of the worker is set to $\beta_w = 0.5$.

The parameters in the utility function are $\gamma_c = 0.5$ and $\gamma_n = 2$, which imply decreasing marginal utility of consumption and quadratic disutility from labor supply.

The matching function is assumed to be constant returns to scales $m(u_t, v_t) = \mu u_t^\alpha v_t^{1-\alpha}$. Following Mortensen & Pissarides (1994) $\alpha = 0.5$. μ is jointly calibrated with other parameters and steady state values of variables of the model in the following manner. The probability of leaving unemployment in Spain is set to 0.3, following Bover, Arellano & Bentolila (2002), and the probability of a vacancy being filled is assumed to be 0.4 to match the market tightness estimated in Cooley & Quadrini (1999). Looking at quarterly data from 1980 to 2002,⁴ the unemployment rate is set to 15 percent and the proportion of low skilled unemployed to 55 percent of total unemployment. Using these two probabilities and the steady state flow equations from the model we can estimate the steady state values loss of skill, $\lambda = 0.25$, destruction rate, $1 - F(\bar{\eta}_h) = 0.55$, market tightness, $\theta = 0.85$, and the scaling parameter in the matching function, $\mu = 0.51$.

Bover, Arellano & Bentolila (2002) estimate the probability of exiting unemployment as a function of unemployment duration. They estimate that for the Spanish economy this probability is reduced in half for workers who have been unemployed for a whole year. This estimate is consistent with the findings of Jackman & Layard (1991) for the British economy. Therefore we will assume that, since the matching probability is the same for both types of workers and the difference in the transition from unemployment to employment for low skilled workers is marked by the cost of training, this cost will be set so that the probability of a successful match for a low skilled worker is half of that for a high skilled.

The technology shock follows a first order autoregressive process, $z_t = \rho_z z_{t-1} + \epsilon_t^z$ where $\epsilon_t^z \sim N(0, \sigma_z^2)$. $\rho_z = 0.95$, as reported by Prescott (1986) and σ_z^2 is calibrated to match the standard deviation of unemployment, 0.065 for the Spanish economy.

For simplicity, the idiosyncratic shock to the firm is assumed to be distributed as an exponential $\eta \sim \frac{1}{\varphi} e^{-\frac{\eta}{\varphi}}$, where φ is jointly estimated along with all remaining steady state variables of the economy through the steady state equilibrium of the model.

⁴OECD Internationaly Statistics Yearbook.

Table 1: Benchmark Parameters of the Model

Exogenous parameters				
$\beta = 0.98$	$\gamma_c = 0.5$	$\gamma_n = 2$		
$v = 1$	$\alpha = 0.5$	$\beta_w = 0.5$		$\rho_z = 0.95$
Endogenous parameters				
$\lambda = 0.25$	$t = 0.04$	$k = 0.0028$		$b = 0.0045$
$A = 0.046$	$\mu = 0.51$	$\varphi = 0.01$		$\sigma_z = 0.01$

4 Results

This section presents the main results of the paper. Now that the model has been explained, it is possible to understand how this model helps reconcile theory and empirical evidence. In the introduction to the paper we noted the high persistence of unemployment over the business cycle and its different behavior in the US and Europe, specially with the extreme case of Spain. Here we present more concrete empirical evidence on these facts and we show how this model can explain them through the introduction of the loss of skill of workers and the subsequent costly training.

4.1 One vs. Two Skills

Figure 1 shows the response, in deviations from the steady state, of the main variables of the model to a one time negative technology shock. We can see in these graphs the effects of the loss of skill of workers on unemployment. We observe that on impact, high skilled unemployment increases, reaches its highest level in the second period and starts returning to the steady state after that. However low skilled unemployment increases slower and only peaks after eight periods, starting then a slow decline towards the steady state.

Thinking in terms of the model, a negative technology shock has two effects on unemployment. The first effect is the immediate destruction of jobs. This happens because of the cut in profits and the unprofitability of jobs with too high an intermediate cost. Workers who lose their job become high skilled unemployed, since the loss of skill only comes with time. The second effect, which is also a consequence of this decrease in profits, is the non-consummation of some matches which took place in the previous period. This affects both high and low skilled unemployment, raising both rates from their steady state levels. The increase in high skilled unemployment is dominated by the destruction of jobs effect and that is the reason of its fast decline to the new steady state. However, the most important mechanism driving up low skilled unemployment is the flow from high to low skilled unemployment due to the loss of skill. Since high skilled unemployment increases substantially due to the negative shock, the market becomes tighter making more difficult for everybody to find jobs. This increases the duration of unemployment and raises the chances of losing the skill. The slow flow from one pool of unemployment to the other and the fact that low skilled unemployed workers need to be retrained before starting to work, which reduces their chances

Figure 1: Impulse Responses to a Negative Technology Shock - Benchmark Calibration.

of having a successful match, steadily increases the low skilled population out of work. When high skilled unemployment decreases enough, the trend in low skilled unemployment is reverted and its level starts decreasing.

Total unemployment shows a slow transition to the steady state. This variable is the sum of the two types of unemployed workers and its sluggish return back to the steady state is explained by the combination of several factors. First, the increase in high skilled unemployment after the shock, which increases the labor market tightness and reduces the probability of matching with a firm. Second, the increase in the probability of becoming low skilled unemployed. And finally the problem that once being low skilled it is much harder to find a job since the worker needs to be retrained.

Employment decreases after the shock, mirroring the increase in unemployment since there are no inactive workers in the economy.

Hours of work decrease on impact and slowly return to the long run levels. The reduction in the technology level of the economy will decrease the marginal product of labor and given the disutility suffered by the worker from supplying labor, it will make leisure more attractive and reduce the number of hours of work.

The decrease in technology, employment and hours reduce output and consequently consumption. Slowly all these variables go back to the steady state as the direct effects of the shock disappear and so do their indirect consequences through the sluggish movements in the labor market.

After the initial analysis of each the variables of the model, we move to study the importance of the loss of skill mechanism in the response of unemployment to shocks to the economy. Figure 2 shows the response to a negative technology shock of two different versions of the model. The first one is a model in which it is costly to retrain the workers who have lost the skill and therefore, the probability of having successful matches for this pool of workers is lower than for high skilled unemployed. The second is a model with no cost of training, which implies a framework in the style of Mertz (1995) and Andolfatto (1996), which, as was noted in the introduction, are unable to successfully replicate the persistence of unemployment and output over the cycle. We can see in the graph how the loss of skill of workers and their lower transition back to work makes unemployment much more persistent than in the model without this mechanism. Unemployment picks after 4 quarters in the model without cost of retraining, whereas it takes 8 quarters in the one with cost. Also note that the transition back to the steady state is slower in the model with cost than in the one without it, implying, as is shown in Table 2, a higher persistence in unemployment.

Figure 2: Impulse Responses to a Negative Technology Shock in a Model with and without Retraining Cost - Benchmark Calibration.

4.2 Cross Country Analysis

The previous section showed how the model proposed in this paper improves the performance in the response to a shock compared to a model which does not take into account the cost of training the workers who have lost their skills. This section tries to analyze the ability of the model to explain cross country differences in the persistence of the business cycle, specifically in the persistence of unemployment and output.⁵ The benchmark simulations were done by calibrating the model to an extreme case, the Spanish economy, which is characterized by a high level of unemployment and an elevated proportion of long term unemployment. Now the other extreme is taken and the model is put into test by analyzing its ability to explain the persistence of the business cycle for the US economy, where unemployment is low and so is the fraction of long term unemployment.⁶

The case for the Spanish economy is shown in Table 2. We can see how the model which takes into account the loss of skill of workers is able to create much more persistence on unemployment and some more persistence on output. The Spanish economy shows a serial correlation decreasing from 0.95 for the first order to 0.44 for the fifth order autocorrelation. The model presented in this paper is closer to these facts than the model with no retraining cost, from autocorrelations decreasing from 0.92 to 0.14 for first to fifth order. Given that the model is closer to the data in the case of the US economy, whose results are shown in Table 3, it could be considered that the loss of skill problem is one among several affecting the Spanish economy. Some of the characteristics of this economy not taken into account in this model could be related to institutions, the cost of hiring and firing, minimum wage regulations or unemployment benefit systems. The performance of the model would improve if these were included, but they were not explicitly modeled for simplicity and to be able to isolate the effect of the loss of skill on the persistence of unemployment.

⁵The serial correlations reported in Tables 2 and 3 for the model of this paper are obtained by simulating the economy 100 times for 300 periods each, where every period the economy receives a shock drawn from the distribution $N(0, \sigma_z^2)$. Serial correlations of unemployment and output are computed after each simulation and averaged out over the 100 simulations.

⁶The new calibration is explained in the appendix.

Table 2: Unemployment and Output Serial Correlations

	$\rho(u_t, u_{t-1})$	$\rho(u_t, u_{t-2})$	$\rho(u_t, u_{t-3})$	$\rho(u_t, u_{t-4})$	$\rho(u_t, u_{t-5})$
Spanish Data ⁷	0.95	0.86	0.73	0.59	0.44
Model with Retraining Cost	0.92	0.74	0.53	0.33	0.14
Model with no Retraining Cost	0.90	0.68	0.44	0.21	0.03

	$\rho(y_t, y_{t-1})$	$\rho(y_t, y_{t-2})$	$\rho(y_t, y_{t-3})$	$\rho(y_t, y_{t-4})$	$\rho(y_t, y_{t-5})$
Spanish Data	0.81	0.69	0.55	0.41	0.37
Model with Retraining Cost	0.77	0.53	0.32	0.15	0.01
Model with no Retraining Cost	0.76	0.51	0.30	0.12	-0.01

Finally, Table 3 shows the results of the simulations for the US economy and compares them with the data and the results of the two seminal works in the literature which embed a search and matching labor market into an RBC model, but which do not consider the loss of skill mechanism. The first thing to note is that the model is able to reproduce closely the persistence in unemployment observed in the data. It also performs better than Mertz (1995) and Andolfatto (1996), given that the latter needs to assume a steady state unemployment rate of 40 percent and my calibration is done to target 6 percent unemployment rate.

Table 3: Unemployment Serial Correlations for the US Economy.

	$\rho(u_t, u_{t-1})$	$\rho(u_t, u_{t-2})$	$\rho(u_t, u_{t-3})$	$\rho(u_t, u_{t-4})$
US Data	0.89	0.69	0.46	0.23
Model with Retraining Cost	0.87	0.63	0.38	0.17
Mertz (1995)	0.68	0.42	0.23	N/A
Andolfatto (1996)	0.87	0.65	0.46	0.23

These results show the ability of the model not only to create higher persistence in unemployment and output than a model which does not consider the loss of skill, but it is also able to reproduce the differences in business cycle persistence in economies as different as Spain and the US.

⁷The data for both the Spanish and US economies were obtained in the online version OECD International Statistics Yearbook. The unemployment rate is the quarterly Standardized Unemployment Rate and output is the Real Output at 1995 dollars. All the series were taken logs and detrended using a HP filter with parameter 1600.

5 Conclusions

This paper introduces the loss of skill as a mechanism which increases the ability of business cycle models to explain the persistence of unemployment and other macroeconomic variables.

The model presented here combines a RBC model with a search and matching labor market, which has the additional characteristic that workers can lose their skill if they are unemployed for an extended period of time. These workers need to be retrained in order to be productive and if this training is costly, it will have an effect on their chances of finding work. When a negative shock hits the economy, raising unemployment and decreasing the overall probability of becoming employed, the loss of skill of workers who are unemployed for a long period of time further decreases the average possibilities of returning to work and raises the overall duration of unemployment. This mechanism will make unemployment more persistent than in a model which ignores it.

The simulations of the paper show quantitatively the effects on the persistence of the main macroeconomic variables of the model in two ways. First, unemployment persistence, measured through its serial correlations, is increased by the introduction of the loss of skill mechanism. This is proven by comparing the model with costly loss of skill against one with costless deterioration of skills. The autocorrelations of both unemployment and output increase substantially, getting closer to the data, when the loss of skill has an effect on the probability of finding work.

Second, the model is able to replicate the different cyclical persistence of unemployment across countries. This exercise is done by taking the extreme cases of the US and Spain. The results show that the model can generate the persistence observed in these diverse countries, and implies that the loss of skill of workers is a problem which has a greater effect on economies with high levels of unemployment and high proportions of long-term unemployment.

6 References

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Figure 3: Impulse Responses to a Negative Technology Shock - Calibration for the US Economy

Figure 4: Impulse Responses to a Negative Technology Shock - Calibration for the US Economy

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7 Appendix

Parametrization of the Model for the US Economy.

The calibration for the US economy is done following the same procedure as in the benchmark calibration, but targeting the empirical evidence of this country. Hence we will parametrize the model in order to obtain the following steady state values. 6 percent unemployment rate and 10 percent long term unemployment. The probability of an unemployed worker to leave unemployment is 0.6, as reported by Cole and Rogerson (1996), and the probability of a firm to fill a vacancy is 0.7, as stated in Den Haan, Ramey & Watson (1997). The calibrated parameters are shown in Table A1.

Table A1: Parameters of the Model for the US Economy.

Exogenous parameters				
$\beta = 0.98$	$\gamma_c = 0.5$	$\gamma_n = 2$		
$v = 1$	$\alpha = 0.5$	$\beta_w = 0.5$	$\rho_z = 0.95$	
Endogenous parameters				
$\lambda = 0.03$	$t = 0.05$	$k = 0.0028$	$b = 0.0051$	
$A = 0.053$	$\mu = 0.71$	$\varphi = 0.011$	$\sigma_z = 0.017$	

Derivations

Surplus

The surplus of the match is defined as $S_{it}(\eta_t) = J_{it}(\eta_t) + N_{it}(\eta_t) - U_{it}$. Using Equations (3), (7) and (5) we can find the expression for the value of a surplus with a high skilled worker (3):

$$\begin{aligned}
 S_{ht}(\eta_t) &= A_t L_t^v - \eta_t - w_{ht}(\eta_t) L_t + \beta E_t F(\bar{\eta}_{ht+1}) (1 - \beta_w) \tilde{S}_{ht+1} + \\
 &\quad + w_{ht}(\eta_t) L_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} + \beta E_t \left[F(\bar{\eta}_{ht+1}) \beta_w \tilde{S}_{ht+1} + U_{ht+1} \right] - \\
 &\quad - b_{ht} - \beta E_t \left[p_t F(\bar{\eta}_{ht+1}) \beta_w \tilde{S}_{ht+1} + \lambda (U_{ht+1} - U_{ht+1}) + U_{ht+1} \right]. \\
 S_{ht}(\eta_t) &= A_t L_t^v - \eta_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{ht} + \\
 &\quad + \beta E_t F(\bar{\eta}_{ht+1}) (1 - p_t \beta_w) \tilde{S}_{ht+1} - \beta (1 - p_t F(\bar{\eta}_{ht+1})) \lambda (U_{ht+1} - U_{ht+1}).
 \end{aligned}$$

where $\tilde{S}_{it+1} = \frac{1}{F(\bar{\eta}_{it+1})} \int_{\eta_{\min}}^{\bar{\eta}_{it+1}} S_{it+1}(\eta_{t+1}) dF(\eta_{t+1})$.

We can obtain the surplus of the match with a low skilled worker (10) by using equations (4), (8) and (6):

$$\begin{aligned}
S_{lt}(\eta_t) &= A_t L_t^v - \eta_t - t_t^f - w_{lt}(\eta_t) L_t + \beta E_t F(\bar{\eta}_{ht+1}) (1 - \beta_w) \tilde{S}_{ht+1} + \\
&\quad + w_{lt}(\eta_t) L_t - t_t^w - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} + \beta E_t \left[F(\bar{\eta}_{ht+1}) \beta_w \tilde{S}_{ht+1} + U_{ht+1} \right] - \\
&\quad - b_{lt} - \beta E_t \left[p_t F(\bar{\eta}_{lt+1}) \beta_w \tilde{S}_{lt+1} + U_{lt+1} \right]. \\
S_{it}(\eta_t) &= A_t L_t^v - \eta_t - t_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{it} + \\
&\quad + \beta E_t F(\bar{\eta}_{ht+1}) \tilde{S}_{ht+1} - \beta p_t F(\bar{\eta}_{lt+1}) \beta_w \tilde{S}_{lt+1} - \beta (U_{lt+1} - U_{ht+1}).
\end{aligned}$$

Wage determination

Wage is determined as a Nash bargaining process over the surplus where β_w is the bargaining power of the worker. The result of the bargaining will yield a constant fraction of the surplus for each party, $J_{it}(\eta_t) = (1 - \beta_w) S_{it}(\eta_t)$ and $(N_{it}(\eta_t) - U_{it}(\eta_t)) = \beta_w S_{it}(\eta_t)$. Using the first of the two expressions together with equations (3) and (9), we obtain the wage for the high skilled worker, (11):

$$\begin{aligned}
w_{ht}(\eta_t) L_t &= \beta_w \left[A_t L_t^v - \eta_t + \beta (1 - \beta_w) p_t F(\bar{\eta}_{lt+1}) \tilde{S}_{ht+1} \right] + \\
&\quad + (1 - \beta_w) \left[\frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} + b_{ht} + \beta (1 - p_t F(\bar{\eta}_{ht+1})) \lambda E_t (U_{lt+1} - U_{ht+1}) \right]
\end{aligned}$$

Similarly, the low skilled wage (11) is obtained by using equations (4) and (10) in combination with $J_{ht}(\eta_t) = (1 - \beta_w) S_{ht}(\eta_t)$

$$\begin{aligned}
w_{lt}(\eta_t) L_t &= \beta_w \left[A_t L_t^v - \eta_t + \beta (1 - \beta_w) p_t F(\bar{\eta}_{lt+1}) \tilde{S}_{lt+1} \right] + \\
&\quad + (1 - \beta_w) \left[\frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} + b_{lt} + \beta E_t (U_{lt+1} - U_{ht+1}) \right]
\end{aligned}$$

Reservation Cost

The reservation cost is the cost level above which the match will be destroyed, and hence will be the one which makes the surplus of the match equal to zero. $S_{it}(\bar{\eta}_t) = J_{it}(\bar{\eta}_{it}) + N_{it}(\bar{\eta}_{it}) - U_{ht} = 0$. Evaluating the surpluses for high and low skilled matches, (9) and (10) at $\bar{\eta}_{ht}$ and $\bar{\eta}_{lt}$, we obtain the threshold costs:

$$\begin{aligned}
S_{ht}(\eta_t) &= A_t L_t^v - \eta_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{ht} + \\
&\quad + \beta E_t F(\bar{\eta}_{ht+1}) (1 - p_t \beta_w) \tilde{S}_{ht+1} - \beta (1 - p_t F(\bar{\eta}_{ht+1})) \lambda E_t (U_{lt+1} - U_{ht+1})
\end{aligned}$$

evaluating the above expression at $\bar{\eta}_{ht}$

$$\begin{aligned}
0 &= A_t L_t^v - \bar{\eta}_{ht} - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{ht} + \\
&\quad + \beta E_t F(\bar{\eta}_{ht+1}) (1 - p_t \beta_w) \tilde{S}_{ht+1} - \beta \lambda E_t (U_{lt+1} - U_{ht+1})
\end{aligned}$$

rearranging

$$\begin{aligned}\bar{\eta}_{ht} = & A_t L_t^v - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{ht} + \\ & + \beta E_t F(\bar{\eta}_{ht+1}) (1 - p_t \beta_w) \tilde{S}_{ht+1} - \beta (1 - p_t F(\bar{\eta}_{ht+1})) \lambda E_t (U_{lt+1} - U_{ht+1})\end{aligned}$$

For the low skilled:

$$\begin{aligned}S_{lt}(\eta_t) = & A_t L_t^v - \eta_t - t_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{lt} + \beta E_t F(\bar{\eta}_{ht+1}) \tilde{S}_{ht+1} - \\ & - \beta p_t E_t F(\bar{\eta}_{lt+1}) \beta_w \tilde{S}_{lt+1} - \beta E_t (U_{lt+1} - U_{ht+1})\end{aligned}$$

evaluating the above expression at $\bar{\eta}_{lt}$

$$\begin{aligned}0 = & A_t L_t^v - \bar{\eta}_{lt} - t_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{lt} + \beta E_t F(\bar{\eta}_{ht+1}) \tilde{S}_{ht+1} - \\ & - \beta p_t E_t F(\bar{\eta}_{lt+1}) \beta_w \tilde{S}_{lt+1} - \beta E_t (U_{lt+1} - U_{ht+1})\end{aligned}$$

rearranging

$$\begin{aligned}\bar{\eta}_{lt} = & A_t L_t^v - t_t - \frac{a_n}{1 + \gamma_n} L_t^{1+\gamma_n} \frac{1}{u'(C_t)} - b_{lt} + \beta E_t F(\bar{\eta}_{ht+1}) \tilde{S}_{ht+1} - \\ & - \beta p_t E_t F(\bar{\eta}_{lt+1}) \beta_w \tilde{S}_{lt+1} - \beta E_t (U_{lt+1} - U_{ht+1})\end{aligned}$$