Expectation Heterogeneity and Excessive Price Sensitivity in the Land Market

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June 1998, This version: September 1998

Keywords: Land Market, Expectation Heterogeneity, Price Sensitivity, Long-run and short-run rational expectations

JEL Classification Number: D40, D43, D82, D84, E44, G12

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*Prepared for the 1998 Japanese Economic Association Nakahara Prize Lecture, September 13, 1998. I am indebted to the anonymous referee and seminar participants at the Universities of Paris I, Uppsala, Copenhagen, Tokyo, Keio, and Clunb for their comments and suggestions.
ABSTRACT

This paper analyzes price behavior in the land market in which the Walrasian auctioneer is absent. Sellers are informed about the land value and post their asking prices. Buyers are uninformed, have different opinions, and decide whether to accept the sellers’ offers or not. In this market, sellers are “price-makers” having probabilistic market power: they can influence the sale probability through their asking price. Price may be excessively sensitive to unexpected changes in market conditions. Moreover, buyer heterogeneity may accentuate the excessive price sensitivity. It has been shown that the more heterogeneous their opinions, the more sensitive are prices.
1. Introduction

The last decade has witnessed turbulent price behavior in the Japanese land market. Figure 1 shows the rate of change in real residential land prices of the six largest cities between 1956 and 1995 juxtaposed onto with the real GDP growth minus the change in the real interest rate, which is taken as a proxy of the “market fundamentals” that influence land prices.¹

This figure reveals the extraordinary behavior of land prices between 1985 and 1995. A modest increase in the market fundamentals around 1984 triggered a dramatic upsurge of land prices in Tokyo in 1985 and 1986 that spread to other large cities. The average annual rate of change reached almost thirty percent in 1989. However, after 1989, this peak and the small dip that followed in the movement of the market fundamentals was accompanied by a precipitous decline in land prices.²

The Japanese example is not at all unique. In the recent past, the United Kingdom, Norway, Sweden, Australia and other countries, including the State of Hawaii in the United States, experienced a real-estate boom that was followed by a sharp downfall in the 1980s. Moreover, the 1997 turmoil of several East Asian economies such as

¹The real interest rate here is the loan rate of banks, minus the rate of change in the consumer price index. Here the change in the real interest rate is used instead of the rate of change, in order to avoid the issue of negative interest rate around 1974.

²See Nishimura (1995) for more detailed account of the history of Japanese land prices after the second World War. Ito (1992, Chapter 14) provides us with a concise summary of the Japanese literature on this subject in English.
Thailand and Malaysia was added to this long list of real estate “boom and bust”
victims.

The conventional interpretation of such turbulent price behavior is based on so-
called rational bubbles.\textsuperscript{3} The rational-bubble interpretation assumes that the land
market can be characterized as a Walrasian asset market in which (a) buyers and
sellers are price-takers, (b) price equates demand and supply continuously, and (c)
buyers and sellers have the same rational expectations. In this market, there must
be no unexploited arbitrage opportunity in equilibrium. This equilibrium condition
determines the relationship between the current price and the future price as well as
the rent and the interest rate, which characterizes the evolution of the price. Based on
this characterization, the rational-bubble interpretation argues that the price of land
deviates from its fundamental value and keeps going up since investors assume that
other investors assume that the price is going up. Their expectations are self-fulfilling
up to some point and then collapse, leading to a sharp decline in price.

There are, however, serious problems in this conventional rational-bubble inter-
pretation. The argument is based on the no-arbitrage-opportunity condition, which
implies market efficiency. However, the land market is not always efficient.\textsuperscript{4} In Figure
2, the semiannual excess rate of return on a typical “blue-chip” commercial property
is depicted between 1963 and 1996. The excess rate of return must not be serially

\textsuperscript{3}However, some economists object to this characterization of “bubbles”. See Garber (1989).

\textsuperscript{4}For other countries than Japan, see, for example, Case and Shiller (1989). Problems concerning
tests of rational bubbles are extensively discussed in Flood and Hodrick (1990).
correlated under the hypothesis of the weak form of efficient markets. However, this figure reveals a strong positive serial correlation after 1985. Moreover, the rational bubble argument provides us with no explanation of why it started in the 1980s, not in the 1970s. It fails to supply an account of what caused its subsequent collapse. Also, it does not explain the turbulent prices in the 1950s.

This paper proposes an alternative theory of turbulent price behavior in the land market based on the following observation: in the land market, there is no Walrasian auctioneer posting price. Instead, the seller posts his asking price in newspapers, magazines and brokers’ networks to search for a buyer. In some cases, buyers also post their bid prices. It is commonly observed that if the seller’s asking price is low, it is relatively easy to find a buyer, but if that asking price is high, it takes considerable time and effort. The price-posting seller chooses his asking price by taking account of this probability relationship and his own urgency to sell.

The land market differs from a Walrasian market of price-taking investors with homogeneous expectations in two respects. Firstly, sellers are not price-takers but price-makers. By changing the asking price, the seller can set the probability of a successful sale. Secondly, expectations about the land’s intrinsic value differ among buyers. In fact, if all buyers had the same expectations, the probability of a successful sale would be unity if the asking price were equal to or lower than the commonly expected value, and zero otherwise. Thus, the observation that the sale probability depends on the asking price implies expectation heterogeneity.
The hallmark of this market is *probabilistic market power*: sellers can influence their sale probability and thus their expected profits by changing their asking price. In this sense, the land market is more similar to a monopolistically competitive market of differentiated products than a well-organized financial market in which the latter can be characterized as a Walrasian market.\(^5\)

The main purpose of this paper is to show prices in a land market may be excessively sensitive to unexpected changes in market conditions. Moreover, it is shown that price sensitivity may depend on the expectation heterogeneity of buyers: the less concentrated buyers’ expectations are, the more sensitive are prices. The theory developed in this paper provides a clue in explaining the timing and magnitude of the turbulent price behavior both in the 1950s and 1980s. The upsurge of land prices was triggered by the influx of uninformed buyers to metropolitan areas due to population movement from rural areas in 1950s, and business concentration toward Tokyo under financial liberalization and globalization in the 1980s.

The paper is organized as follows: Section 2 presents a model of the land market without a Walrasian auctioneer in which buyers are uninformed and have heterogeneous opinions. In this section, the effect of buyer heterogeneity on the market price is delineated. In Section 3, learning from price is incorporated into the model, and it

\(^5\)In finance, there is a sizable literature investigating the effect of the micro-structure of trading on asset prices and trading volumes (see, for example, Laffont and Maskin (1990). O'Hara (1995) surveys this literature). Recently, Madrial and Scheinkman (1997) go beyond the usual Walrasian framework and analyze the effect of imperfect competition among market makers in the New York Stock Exchange. The present paper has the same motivation as Madrial and Scheinkman in recognizing that analysis of the non-Walrasian market structure is essential in understanding observed asset price behavior.
is demonstrated that the main result of Section 2 still holds true even under sophisticated information processing by buyers. Section 4 comments on the limitations of the present analysis and its possible extensions.

2. A Model of the Land Market

To avoid a confounding issue of intertemporal speculation and to concentrate attention on static price formation, a “temporary equilibrium” approach is adopted in which the next period’s market conditions are treated as exogenous. In particular, the next period’s land value is treated as a given parameter, although in a full dynamic equilibrium it is endogenously determined.

Consider a market of land: there are $N$ risk-neutral sellers, each owning one piece of land, while there are $M$ risk-neutral buyers, each considering purchase of one piece of land. The market is not centralized, and sellers and buyers have to search for their trading partner. Search costs are assumed to be substantial, and there is at most one random encounter per market participant. For simplicity, it is assumed that $M > N$ so that the seller expects to meet at most one buyer.\(^6\)

The land is heterogeneous. Let $x_i + X_i$ be the intrinsic value of the $i$th seller’s land, which is the present discounted value of the future benefits from owing the land. Here $X_i$ is the commonly-known component and $x_i$ is the unexpected component. The

\(^6\)The assumptions of one visit per buyer and that $N < M$ are not essential for the qualitative result of this paper. For example, one may assume that the number of buyers as a random variable following a Poisson distribution like Wilde (1977). Qualitative results do not change by such complication.
unexpected component $x_i$ consists of two parts:

$$x_i = y + w_i,$$  \hspace{1cm} (2.1)

where $y$ is the unexpected part of the average intrinsic value and $w_i$ is the idiosyncratic part.

To make the analysis tractable, it is assumed that (a) sellers have perfect information about $x_i$ while buyers’ information is imperfect, and (b) informed sellers post their asking prices, while buyers who are uninformed determine whether or not to accept the offer.\(^7\) Qualitative results do not depend on this asymmetric assumption of information and price posting. The essential components in the following analysis are that (a) there are both uninformed and informed investors and (b) informed investors quote a price, and uninformed investors determine whether or not to accept the market price.

\subsection*{2.1. Uninformed buyers with heterogeneous opinion}

Let us now consider buyers: all buyers are assumed to have access to the same information. However, the information is imperfect, and \textit{buyers differ in the interpretation of}...
the same information. Buyers may have different models of the future economy, and/or they may differ from one another in their ability to process the information. Thus, buyers may have a different opinion even though they have the same information.\textsuperscript{8} Moreover, both buyers and sellers know that this difference exists among the buyers’ opinions. That is, the distribution of opinion among buyers is common knowledge.\textsuperscript{9}

The $j$th buyer’s opinion is summarized in the following way: he assumes $y \sim N(y_j^*, \sigma_y^2)$ and $w_i \sim N(0, \sigma_w^2)$, where $N(\mu, \sigma^2)$ denotes a Normal distribution with mean $\mu$ and variance $\sigma^2$. Thus, buyers have a different opinion about the mean of $y$. The distribution function of $y_j^*$ among buyers is denoted by $F(y)$, that is:

$$F(y) = \Pr \left( y_j^* < y \right).$$

When a buyer meets a seller, the buyer receives a price quotation $p_i + X_i$ from the seller and decides whether he buys from the seller or not. Here, $p_i$ is the price part in excess of the commonly known part $X_i$. The buyer is assumed to be risk neutral. Thus, if his estimate of the intrinsic value $x_i + X_i$, $E^j (x_i + X_i)$ exceeds the price $p_i + X_i$, the buyer buys. $E^j (x_i + X_i)$ can be called the buyer’s reservation price.

If he is sophisticated, or in other words, “rational”, he will use this price quotation to update his opinion to get the estimate of $x_i$ since he knows the seller has perfect information about $x_i$. Then, he will make the purchase decision relying on this updated

\textsuperscript{8}Varian’s (1989) definition is used in distinguishing opinion from information.

\textsuperscript{9}MacDonald and Marsh (1996) show that there is, in fact, substantial difference in opinion in the context of foreign exchange markets.
opinion. However, in order to concentrate attention on buyer heterogeneity per se, this learning process from price is ignored in this section. Full investigation of the effect of rational expectation formation is postponed until the next section.

With the “unsophisticated buyer” assumption, the buyer’s reservation price, that is, his estimate of the intrinsic value, is $E^j (x_i + X_i) = E^j (y + u_i + X_i) = y_j^* + X_i$. Consequently, the $j$th buyer having opinion $y_j^*$ buys the land if $p_i + X_i \leq E^j (x_i + X_i)$, or equivalently, $p_i \leq y_j^*$.

2.2. Informed sellers determining asking price

Under these assumptions, the seller knows the true value of his land, the buyer behavior, and the opinion distribution among buyers. However, the seller does not know the particular opinion of the buyer that the seller meets. By taking account of buyer behavior and the distribution of opinions, the seller can calculate the probability of a successful sale when his asking price is $p_i$.

Let $\phi(p_i)$ be the probability of a successful sale. Since meeting buyers is random, the probability of one buyer visiting the seller is $M/N$. This particular buyer buys the land if $p_i \leq y_j^*$. Consequently, the probability of a successful sale, $\phi(p_i)$, is

$$\phi(p_i) = \frac{M}{N} \left[ 1 - \Pr\left( y_j^* < p_i \right) \right] = \frac{M}{N} \left[ 1 - F(p_i) \right].$$

It is assumed that the seller knows $x_i$. Taking account of the effect of his price on the sale probability, the seller determines his optimal price that maximizes his
expected profit:

\[
\begin{align*}
\max_{p_i} \quad & \text{Expected Profit}_i = \phi(p_i)(p_i + X_i) + [1 - \phi(p_i)](x_i + X_i) \\
& = \phi(p_i)(p_i - x_i) + (x_i + X_i).
\end{align*}
\] (2.2)

Since it is always possible not to sell the land, the optimal price must not be lower than the intrinsic value. Thus, the optimal price must satisfy \( p_i \geq x_i \).

2.3. Market equilibrium

The first order condition of the seller’s expected profit maximization is transformed into the following relationship:

\[
p_i \left(1 + \frac{\phi(p_i)}{\phi'(p_i)p_i} \right) = x_i.
\] (2.3)

This equation determines the market price of the land.

There is an similarity between this pricing formula and the familiar monopoly pricing formula. In fact, if \( \phi(p_i) \) were a demand function, this would be exactly the optimal price formula for a monopolist who has constant marginal cost \( x_i \).

Thus, the price-posting seller in the land market has “market power” like a monopolist in the product market although the nature of the market power is different. In the product market, a monopolist can influence the demand for his product by changing
his price. In the land market, a price-posting seller can influence the probability of a successful sale by changing his price. Thus, both a monopolist and a price-posting land-seller determine their (expected) profit through their price. Since the ability to influence profit through price is the essence of market power, their market power is qualitatively the same. In this sense, the price-posting seller in the land market can be described as having probabilistic market power.

In general, it is difficult to get a tractable expression for $\phi(p_i)/\phi'(p_i)$. However, we can obtain a neat formula for one particular distribution which can be called the Power-Cusp distribution since it is cusp-shaped and based on a power function. It has been found that this formula can be utilized as a yardstick in analyzing other distributions. Thus, we first consider the Power-Cusp distribution and then analyze the case of the Normal distribution.

**Case 1: Power-Cusp Distribution.** Suppose that $y^*_i$ is distributed around $y^*_0$ like a cusp in the following way. The distribution function is

$$F(y) = \begin{cases} 
\frac{1}{2} \left(1 - (y - y^*_0)\right)^{-a} & \text{if } y \leq y^*_0, \\
1 - \frac{1}{2} \left(1 + (y - y^*_0)\right)^{-a} & \text{if } y^*_0 \leq y
\end{cases}, \quad (2.4)$$

and the corresponding density function is

$$f(y) = \begin{cases} 
\frac{1}{2} \left(1 - (y - y^*_0)\right)^{-a-1} a & \text{if } y \leq y^*_0, \\
\frac{1}{2} \left(1 + (y - y^*_0)\right)^{-a-1} a & \text{if } y^*_0 \leq y
\end{cases}. \quad (2.5)$$
Figure 3 depicts this distribution for a large \( a \) \( (a = 3) \) and a small \( a \) \( (a = 1.1) \) when \( y_0^* = 0 \). Since the density curve looks like a cusp and the distribution function is based on a power function, we named this family of distributions the Power-Cusp distribution. It is evident from this figure that large \( a \) implies a high concentration of \( y_j^* \) toward mean \( y_0^* = 0 \).

Throughout this paper, it is assumed that (a) \( 1 < a \) and (b) \( y_0^* < x_i + (1/a) \). The assumption (a) is necessary for equilibrium to exist in this land market. The second assumption (b) implies that the mean of the buyers’ opinion is not substantially higher than the true value \( x_i \). The latter assumption is not necessary for the following analysis, but it considerably reduces complexity.

In this Power-Cusp distribution, we have \( \phi(p_i) = \frac{M}{N} \left[ 1 - \frac{1}{2} \{ 1 - (p_i - y_0^*) \}^{-a} \right] \) for \( p_i \leq y_0^* \) and \( \phi(p_i) = \frac{M}{N} 2 \{ 1 + (p_i - y_0^*) \}^{-a} \) for \( y_0^* \leq p_i \). However, it is straightforward, though cumbersome, to show that the optimal price \( p_i \) is not smaller than \( y_0^* \) under the assumption (b) above (see Appendix A). Therefore, we obtain \( \phi(p_i)/\phi'(p_i) = -\frac{1}{a} (1 + (p_i - y_0^*)) \) at the optimal price. Consequently, the optimal price is\(^{10}\)

\[
p_i = \frac{1}{1 - (1/a)} x_i + \frac{1}{a - 1} (1 - y_0^*). \tag{2.6}
\]

In this case, the price sensitivity is \( \partial p_i/\partial x_i = \{ 1 - (1/a) \}^{-1} \).

\(^{10}\) Individual rationality requires \( p_i \geq x_i \), which implies \( y_0^* - 1 \leq x_i \). This condition is satisfied under the assumptions (a) and (b). Under the same assumptions, the second-order condition is also satisfied at the optimal price.
Case 2: Normal distribution. Let us now consider the case when the distribution of $y_j^*$ is Normal, that is, $y_j^* \sim N(y_0^*, \sigma_0^2)$. Since the Normal distribution function is analytically intractable, we use the following approximation of the Normal distribution function:

$$\int_{-\infty}^{y} \frac{1}{\sigma_0\sqrt{2\pi}} \exp \left(-\frac{(y - y_0^*)^2}{2\sigma_0^2}\right) \, du \approx 1 - \frac{1}{1 + \exp \left\{ \frac{16}{\sigma_0} (y - y_0^*) \right\}}. \quad (2.7)$$

The distribution function $F(p) = 1 - \left[ 1 + \exp \left\{ \frac{16}{\sigma_0} (p - y_0^*) \right\} \right]^{-1}$ belongs to the family of Gibbs distribution functions.\footnote{See Aoki (1996) for properties of this type of distribution functions and its application to economic models.} Figure 4 shows the density function of both distributions for $y_0^* = 0$ and $\sigma_0 = 1$. This figure shows that (2.7) is in fact a good approximation.

Under this approximation, we have

$$\frac{\phi(p_i)}{\phi'(p_i)} = -\frac{\sigma_0}{1.6} \left[ 1 + \exp \left\{ -\frac{1.6}{\sigma_0} (p_i - y_0^*) \right\} \right].$$

Consequently, the optimal price is determined by

$$p_i - \frac{\sigma_0}{1.6} \left[ 1 + \exp \left\{ -\frac{1.6}{\sigma_0} (p_i - y_0^*) \right\} \right] = x_i. \quad (2.8)$$

This implicitly determines the optimal price $p_i$ as a function of $x_i$, although it is not possible to get an explicit formula for $p_i$.\footnote{See Aoki (1996) for properties of this type of distribution functions and its application to economic models.}
2.4. Buyer heterogeneity and the excessive sensitivity of price

We now consider the effect of the heterogeneity of buyers’ opinions. If information is perfect, the price of land is equal to its intrinsic value \( X_i + x_i \). We have \( p_i = x_i \) in this case, so that \( dp_i/dx_i = 1 \).

Let us first consider the Power-Cusp opinion distribution. In this case we have from (2.6)

\[
\left[ \frac{dp_i}{dx_i} \right]_{\text{No Learning}} = \frac{1}{1 - (1/a)}.
\]  

(2.9)

Thus, price sensitivity depends on the parameter \( a \).

As is evident in Figure 3, the parameter \( a \) is a measure of opinion concentration. Therefore, the equilibrium price sensitivity (2.9) shows that increased heterogeneity (i.e., an decrease in \( a \)) increases the land-price sensitivity to the unexpected change in the intrinsic value \( x_i \). Moreover, the coefficient of \( x_i \) is always greater than unity. This means that if the buyers are heterogeneous in their opinion, land prices are more sensitive in the heterogeneous-opinion case than in the perfect-information one. The more dispersed the buyers’ opinion, the more sensitive land prices are. Thus, buyer heterogeneity leads to excessive sensitivity of land prices.

An interesting property of price sensitivity is found in (2.9). The effect of concentration is asymmetric between a small \( a \) and a large \( a \). If \( a \) is small (close to unity), a small change in \( a \) produces a large change in the sensitivity. By contrast, if \( a \) is large, the sensitivity is close to unity and a change in \( a \) does not significantly change the sensitivity.
Let us now consider the case of the Normal distribution. The Normal distribution is bell-shaped. Firstly, the density slope is gentle in the neighborhood of the mean. This part can be approximated by the case of a small $a$ in the Power-Cusp distribution. Secondly, the slope becomes steep as one moves further away from the mean. This part is similar to the case of a large $a$ in the Power-Cusp distribution. Thus, we can expect that the price sensitivity of the Normal opinion distribution case depends on whether a gentle slope or a steep one is relevant in price determination.

The optimal price formula of the Normal opinion distribution case is shown in Figures 5 and 6.\(^{12}\) As expected, it depends on the mean of the opinions. Figure 5 depicts the case in which buyers are, on average, optimistic ($y_0^* = 2$), while Figure 6 represents the case in which they are, on average, pessimistic ($y_0^* = -2$). Both figures show three cases of standard deviation of opinion ($\sigma = 0.16$: thin line, $\sigma_0 = 0.52$: medium line, $\sigma = 1.6$: thick line).

These figures show the interesting properties of the Normal opinion distribution. If buyers are on average optimistic, the price sensitivity of the Normal opinion distribution case is qualitatively the same as that of the small $a$ case in the Power-Cusp-opinion distribution; the higher the variance (i.e., the lower the concentration), the more sensitive the price.

As mean $y_0^*$ increases from zero, the opinion density curve moves to the right.

\(^{12}\)These figures are drawn in the following way: note that (2.8) defines $x_i$ as a function of $p_i : x_i = G(p_i)$. Then, the graph $x_i = G(p_i)$ is drawn on the $(x, p)$ plane and then flipped on the $(p, x)$ plane to get these figures.
This implies that for $x_i$ around zero, the optimal price (which is greater than $x_i$) is likely to be located in the range of the gentle density slope. This part can be approximated by the Power-Cusp distribution with a small $a$. Thus, price sensitivity increases substantially when opinion variance increases (that is, when opinion is less concentrated).

On the other hand, let us suppose that buyers are on average pessimistic. In this case, the opinion density curve is moved to the left. This shift implies that the optimal price for $x_i$ around zero is likely to be far away from the mean, that is, in the range of the steep density slope. This part can be approximated by the Power-Cusp distribution with a large $a$. Therefore, price sensitivity is close to unity, and an increase in opinion variance does not significantly change price sensitivity.

The result of this section can be intuitively explained in the following way: in a land market with heterogeneous buyer opinion, the price-posting seller can expect a positive probability of a successful sale even if he asks for a higher price than the land's intrinsic value. Moreover, when buyer heterogeneity increases, the seller can increase his expected profit by increasing his price for a given change in the intrinsic value; the less concentration in opinion, the more people are found in the both tails, implying that the more people are optimistic as well as the more people are pessimistic and the less people are around the mean. Although a price increase reduces the sale probability, the probability reduction is smaller if the buyers’ opinion is more dispersed. Consequently, the seller’s optimal price is higher when the heterogeneity of buyers is greater.
3. Learning from Price and Price Sensitivity

In the previous section, buyers were not sophisticated in the sense that they did not incorporate asking price information into their expectations. Then, it could be argued that the excessive sensitivity result is due to this particular assumption of non-rational expectations. Bayesian buyers learn the intrinsic value through the sellers’ asking price which considerably diminishes the ex-post heterogeneity so as to reduce the price sensitivity. In this section, we investigate the case where buyers use Bayesian updating and show that qualitatively the same result is obtained even if buyers are sophisticated information users.

Before proceeding with the formal analysis, an intuitive discussion may be helpful in understanding the effect of learning from price in the land market. Although buyers’ learning reduces their ex post heterogeneity of opinion, there is a new source of excessive sensitivity which is inherent in the case of sophisticated (Bayesian) buyers. Since the buyer tries to get information from the seller’s offer, the seller can influence the buyer’s perception of the market by changing his offer. In general, the buyer thinks that a high asking price is a sign of high intrinsic value and vice versa. Thus, the seller’s optimal price ceteris paribus increases further under these conditions than in the case of unsophisticated buyers; since the seller’s higher asking price may lead the sophisticated buyer to think the land’s value is higher, making room for further price raise. Thus, learning from price has two offsetting effects on price sensitivity, and the overall effect is ambiguous. In fact, these two effects exactly offset each other.
in the case of the Power-Cusp opinion distribution, which is considered in this section.

It should be noted that this type of information manipulation is not present in the Walrasian market, and is one of the distinctive characteristics of the market of price-posting sellers.

3.1. Bayesian Nash equilibrium with heterogeneous priors

Let us consider a Bayesian Nash equilibrium of the land market where (1) buyers have different prior distributions but these prior distributions are common knowledge to both buyers and sellers, (2) buyers update their expectations using sellers’ asking price and based on these updated expectations, buyers determine optimally whether to buy or not, and (3) sellers, knowing buyer behavior, decide on their asking price by maximizing their expected profit. This equilibrium definition is a natural extension of the commonly used Bayesian Nash equilibrium to the case of heterogeneous priors.

For analytic tractability, buyer-opinion distribution is assumed to be of the Power-Cusp type (2.4). The discussion in the previous section suggested that the Normal and other distributions can be approximated locally by a piece-wise Power-Cusp distribution.

3.2. Sellers with a heterogeneous urgency to sell

To make equilibrium non-trivial in the case of sophisticated buyers, we need heterogeneity on the side of the sellers. Let suppose for a moment that sellers are homogeneous. In this case, the seller’s optimal price might perfectly reveal the true intrinsic
value. To see this, let us consider the optimal asking price of the previous section (2.6). Since buyers know \( p_i \) and \( y_i^* \) as well as \( a \), they can perfectly infer an unexpected change in the true intrinsic value, \( x_i \).

To avoid this conceptual problem, we assume that sellers have a different urgency to sell. In fact, this characteristic is distinctive of the land market. Unlike financial markets where the majority of participants are institutional investors, the majority of buyers and sellers in the land market are ordinary people who do not have good access to capital markets. Some sellers, such as those who are selling their land to pay inheritance taxes, may incur large costs when they fail to sell. Such urgency differs among sellers.

In contrast, some land owners may get non-pecuniary idiosyncratic benefits from owing a particular piece of land. In order to convince them to sell their land, price should be higher than the intrinsic value by the amount of compensation for the idiosyncratic benefits.

Thus, we consider the following expected profit maximization problem of the seller in this section.

\[
\begin{align*}
\max_{p_i} \quad & \text{Expected Profit}_i \\
= & \phi(p_i) (p_i + X_i) + [1 - \phi(p_i)] (x_i + X_i - \delta_i) \\
= & \phi(p_i) \{p_i - x_i + \delta_i\} + (x_i + X_i - \delta_i)
\end{align*}
\]
where the urgency to sell is represented by $\delta_i$,\textsuperscript{13} which is the penalty that the $i$th seller incurs when the seller fails to sell the land. The urgency to sell is assumed to satisfy $\delta_i \sim N(\delta_0, \sigma^2_s)$. Here, a negative penalty is allowed, which corresponds to the case of non-pecuniary idiosyncratic benefits of land holding.

### 3.3. Sophisticated buyers with diverse opinions

Consider now the expectation formation process of buyers. Suppose that the buyer assumes that the structure of the market is such that

$$p_i = z + \omega (x_i - \delta_i) \quad (3.1)$$

where $z$ and $\omega$ are undetermined coefficients. In the Bayesian Nash (rational expectation) equilibrium, the buyer’s subjective value of $z$ and $\omega$ coincides with their actual value.

It should be noted that this short-run rational expectation assumption is consistent with opinion heterogeneity. The differences in opinion lie in the future benefits of owing a particular piece of land. Economic agents may have very different opinions about the future. The rational expectation assumption here implies that economic agents agree the short-run price behavior in the market. Advancement in information technology makes a vast amount of short-run information available to economic agents so that

\textsuperscript{13}Another way to incorporate the urgency to sell is to make sellers risk averse. Since qualitative results are similar, we retain this simple formulation.
this short-run rational expectation assumption may not be so unrealistic.

Here, long-run and short-run rational expectations should be properly distinguished from each other. The long-run rational expectation assumption, which is often made in macroeconomics, assumes that all economic agents know the true structure of the *future* economy as well as the current one. This assumption implies homogeneous expectations. By assuming opinion heterogeneity, we reject long-run rational expectations.

The *j*th buyer observing *p*\(_i\) updates his expectation of *x*\(_i\) using a familiar signal-extraction procedure (see Appendix B.1)

\[
E^j_i \mid (p_i) = (1 - \theta) y^*_j + \theta \omega^{-1} (p_i - z),
\]

where

\[
1 > \theta \equiv \frac{\sigma^2_x}{\sigma^2_x + \sigma^2_\theta} > 0
\]

in which \(\sigma^2_x = \sigma^2_y + \sigma^2_\omega\) (see (2.1)). \(E^j_i \mid (p_i)\) is the expectation of *x*\(_i\) conditional on \(p_i\), based on the *j*th buyer’s probabilistic view of the market.

The buyer buys if \(p_i \leq E^j_i \mid (p_i)\). Consider a marginal buyer having \(y^{**}\) who is indifferent between buying and not buying. Then we have

\[
(1 - \theta) y^{**} + \theta \omega^{-1} (p_i - z) = p_i,
\]

The buyer, whose \(y^*_j\) is greater than \(y^{**}\), buys the land. Consequently, the buyer
having \( y^*_j \) such that \( y^*_j \geq y^{**}(p_i) \) buys the \( i \)th land, where the critical value \( y^{**}(p_i) \) is

\[
y^{**}(p_i) = r p_i + t : r = \frac{1 - \theta \omega^{-1}}{1 - \theta}; t = \frac{\theta \omega^{-1}}{1 - \theta} z. \tag{3.4}
\]

The probability of a successful sale is from (2.4)

\[
\phi(p_i) = \frac{M}{N} \frac{1}{2} \left\{ 1 + (y^{**}(p_i) - y^*_0) \right\}^{-\alpha}.
\]

Solving for the seller’s optimal price rule (see Appendix B.2), we have

\[
p_i = \frac{1}{1 - (1/a)} (x_i - \delta_i) + \frac{1}{(a - 1) r} (1 + t - y^*_0). \tag{3.5}
\]

3.4. Market equilibrium

In the Bayesian Nash equilibrium, the buyer’s perceived structure (3.1) must coincide with the actual one (3.5). Thus, we require \( \omega \) and \( z \) to satisfy

\[
\omega = \frac{1}{1 - (1/a)}; z = \frac{1}{(a - 1) r} (1 + t - y^*_0). \tag{3.6}
\]

Therefore, the equilibrium price is (see Appendix B.3)

\[
p_i = \frac{1}{1 - (1/a)} (x_i - \delta_i) + \frac{1}{(a - 1) (1 - y^*_0)}. \tag{3.7}
\]
3.5. Expectation heterogeneity and excessive price sensitivity

Let us now consider the effect of buyer expectation heterogeneity in the case of sophisticated buyers. Comparing (2.6) and (3.7), we know the equilibrium price is exactly the same regardless of whether buyers are sophisticated or not. In particular, we have from (3.7)

\[
\left[ \frac{dp_i}{dx_i} \right]_{\text{Learning}} = \frac{1}{1 - (1/a)}. \tag{3.8}
\]

Thus, the qualitative results in the previous section also hold in the case of sophisticated buyers.

As suggested in the beginning of this section, learning from price has two mutually offsetting effects. On the one hand, learning from price implies that buyer heterogeneity becomes less important. The buyer’s optimal forecast (3.2) is the weighted average of his prior opinion and the asking price of the seller. Thus, the ex post expectations are more homogeneous than ex ante opinions. This property of rational expectation formation has the same effect as increased concentration, so that rational expectation formation reduces price sensitivity.

On the other hand, learning from price introduces the possibility of manipulation. By choosing the asking price, the seller can influence the perception of the buyer. This effect is also represented in the optimal forecast (3.2). By choosing a higher price, the seller can raise the buyer’s forecast of the intrinsic value. This manipulation effect introduces an additional incentive to raise price, which is absent in the model of unsophisticated buyers. Thus, this raises price sensitivity.
In the case of the Power-Cusp opinion distribution, these two effects exactly offset each other, and the result is the same as in the unsophisticated buyer case. Although the exact offsetting of two effects is specific to the Power-Cusp distribution, the result of this section strongly suggests that learning from price does not significantly change the result of excessive price sensitivity in the previous section, even if we consider other plausible opinion distributions.

4. Concluding Remarks

This paper has shown that the land market may exhibit excessively sensitive price behavior if there are a number of uninformed buyers having heterogeneous opinions. It has also been revealed that the more diverse buyers’ opinions are, the more sensitive land prices are. The result is in sharp contrast with product markets in which imperfect and diverse information implies rigidity rather than volatility.  

According to this model, the culprit behind real estate booms and busts in the 1980s is the influx of relatively uninformed investors into the real estate markets. In the case of Japan, Sweden, and other industrial countries, the surge of these new investors was brought on by internationalization and liberalization of financial markets. It

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14See, for example, Nishimura (1992, 1998).

15Nishimura, Watanabe and Iwatsubo (1999a, 1999b) tested the validity of the heterogeneous expectations model in the Tokyo Stock Exchange and in the Japanese urban real estate market. In the case of the Tokyo Stock Exchange, they (1999a) used the variance of three-month-ahead foreign exchange forecasts among institutional investors as a proxy of expectation heterogeneity among stock-market participants, and found that expectation heterogeneity significantly increased price sensitivity between May, 1985 and December, 1997. In the case of the real estate market, they (1999b) used dispersion in six-month-ahead interest-rate forecasts among corporations as a proxy of market participants’ expectation heterogeneity, and found that price sensitivity of residential
should be noted that even in many industrialized countries, the real estate markets are characterized by substantial transaction costs and susceptible to this kind of shocks.

The model also suggests that this type of volatile price behavior may happen in any market with non-negligible transaction costs. Thus, volatile price behavior is likely to be observed in stock markets of developing countries as well as in their real estate markets, as we observed in South East Asian financial crises in 1997.

There are, however, several limitations to this model. Firstly, the model is a static “temporary equilibrium” model, so that intertemporal price behavior is not explicitly analyzed. Specifically, the heterogeneous opinions on the side of buyers is assumed, and the evolution of these opinions over time is not articulated. In order to fully incorporate this factor, we must consider a dynamic model in which heterogeneous opinions are endogenized.\textsuperscript{16}

Secondly, we assume that sellers make take-it-or-leave-it type offers. The implications of such offers is that there is no negotiation on the terms of trade among buyers and sellers. This assumption is made to simplify the analysis, but it is unsatisfactory since intense negotiation usually takes place in real estate markets. In a more general model of real estate markets, we must consider a Bayesian Nash equilibrium of a market with pair-wise bargaining under incomplete information.

\textsuperscript{16}It should also be noted that, in a fully dynamic model, buyers today may be the sellers in the future and \textit{vice versa}. Strong asymmetry imposed upon the model between buyers and sellers cannot be maintained in this case.
References


Appendix A: Derivation of Condition \( y_0^* < x_i + (1/a) \) as a Sufficient Condition for \( p_i \geq y_0^* \) (Section 2)

Let \( p_i \) be the optimal price. If \( y_0^* \leq x_i \), we immediately get \( y_0^* \leq x_i \leq p_i \), since individual rationality requires \( x_i \leq p_i \). Thus, we have \( y_0^* \leq p_i \) if \( y_0^* \leq x_i \). Consequently, it is sufficient in the following to consider the case that \( x < y_0^* \).

Suppose \emph{ad absurdum} that the optimal price \( p_i \) satisfies \( p_i < y_0^* \). Since the density curve of \( f \) is symmetric and single-peaked around \( y_0^* \), we have

\[
 f (p_i) < f (y_0^*).
\]

Moreover, we have by definition

\[
 F (p_i) < F (y_0^*).
\]

Since \( \phi (p_i) = (M/N) [1 - F (p_i)] \) and \( \phi ' (p_i) = - (M/N) f (p_i) \), we have from the above relations

\[
 0 > \phi ' (p_i) > \phi ' (y_0^*)
\]

and

\[
 \phi (p_i) > \phi (y_0^*) > 0. \quad \text{(A.1)}
\]
It is assumed that $p_i < y_0^*$, and we know $x_i \leq p_i$. Consequently, we obtain

$$0 > \phi'(p_i)(p_i - x_i) \geq \phi'(y_0^*)(p_i - x_i) > \phi'(y_0^*)(y_0^* - x_i). \quad (A.2)$$

Combining (A.1) and (A.2) we have

$$\phi'(p_i)(p_i - x_i) + \phi(p_i) > \phi'(y_0^*)(y_0^* - x_i) + \phi(y_0^*) \quad (A.3)$$

Let us define

$$\pi(p_i) = \phi(p_i)(p_i - x_i),$$

which is the expected profit in excess of the land’s intrinsic value $x_i + X_i$. Then we have from (A.3)

$$\pi'(p_i) = \phi'(p_i)(p_i - x_i) + \phi(p_i) > \phi'(y_0^*)(y_0^* - x_i) + \phi(y_0^*). \quad (A.4)$$

It should be noted that we have by definition $\phi(y_0^*) = (M/N)(1/2)$ and $\phi'(y_0^*) = -(M/N)(1/2)a$. Thus

$$\phi'(y_0^*)(y_0^* - x_i) + \phi(y_0^*) = (M/N)(1/2)a(-y_0^* - x_i + a^{-1})$$
Thus, under the condition that \( y_0^* < x_i + (1/a) \), we have

\[
\phi' \left( y_0^* \right) (y_0^* - x_i) + \phi \left( y_0^* \right) > 0.
\]

Consequently, we have from (A.4)

\[
\pi' (p_i) > 0,
\]

which contradicts the claim that \( p_i \) is the optimal price. Therefore, the optimal price satisfies \( y_0^* \leq p_i \).

Appendix B: Derivation of Equilibrium in Section 3

B.1. The Buyer’s Expectation Formation

Let us define information \( A \) as

\[
A = \omega^{-1} \left( p_i - z - \omega y_j^* \right).
\]

Then, from (3.1) we have

\[
A = (x_i - y_j^*) - \delta_i
\]

Note that the buyer knows \( (x_i - y_j^*) = (y - y_j^* + w_i) \sim N (0, \sigma_x^2) \) where \( \sigma_x^2 = \sigma_y^2 + \sigma_w^2 \).
\( \delta_i \sim N(0, \sigma_\delta^2) \). Then, from the standard procedure of the signal extraction problem, we know \( E^j (x_i - y_j^* \mid A) = \theta A \), where \( \theta = \sigma_x^2 / (\sigma_x^2 + \sigma_\delta^2) \). Consequently, we have

\[
E^j (x_i \mid p_i) = (1 - \theta) y_j^* + \theta \omega^{-1} (p_i - z).
\]

**B.2. Derivation of the Optimal Price Rule in Section 3**

In Section 3 we have assumed that the Power-Cusp distribution is the buyer-opinion distribution. Then, sales probability is

\[
\phi(p_i) = \frac{M}{N} \frac{1}{2} (1 + r p_i + t - y_0^*)^{-a}
\]

(B.1)

where \( r \) and \( t \) are given in (3.4).

From (B.1), we have

\[
\frac{\phi(p_i)}{\phi'(p_i)} = -\frac{1}{ar} (1 + r p_i + t - y_0^*).
\]

Consequently, the optimal price rule is

\[
p_i = \frac{1}{1 - (1/a)} (x_i - \delta_i) + \frac{1}{(a - 1) r} (1 + t - y_0^*)
\]

(B.2)

**B.3. Derivation of Rational Expectations Equilibrium**

From (3.6), we have
\[ z = \frac{1}{(a - 1) r} (1 - y_0^*) + \frac{1}{(a - 1) r^t} \]  \hspace{1cm} (B.3)

Substituting (3.4) into (B.3) and rearranging terms, we have

\[ z = \frac{1 - \theta}{a - 1 - a \theta \omega^{-1}} (1 - y_0^*) \]

Since \( \omega^{-1} = 1 - (1/a) \), we have with some calculation

\[ z = \frac{1}{a - 1} (1 - y_0^*) . \]
Figure 1.
Metropolitan Residential Land Price and Its “Fundamentals”:
The Annual Rate of Change 1956-1995

Figure 2.
Semianual Excess Returns on “Blue-Chip” Commercial Property

Figure 3.
Power-Cusp distribution: Solid line ($a = 1.1$) and dotted line ($a = 6$)
Figure 4.
Normal Distribution (solid line) and Gibbs Distribution (dotted line)
Figure 5.
Optimal Price Formula When More Buyers Are Optimistic ($y^*_0 = 2$)

$\sigma = 0.16$: thin, $\sigma_0 = 0.52$: medium, $\sigma = 1.6$: thick
Figure 6:
Optimal Price Formula When More Buyers Are Pessimistic ($y_0^* = -2$)
$\sigma = 0.16$: thin, $\sigma = 0.52$: medium, $\sigma = 1.6$: thick