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Extraneous Shocks and International Linkage of Business Cycles in a Two-Country Monetary Model

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Extraneous Shocks and International Linkage of Business Cycles in a Two-Country Monetary Model *

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Abstract

This paper analyzes how extraneous shocks can be the source of world business cycles. Using a two-country monetary model with the cash-in-advance constraint, we find that there exist stationary sunspot equilibria when either the relative risk aversion of the utility function or external effects in production are large. In both cases, stationary sunspot equilibria are a more likely outcome for the world aggregate output than for country-specific output. Therefore, even if the fundamental value shows small cross-country output correlations, extraneous shocks can cause large synchronization of business cycles under rational expectations.

JEL numbers: E30, F40
Key words: extraneous shocks, business cycles, international transmission

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1. Introduction

The source of international business cycles is an extensively debated topic in international macroeconomics. In previous literature, a large number of empirical studies have noted an approximate synchronization between different national business cycles and have hypothesized the existence of “world business cycles.”¹ The empirical evidence is, however, somewhat paradoxical because other empirical studies have shown that the international transmission of economic fluctuations is not large.² One plausible explanation for the empirical evidence is the common exogenous shock hypothesis: that business cycle synchronization is caused by exogenous global shocks such as oil shocks.³ In particular, oil price shocks in 1973 and 1979 had substantial effects on economies around the world. However, it is hard to believe that we have experienced highly frequent fundamental global shocks that caused business cycle synchronization. In addition, several empirical studies report that correlations of output across countries are much larger than those of productivity.⁴

In this paper, we investigate whether sunspots can be another source of world business cycles even if agents form rational expectations, using a two-country monetary model with country-specific cash-in-advance constraints.⁵ Assuming small international transmission of economic fluctuations, we explore the dynamic property of this two country monetary model and

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¹ In two-country equilibrium frameworks, examples of these studies include Backus, Kehoe, and Kydland (1992), Devereux, Gregory, and Smith (1992), and Stockman and Tessar (1995).
² For example, Fair (1982) provides quantitative estimates of international linkages from his econometric model. In his tables, we can easily see that international linkages are very small except for linkages from the United States to Canada and from Germany to other European countries. Oudiz and Sachs (1984) review evidence on policy multipliers in two large-scale econometric models, the Japanese Economic Planning Agency (EPA) model and the Federal Reserve Board’s Multicountry model (MCM). They conclude that the cross-country policy multipliers are generally quite small, although the United States has some effect on West Germany and Japan.
³ For example, running a vector autoregression, Dellas (1986) found that the lagged effects of one economy on another were less significant than the contemporaneous effects and concluded that common shocks explain the existence of world business cycles.
⁴ See, for example, Costello (1993), Backus, Kehoe, and Kydland (1995), and Stockman and Tessar (1995).
⁵ This type of cash cash-in-advance constraints was extensively studied in open macroeconomics [e.g., Helpman (1981), Lucas (1982), Aschauer and Greenwood (1983), Grilli and Roubini (1992), and Ch.8A in Obstfeld and Rogoff (1997)].
the implications of extraneous (non-fundamental) shocks for world business cycles.

In the following two-country model, productivity shocks always have strong impacts on the domestic output. However, international transmissions of the productivity shocks are small under reasonable parameters. Therefore, unless productivity shocks are highly correlated across countries, it is unlikely that the fundamental value of output in country 1 has a strong correlation with that in country 2.

When we investigate the dynamic property of this monetary model, we find that there exist stationary sunspot equilibria when either the relative risk aversion of the utility function or positive external effects in production are large. In both cases, stationary sunspot equilibria are the more likely outcome for the world aggregate output than for country-specific output because a rise of the expected future domestic output has a positive impact on both domestic and foreign current outputs. However, even when the international linkage is small, extraneous uncertainty tends to cause strong synchronization of business cycles. The result holds even if two countries have asymmetric economic structures in production functions, although it is not clear when two countries have asymmetric economic structure in utility functions.

One closely related study is by Guo and Sturzenegger (1998). They examine how sunspots can create international business cycles in the dynamic general equilibrium model with increasing returns to scale. Selover and Jensen (1999) also investigate international business cycle synchronization in a non-linear dynamic model. In contrast with our paper, none of them explored how sunspots can create world business cycles in a monetary economy. In previous literature, several authors point out that there can exist sunspot equilibria in cash-in-advance models [see Woodford (1994)]. In particular, Woodford (1991) and King, Wallace and Weber (1992) find the existence of chaos in the cash-in-advance model. In the dynamic stability of the other closed monetary models, Benhabib and Day (1982) and Grandmont (1985) characterize the chaotic phenomena in overlapping generation models. In addition, Matsuyama (1990, 1991) and Fukuda (1993a, 1997) find that models of money in the utility function can produce sunspots and chaotic dynamic paths.

Weil (1991) and Fukuda (1994) explore the same issue in models of money in the utility function.

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6 In the optimal growth model, Nishimura and Yano (1993) investigate the existence of endogenous cycles in the world economy. However, they pay little attention to the issues of world business cycles discussed in this paper.


8 Weil (1991) and Fukuda (1994) explore the same issue in models of money in the utility function.
In general, a sunspot equilibrium depends on extraneous uncertainty only because agents believe it to be so. One may thus interpret our result as implying that agents’ psychology can play an important role in explaining world business cycles. If this is the case, our theoretical results are consistent with several previous empirical studies that stressed the role of investor psychology in explaining the international transmission of exogenous shocks.9

The paper proceeds as follows. The next section presents a basic two-country model with cash-in-advance constraints, while section 3 derives its equilibrium conditions. Section 4 examines the existence of sunspot equilibria in our model and section 5 presents some simulation results. Section 6 extends our model to different production functions in two countries. Finally, section 7 summarizes our main results and refers to their possible extension.

2. The model

We consider a world economy with two countries, country 1 and country 2. Each country has identical agents with constant population. There are two, perishable, tradable consumption goods, A and B. The consumption good A is produced only in country 1 while the consumption good B is produced only in country 2. However, all agents consume both of the consumption goods.

Each representative agent in country $i$ ($i = 1, 2$) maximizes the following expected utility function:

$$E_t \sum_{j=0}^{\infty} \beta^j [U(c_{Ai}^{t+j}, c_{Bi}^{t+j}) - V(n_{i}^{t+j})], \quad 0 < \beta < 1,$$

(1)

where $E_t$ is the conditional expectation operator based on information at period $t$, $c_{Ai}^{t}$ is consumption in country $i$ of the good A, $c_{Bi}^{t}$ is consumption in country $i$ of the good B, and $n_{i}^{t}$ is labor input in country $i$ at period $t$. The utility functions and the discount factor $\beta$ are common to both countries. In the following analysis, we specify the utility functions as follows:

$$U(c_{Ai}^{t}, c_{Bi}^{t}) \equiv (c_{Ai}^{t} \alpha c_{Bi}^{t} \gamma / \gamma), \quad \text{where} \quad 1/2 < \alpha < 1, \quad \gamma < 1, \quad \gamma \neq 0,$$

(2a)

$$V(n_{i}^{t}) \equiv n_{i}^{t+\delta} / (1+\delta), \quad \text{where} \quad \delta > 0.$$

(2b)

9 For example, Shiller, Kon-Ya, and Tsutsui (1991) show that international stock market crash in the October 1987 was attributable to changes in investor psychology both in the United States and Japan. Engle, Ito, and Lin (1990) investigate the international transmission of intra-daily asset volatility in the foreign exchange market and conclude that various market failures such as fads, bubbles, and bandwagons can be the explanation for their finding.
The utility function (2a) represents the case where an elasticity of substitution in consumption between the good A and the good B equals to one. The condition that $\alpha > 1/2$ indicates that the agent places more weight on the domestic consumption good than on the foreign consumption good.

Define $p^A_t$ as the price of the good A in terms of country 1 currency, $p^B_t$ as the price of the good B in terms of country 2 currency, and $e_t$ as the nominal exchange rate in terms of country 1 currency, where subscript $t$ denotes time period $t$. Also define $B^i_t$ as the net saving in country $i$, $r^i_t$ as real interest rate in country $i$, $y^i_t$ as income in country $i$, and $T^i_t$ as the real lump-sum tax (or transfer if negative) in country $i$ ($i = 1, 2$). Then, since the consumption-based country A’s currency price index is $P^1_t = (p^A_t)^\alpha (e_t p^B_t)^{1-\alpha}$, the budget constraint in country 1 in terms of country 1 currency is written as

$$p^A_t c^A_1 + e_t p^B_t c^B_1 + M^{11}_{t+1} + e_t M^{21}_{t+1} - P^1_t T^1_t + P^1_t B^1_{t+1} \leq p^A_t y^1_t + M^{11}_t + e_t M^{21}_t + P^1_t (1+r^1_t) B^1_t.$$  

(3a)

Similarly, noting that the consumption-based country B’s currency price index is $P^2_t = (p^A_t/e_t)^{1-\alpha} (p^B_t)^\alpha$, the budget constraint in country 2 in terms of country 2 currency is written as

$$\left(\frac{1}{e_t}\right) p^A_t c^A_2 + p^B_t c^B_2 + (1/e_t)M^{12}_{t+1} + M^{22}_{t+1} - P^2_t T^B_t + P^2_t B^2_{t+1} \leq p^B_t y^2_t + (1/e_t)M^{12}_t + M^{22}_t + P^2_t (1+r^2_t) B^2_t,$$

(3b)

where $M^{1i}_t$ is the amount of currency 1 held by the agent in country $i$ at the beginning of period $t$, and $M^{2i}_t$ is the amount of currency 2 held by the agent in country $i$ at the beginning of period $t$. The net savings $B^i_t$ and taxes $T^i_t$ ($i = 1, 2$) are expressed in real terms and are assumed to be denominated in units of the index of total real consumption [see, for example, Obstfeld and Rogoff (pp.595-597)].

In the following analysis, we assume that $y^i_t$ is equal to the total output of the good A when $i = 1$ and is equal to the total output of the good B when $i = 2$. We also assume that the production function in each country is written as follows:

$$y^i_t = H \exp(w^i_t) n^i_\varepsilon n^i_\eta, \quad 0 < \varepsilon < 1, \; 0 < \eta,$$

(4)
where $N_i^t$ is the average labor input in country $i$ at period $t$ and $w_i^t$ is the zero-mean productivity shock in country $i$ at period $t$. In the following analysis, we assume that $w_i^t$ follows AR(1) process:

$$w_i^t = \rho w_i^{t-1} + \xi_i^t, \quad 0 < \rho < 1,$$

where $\xi_i^t$ follows the normal distribution $N(0, \sigma^2)$ and is independent over time.

A key characteristic in the above production function is that the output in country $i$ depends not only on the individual labor input in country $i$, $n_i^t$, but also on the average labor input in country $i$, $N_i^t$. It is based on an implicit assumption that the individual production in country $i$ has spillover effects on the average production in country $i$ but no spillover effect on the production in country $j$ ($j \neq i$). The existence of intranational spillover effects has widely been discussed in the literature of endogenous growth [see, for example, Grossman and Helpman (1990)]. In particular, since the influential paper by Coe and Helpman (1995), a large number of empirical studies have found strong evidence of intranational knowledge spillovers but limited evidence of international knowledge spillovers.\(^{11}\) We can interpret that our production function simply formulated their evidence without capital. Because $0 < \varepsilon < 1$, the production function (4) reveals decreasing returns to scale in terms of individual labor input. However, when $\varepsilon + \eta > 1$, the aggregate production function gives increasing returns to scale in labor.

Concerning cash-in-advance constraints, we assume that agents must set aside currency 1 in advance to purchase the good A and currency 2 in advance to purchase the good B.\(^{12}\) The cash-in-advance constraints are thus given by

$$p_i^A c_i^A t \leq M_i^A t, \quad (6a)$$
$$p_i^B c_i^B t \leq M_i^B t. \quad (6b)$$

To the extent that the transversality conditions are satisfied\(^{13}\), the above constrained optimization

\(^{10}\) If we denote labor input of firm $z$ in country $i$ at period $t$ by $n(z_i^t)$, then $N_i^t = \sum_{z=1}^{\Omega} n(z_i^t)$, where $\Omega$ is the number of firms in country $i$.
\(^{11}\) See Branstetter (2001) for overview.
\(^{12}\) Zhou (1997) shows that this type of cash-in-advance constraint can be derived in a search theoretic monetary model.
\(^{13}\) In our model specification, the transversality conditions are satisfied when $\lim_{j \to \infty} \beta^j M_{ij}^{t+1} = 0$ and $\lim_{j \to \infty} \beta^j e_{ij}^{t} M_{ij}^{t+1} = 0$. 

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The problem for the agent in country 1 is maximized by differentiating the following Lagrangean by $c^{A1}_t$, $c^{B1}_t$, $n^{1}_t$, $M^{11}_t$, $M^{21}_t$, and $B^{1}_t$:

$$L = E_i \sum_{j=0}^\infty \beta^j \{ U(c^{A1}_t, c^{B1}_t) - V(n^{1}_t) \}$$

$$- \lambda_{t+j} [p^{A}_t c^{A1}_t + e_{t+j} p^{B}_t c^{B1}_t + M^{11}_t + e_{t+j} M^{21}_t + p^{A}_t T^{1}_t + p^{B}_t B^{1}_t + p^{A}_t H \exp(w^{1}_t)] n^{1}_t e^{1}_t N^{1}_t \eta - \lambda_{t+j} \phi_{t+j} (p^{A}_t c^{A1}_t - M^{21}_t) \chi_{t+j} (p^{B}_t c^{B1}_t - M^{11}_t),$$

where $\lambda_t$, $\phi_t$, and $\chi_t$ are Lagrange multipliers at period $t$ (see Appendix 2 for a table of the parameters’ definitions).

In our model under uncertainty, the cash-in-advance constraints are not necessarily binding in the presence of large unexpected productivity shocks. However, assuming that the unexpected productivity shocks are not large, the following analysis focuses on the case where the cash-in-advance constraints ($6a, b$) are always binding.\(^\text{14}\) The first-order conditions for the agent in country 1 are then written as

$$\alpha(c^{A1}_t c^{B1}_t)^{1-\alpha} (c^{A1}_t c^{B1}_t c^{A1}_t c^{B1}_t)^{\gamma-1} = \lambda_{t} p^{A}_t + \phi_{t} p^{B}_t, \quad (7a)$$

$$(1-\alpha) (c^{A1}_t c^{B1}_t)^{1-\alpha} = \lambda_{t} e_{t} p^{B}_t + \chi_{t} p^{B}_t, \quad (7b)$$

$$n^{1}_t = \lambda_{t} p^{A}_t H \exp(w^{1}_t) e^{1}_t N^{1}_t \eta, \quad (7c)$$

$$\lambda_{t} = \beta E_t (\lambda_{t+1} + \phi_{t+1}), \quad (7d)$$

$$e_{t} \lambda_{t} = \beta E_t (e_{t+1} \lambda_{t+1} + \chi_{t+1}), \quad (7e)$$

$$\lambda_{t} p^{1}_t = \beta (1+r^{1}_t) E_t (\lambda_{t+1} p^{1}_t) \quad (7f)$$

Noting that $y^{1}_t = H \exp(w^{1}_t) n^{1}_t N^{1}_t \eta$ and $n^{1}_t = N^{1}_t$ in equilibrium, (7a) – (7c) lead to

$$c^{A1}_t c^{B1}_t = [\alpha/(1-\alpha)](e_{t+1} p^{B}_t / p^{A}_t), \quad (8a)$$

$$(y^{1}_t)^{\mu-1} = \beta \alpha H \exp(\mu w^{1}_t) E_t [(c^{B1}_t / c^{A1}_t)^{1-\alpha} (c^{A1}_t c^{B1}_t)^{1-\alpha} p^{A}_t / p^{A}_t)], \quad (8b)$$

where $\mu = (1+\delta)/(\varepsilon + \eta)$.

\(^{14}\) Strictly speaking, the assumption does not hold because the innovation in the productivity shock follows the normal distribution. However, to the extent that its variance is small, the possibility that the unexpected shocks violate the cash-in-advance conditions can be negligible.
Similarly, assuming the transversality and the cash-in-advance conditions, the first-order conditions for the agent in country 2 lead to

\[ c^{B_2}/c^{A_2} = \left[ \alpha/(1-\alpha) \right] \left( p^A/\epsilon_t \right) \left( p^B/P^B_{t+1} \right), \]

\[ (y^2_t)^\mu = \beta \alpha \epsilon H^\mu \exp(\mu w^2_t) E_t \left[ (c^{A_2}_{t+1}/c^{B_2}_{t+1})^{1-\alpha} \left( c^{B_2}_{t+1} \alpha c^{A_2}_{t+1} \gamma^{1-\alpha} \right) \right]. \]

[See Appendix 1 for derivations of (8a), (8b), (9a), and (9b)].

3. Market Equilibrium

In the following analysis, we assume that the monetary authority of each country keeps the total supply of its currency constant over time. We also assume that government revenue through money creation in country 1 is always balanced by the exogenous change in the lump-sum transfer to the agent in country 1. Then, denoting the total nominal supply of currency 1 by \( M^1 \), it holds that \( T_1 = 0 \) and \( M^1 + M^2 = M^t \) for \( i = 1 \) and 2. Therefore, the equilibrium conditions in the good markets, the money markets, and the foreign exchange market lead to

\[ c^{A_1}_t + c^{A_2}_t = y^1_t, \]
\[ c^{B_1}_t + c^{B_2}_t = y^2_t, \]
\[ p^A_t y^1_t = M^1, \]
\[ p^B_t y^2_t = M^2, \]
\[ e_t = M^1/M^2, \quad \text{(or equivalently } p^A_t y^1_t = e_t p^B_t y^2_t). \]

One important property of these equilibrium conditions is that they dichotomize. That is, all real equilibrium quantities are independent of the levels of the nominal money stocks, while the levels of equilibrium prices and the nominal exchange rate are determined by the money equation. Hence, the traditional Quantity Theory (neutrality of money) holds in our model.

Since both \( M^1 \) and \( M^2 \) are constant, (10c) implies that the nominal exchange rate \( e_t \) is constant over time. Thus, the equilibrium conditions (10a, b, c) with (8a) and (9a) lead to

\[ c^{A_1}_t = \alpha y^1_t, \]
\[ c^{B_2}_t = \alpha y^2_t, \]
\[ c^{A_2}_t = (1-\alpha) y^1_t, \]
\[ c^{B_1}_t = (1-\alpha) y^2_t. \]

Substituting (11a, b) into the first-order conditions (8b) and (9b), we obtain

\[ (y^1_t)^\mu = J \exp(\mu w^1_t) E_t \left( y^1_{t+1} \alpha y^2_{t+1} (1-\alpha) \gamma \right), \]

8
\[
(y^2_t)^\mu = J \exp(\mu w^2_t) E_t \left( y^1_{t+1} (1-\alpha\gamma) y^2_{t+1} \alpha^\gamma \right),
\]

(12b)

where \( J \equiv \beta \alpha^{\alpha \gamma} (1-\alpha)^{(1-\alpha)\gamma} e^{H \mu} \).

Equations (12a) and (12b) determine the dynamic system in our model. In order to obtain the fundamental values of \( y^1_t \) and \( y^2_t \), we suppose that

\[
\begin{align*}
  y^1_t &= \Gamma \exp(a\mu w^1_t) \exp(b\mu w^2_t), \\
  y^2_t &= \Gamma \exp(b\mu w^1_t) \exp(a\mu w^2_t),
\end{align*}
\]

(13a)

where \( \Gamma, a, \) and \( b \) are constant unknown parameters. Then, equations (13a) and (13b) imply that

\[
E_t \left( y^1_{t+1} (1-\alpha\gamma) y^2_{t+1} \alpha^\gamma \right) = \Gamma \exp(\mu \sigma^2/2) \exp(\mu \rho (h w^1_{t+1} + k w^2_{t+1})) \exp(\mu \sigma^2/2) \exp(\mu \sigma^2/2) \exp(\mu \rho (h w^1_{t+1} + k w^2_{t+1})),
\]

(14c)

(See Appendix 2 for a table of the parameters’ definitions).

Since \( 1 > |\rho\gamma| \), the unknown parameter \( a \) is always positive in (14a). We can thus see that the productivity shocks always have positive effects on the domestic output. In contrast, the productivity shocks have positive effects on the foreign output when \( \gamma > 0 \) and negative effects on the foreign output when \( \gamma < 0 \). However, since \( b \) is small when either \( 1-\alpha, \rho, \) or \( \gamma \) is close to zero, international transmissions of the productivity shocks are small either when the consumption is highly biased to the home product, when productivity shocks are less persistent, or when the utility function is close to log-linear type. Therefore, when one of these conditions is satisfied, it is unlikely that the fundamental value of output in country 1 has a strong correlation with that in country 2 unless \( w^1_t \) is highly correlated with \( w^2_t \).

For example, Figure 1a-b plot accumulated impulse response functions of \( \log y^1_t \) and \( \log y^2_t \) to a productivity shock in country 1 for two alternative parameters. In calculating the impulse response functions, we set that an innovation in the productivity shock \( \xi^1_t \) is equal to 1 when \( t = 1 \).
and 0 otherwise. In addition, we assume that $\omega_t^2 = 0$ for all $t$ and set that $\alpha = 0.8$, $H=1$, $\varepsilon = 0.8$, $\sigma^2 = 1$, $\sigma_{12} = 0$, and $\beta = 0.9$.\(^{15}\)

Figure 1a shows the accumulated impulse response functions of log $y_1^t$ and log $y_2^t$, for a parameter set that $\rho = 0.9$, $\delta = 2$, $\eta = 0.2$, and $\gamma = -4$. In the figure, the productivity shocks have strong positive effects on the domestic output. However, because $\gamma < 0$, the productivity shocks have negative effects on the foreign output. In addition, the negative effects are very small in their magnitude.

Figure 1b shows the accumulated impulse response functions of log $y_1^t$ and log $y_2^t$, for a parameter set that $\rho = 0.6$, $\delta = 0.4$, $\eta = 1.2$, and $\gamma = 0.8$. As in the figure 1a, the productivity shocks have strong positive effects on the domestic output in figure 1b. In addition, because $\gamma > 0$, the productivity shocks have positive effects on the foreign output. However, the effects on the foreign output are negligible in their magnitude.

4. The Existence of Sunspot Equilibria

In this section, we investigate whether the dynamics (12a) and (12b) can have sunspot equilibria under reasonable conditions. In the analysis, we consider the case where there exists no stochastic shock in the economy. Then, (12a, b) lead to a pair of difference equations for the perfect foresight equilibrium dynamics of two logged output levels:

\begin{align*}
\log y_1^t & = \frac{1}{\mu} \log J + \frac{\alpha \gamma}{\mu} \log y_1^{t+1} + \frac{(1-\alpha) \gamma}{\mu} \log y_2^{t+1}, \\
\log y_2^t & = \frac{1}{\mu} \log J + \frac{\alpha \gamma}{\mu} \log y_2^{t+1} + \frac{(1-\alpha) \gamma}{\mu} \log y_1^{t+1},
\end{align*}

or equivalently,\(^{16}\)

\begin{align*}
\log y_1^t + \log y_2^t & = 2\frac{1}{\mu} \log J + \frac{\gamma}{\mu} (\log y_1^{t+1} + \log y_2^{t+1}), \\
\log y_1^t - \log y_2^t & = \frac{(2\alpha-1) \gamma}{\mu} (\log y_1^{t+1} - \log y_2^{t+1}).
\end{align*}

The steady state equilibrium for the dynamic equations is defined by $\log y_1^t = \log y_1^{t+1}$ and $\log y_2^t = \log y_2^{t+1}$. Thus, when we denote the steady state value of $\log y_1^t$ and $\log y_2^t$ by $\log y_1^*$ and $\log y_2^*$, we obtain that $\log y_1^* = \log y_2^* = \log J/(\mu - \gamma)$, or equivalently, $\log y_1^* + \log y_2^* = 2\log J/(\mu - \gamma)$.

\(^{15}\) The assumption that $\sigma_{12} = 0$ is not crucial in the following simulations because neither $a$ nor $b$ depends on $\sigma_{12}$ in (14a,b).

\(^{16}\) The transformation follows Aoki (1981) and Fukuda (1993b).
and \( \log y^1_\ast - \log y^2_\ast = 0 \).

It is well-known that a one-dimensional map \( x_t = J(x_{t+1}) \) has stationary sunspot equilibria around its steady state \( x^\ast \) if and only if \( |J(x^\ast)| > 1 \) where \( x^\ast = J(x^\ast) \) [see, for example, Blanchard and Kahn (1980), Grandmont (1986), Woodford (1986), and Chiappori, Geoffard, and Guesnerie (1992)]. Thus, equations (16a, b) lead to the following proposition.

**Proposition:** There exist stationary sunspot equilibria of \( \log y^1_t + \log y^2_t \) if and only if

\[
\gamma < -\mu \quad \text{or} \quad \gamma > \mu. \quad (17)
\]

On the other hand, there exist stationary sunspot equilibria of \( \log y^1_t - \log y^2_t \) if and only if

\[
\gamma < -\frac{\mu}{(2-\alpha)-1} \quad \text{or} \quad \gamma > \frac{\mu}{(2-\alpha)-1}. \quad (18)
\]

The above proposition indicates that there exist stationary sunspot equilibria in our model for two alternative cases. The first is the case where \( \gamma \) is small and where the relative risk aversion of the utility function is large. In this case, stationary sunspot equilibria arise reflecting the conflict between intertemporal substitution and income effects. In other words, an expected decline of price levels in future has a negative substitution effect but has a positive income effect on the current demand for the consumption. Hence, its total effect on the current demand for the consumption is always ambiguous when \( \gamma \) is small, causing sunspot equilibria that were obtained in the above proposition.

The second is the case where \( \mu = (1+\delta)/(\varepsilon + \eta) \) is small and where the positive external effects in production are large. In this case, the stationary sunspot equilibria that arise reflect strategic complementarity in production. Thus, under the strategic complementarity, each agent produces more when he expects more aggregate production but less when he expects less aggregate production. Hence, the total output in the economy depends on whether agents’ expectations are bull or bear. This ambiguity also causes the sunspot equilibria that were obtained in the above proposition.

However, because \( 1/2 < \alpha < 1 \), it always holds that \( \mu/(2-\alpha) > \mu \). Therefore, the condition (17) is always satisfied if (18) is satisfied, but the condition (18) is not always satisfied even if (17) is satisfied. In other words, stationary sunspot equilibria are a more likely outcome for the world aggregate output level, \( \log y^1_t + \log y^2_t \), than for the output level difference between two countries,
\( \log y^1_t - \log y^2_t \). In particular, when \(-\mu/(2\alpha-1) < \gamma < -\mu\) or \(\mu < \gamma < \mu/(2\alpha-1)\), extraneous uncertainty affects only the world aggregate output level, \(\log y^1_t + \log y^2_t\), and causes synchronization of business cycles, that is, international business cycles!

The reason why stationary sunspot equilibria are more likely for the world aggregate output level is that a rise in the expected future output has a positive impact on both domestic and foreign current outputs. In fact, because the condition (17) is equivalent to (18) if \(\alpha = 1\), stationary sunspot equilibria are no more likely for the world aggregate output than for country-specific output if there exists no international linkage between two countries.

However, as long as some international linkage between two countries exists, extraneous uncertainty tends to cause synchronization of business cycles even in the case where the productivity shocks have negative effects on the foreign output, that is, when \(\gamma < 0\). In addition, stationary sunspot equilibria are more likely outcome for the world aggregate output level even if the consumption is highly biased to the home product, say, \(\alpha = 0.8\).

For example, suppose that \(\mu = 3\) and \(\alpha = 0.8\). In this case, there exist stationary sunspot equilibria of \(\log y^1_t + \log y^2_t\) if \(\gamma < -3\). However, there exists no stationary sunspot equilibrium of \(\log y^1_t - \log y^2_t\) unless \(\gamma < -5\). On the other hand, suppose that \(\mu = 0.7\) and \(\alpha = 0.8\). In this case, there exist stationary sunspot equilibria of \(\log y^1_t + \log y^2_t\) if \(\gamma > 0.7\). However, because \(\mu/(2\alpha-1) > 1\), there exists no stationary sunspot equilibrium of \(\log y^1_t - \log y^2_t\) for any value of \(\gamma\).

5. Some Stochastic Simulation Results

The purpose of this section is to present some stochastic simulation results to see how extraneous shocks, that is sunspots, can cause international business cycles. In the simulation, we first focus on the case where \(|\gamma/\mu| > 1 > |\gamma(2\alpha-1)/\mu|\). In this case, there exist sunspot equilibria only for \(\log y^1_t + \log y^2_t\). Thus, the general solutions of \(y^1_t\) and \(y^2_t\) for (12a, b) can be written as

\[
\begin{align*}
y^1_t &= \Gamma \exp(q^2/2) \exp(a\mu w^1_t) \exp(b\mu w^2_t) \exp(q v_t), \\
y^2_t &= \Gamma \exp(q^2/2) \exp(b\mu w^1_t) \exp(a\mu w^2_t) \exp(q v_t),
\end{align*}
\]  
(19a)
(19b)

where \(\Gamma\), \(a\), and \(b\) are constant parameters defined by (14a,b,c).

In (19a,b), an extraneous stochastic shock \(v_t\) follows a stationary AR(1) process:

\[
v_t = (\mu/\gamma) v_{t-1} + \phi_t,
\]  
(20)
where $\varphi_t$ follows $N(0,1)$ and is independent over time. Therefore, the dynamic property of the extraneous shock $v_t$ generally depends on some structural parameters. In particular, if $\gamma < 0$ ($\gamma > 0$), $v_t$ has negative (positive) serial correlation. Thus, if the relative risk aversion of the utility function is large, the extraneous shocks tend to alternate their sign, and vice versa.

It is easy to see that when $q = 0$, the solutions are degenerated into the fundamental values of $y_1^t$ and $y_2^t$ defined by (13a,b). If $|\gamma/\mu| < 1$, the stochastic shock $v_t$ would not be stationary, so that the transversality condition would imply that $q = 0$. However, to the extent that $|\gamma/\mu| > 1$, the shock $v_t$ is stationary. Since the solutions (19a, b) and (20) satisfy equations (12a) for an arbitrary value of $q$, the transversality condition cannot uniquely determine the value of $q$. Therefore, as long as the transversality condition is satisfied, the effects of extraneous shocks can dominate those of fundamental shocks, $w_1^t$ and $w_2^t$ when the absolute value of $q$ is relatively large.

Based on the general solutions (19a,b), we will present stochastic simulation results for some specific parameter sets and stochastic processes. The parameter sets used in the simulation are the following two types:

Type 1: $\alpha = 0.8, \rho = 0.9, H = 1, \varepsilon = 0.8, \beta = 0.9, \delta = 2, \eta = 0.2,$
$\sigma^2 = 1, \sigma_{12} = 0, \text{and } \gamma = -4$;

Type 2: $\alpha = 0.8, \rho = 0.6, H = 1, \varepsilon = 0.8, \beta = 0.9, \delta = 0.4, \eta = 1.2,$
$\sigma^2 = 1, \sigma_{12} = 0, \text{and } \gamma = 0.8$.

These two parameter sets are the same as those used for figure 1 (see Appendix 2 for a table of the parameters’ definitions). For stochastic shocks, we assume that all of $\xi_1^t, \xi_2^t$, and $\varphi_t$ follow $N(0,1)$ and are independent of each other. We also assume that $y_1^t = y_2^t$ when $t = 0$.

Two graphs in Figure 2a show simulated log $y_1^t$ and log $y_2^t$ based on type 1 parameter set for two alternative values of $q$. That is, Figure 2a-1 is the simulation results for $q = 0$ and Figure 2a-2 is for $q = 2$. Because extraneous shocks have no effect on output levels when $q = 0$, log $y_1^t$ and log $y_2^t$ show quite different movements in Figure 2a-1. In particular, because the productivity shocks have negative effects on the foreign output when $\gamma < 0$, log $y_1^t$ and log $y_2^t$ have some weak negative correlation in Figure 2a-1. However, when $q = 2$, log $y_1^t$ and log $y_2^t$ have strong positive correlation in Figure 2a-2. In particular, because extraneous stochastic $v_t$ has negative serial correlation when $\gamma < 0$, log $y_1^t$ and log $y_2^t$ have some negative serial correlation in Figure 2a-2.

Two graphs in Figure 2b, on the other hand, show simulated log $y_1^t$ and log $y_2^t$ based on the type 2 parameter set for two alternative values of $q$. That is, Figure 2b-1 is the simulation results for $q$
= 0 and Figure 2b-2 is for \( q = 2 \). Because the productivity shocks have positive effects on the foreign output when \( \gamma > 0 \), \( \log y^1_t \) and \( \log y^2_t \) have some weak positive correlation in Figure 2b-1. However, the degree of correlation is small when \( q = 0 \). On the other hand, when \( q = 2 \), \( \log y^1_t \) and \( \log y^2_t \) have strong positive correlation in Figure 2a-2. In particular, because extraneous stochastic \( v_i \) has positive serial correlation when \( \gamma > 0 \), \( \log y^1_t \) and \( \log y^2_t \) have some positive serial correlation in Figure 2a-2.

Although the above simulation results are based on specific parameter sets, similar results hold as long as \(|\gamma / \mu| > 1 > |\gamma (2\alpha - 1)/\mu|\). In deriving the results, the condition that \(|\gamma / \mu| > 1\) is crucial because it is necessary for the existence of sunspot equilibria. In addition, when \(|\gamma (2\alpha - 1)/\mu| > 1\), there exist sunspot equilibria for both \( \log y^1_t + \log y^2_t \) and \( \log y^1_t - \log y^2_t \). In this case, the general solutions of \( y^1_t \) and \( y^2_t \) for (12a, b) are written as

\[
y^1_t = \Gamma \exp\left[\left(q^2 + s^2\right)/2\right]\exp(a \mu w^1_t) \exp(b \mu w^2_t) \exp(q v_t) \exp(s \theta_t), \quad (21a)
\]
\[
y^2_t = \Gamma \exp\left[\left(q^2 + s^2\right)/2\right]\exp(b \mu w^1_t) \exp(a \mu w^2_t) \exp(q v_t) \exp(-s \theta_t). \quad (21b)
\]

In (21a,b), an extraneous stochastic shock \( \theta_t \) follows an AR(1) process:

\[
\theta_t = \{\mu / [\gamma (2\alpha - 1)]\} \theta_{t-1} + \kappa_t, \quad (22)
\]

where \( \kappa_t \) follows \( N(0, 1) \) and is independent over time.

It is easy to see that when \( q = s = 0 \), the solutions are reduced to the fundamental values of \( y^1_t \) and \( y^2_t \). However, as long as \(|\gamma (2\alpha - 1)/\mu| > 1\), the extraneous shocks \( v_t \) and \( \theta_t \) are stationary. Thus, to the extent that \(|\gamma (2\alpha - 1)/\mu| > 1\), the transversality condition cannot uniquely determine the value of \( q \) and \( s \) in (21a,b).

When \( q \neq 0 \) and \( s = 0 \) in (21a,b), extraneous shocks cause only world business cycles. However, when \( s \neq 0 \), extraneous shocks can affect each country’s output asymmetrically. In particular, when the absolute value of \( s \) is relatively large, sunspots do not necessarily make business cycle correlation across countries.

6. The Case of Asymmetric Structure

Until the last section, we have assumed that two countries have the same utility functions and production functions. Although the assumption greatly simplified our analysis, the symmetric economic structure may be restrictive in deriving business cycle correlation across countries.
The purpose of this section is to investigate whether introducing asymmetry in production functions may change our basic results or not.\textsuperscript{17}

In the following analysis, we assume that the production function in country $i$ ($i = 1, 2$) is written as

$$y_i^t = H_i n_i^{\epsilon_i} N_i^{\eta_i}, \quad 0 < \epsilon_i < 1, 0 < \eta_i.$$  \hspace{1cm} (23)

Except that parameters $H_i$, $\epsilon_i$, and $\eta_i$ are country specific, this production function is essentially the same as what was used in previous sections. For analytical simplicity, we assume that $\epsilon_1 + \eta_1 \geq \epsilon_2 + \eta_2$. We also assume that there is no productivity shock, that is, $w_i^t = 0$ for all $t$.

Under these asymmetric production functions, the dynamic equations (12a,b) are modified as follows:

\begin{align}
(y_1^t)^{\mu_1} & = J_1 E_t (y_{t+1}^{1(\alpha-\gamma)\gamma} y_{t+1}^{2(1-\alpha)\gamma}), \quad \text{(24a)} \\
(y_2^t)^{\mu_2} & = J_2 E_t (y_{t+1}^{1(\alpha-\gamma)\gamma} y_{t+1}^{2(1-\alpha)\gamma}), \quad \text{(24b)}
\end{align}

where $J_i \equiv \beta \alpha^{\alpha(1-\alpha)\gamma} \epsilon_i H_i^{\mu_i}$ and $\mu_i \equiv (1+\delta)/(\epsilon_i + \eta_i)$.

When there exists no stochastic shock in the economy, these two equations lead to a pair of perfect foresight dynamics for two logged output levels:

\begin{align}
\log y_1^t & = (1/\mu_1) \log J_1 + (\alpha \gamma / \mu_1) \log y_{t+1}^1 + [(1-\alpha) \gamma / \mu_1] \log y_{t+1}^2, \quad \text{(25a)} \\
\log y_2^t & = (1/\mu_2) \log J_2 + [(1-\alpha) \gamma / \mu_2] \log y_{t+1}^1 + (\alpha \gamma / \mu_2) \log y_{t+1}^2. \quad \text{(25b)}
\end{align}

The characteristic equation for this two dimensional dynamic system is

$$g(\lambda) \equiv \lambda^2 - \alpha \gamma (\mu_1 + \mu_2) / (\mu_1 \mu_2) \lambda + (2\alpha - 1) \gamma^2 / (\mu_1 \mu_2) = 0. \quad \text{(26)}$$

It holds that $g(0) > 0$, $g(\alpha \gamma / \mu_1) = g(\alpha \gamma / \mu_2) = -(1-\alpha) \gamma^2 / (\mu_1 \mu_2) < 0$, and $g(\gamma / \mu_1) = (1-\alpha) \gamma^2 (\mu_2 - \mu_1) / (\mu_1 \mu_2) \geq 0$. Thus, the characteristic equation has two characteristic roots $\lambda_1$ and $\lambda_2$ such that

\textsuperscript{17} Analytically, it is difficult to evaluate differences in agents’ preferences in our model.
\[\gamma / \mu_1 \geq \lambda_1 > \alpha \gamma / \mu_1 \geq \alpha \gamma / \mu_2 > \lambda_2 > 0 \quad \text{when } \gamma > 0, \]  
(27a) 
\[\gamma / \mu_1 \leq \lambda_1 < \alpha \gamma / \mu_1 \leq \alpha \gamma / \mu_2 < \lambda_2 < 0 \quad \text{when } \gamma < 0. \]  
(27b) 

Define \( f_1 \equiv (\alpha \gamma - \mu_2)/(1-\alpha \gamma) > 0 \) and \( f_2 \equiv (\mu_1 \lambda_1 - \alpha \gamma)/(1-\alpha \gamma) > 0. \) Then, after some tedious algebra in Appendix, equations (25a, b) are rewritten as

\[ \log y_1 t + f_1 \log y_2 t = \text{constant} + \lambda_1 (\log y_1 t+1 + f_1 \log y_2 t+1), \]  
(28a) 
\[ f_2 \log y_1 t - \log y_2 t = \text{constant} + \lambda_2 (f_2 \log y_1 t+1 - \log y_2 t+1). \]  
(28b) 

When \( \mu_1 = \mu_2 = \mu \), it is easy to show that \( \lambda_1 = \gamma / \mu \) and \( \lambda_2 = (2\alpha - 1) \gamma / \mu \), or equivalently \( f_1 = f_2 = 1. \) Therefore, as long as \( \mu_1 = \mu_2 \), introducing asymmetric structure does not change our basic results derived in previous sections. In particular, because \( \mu_1 = \mu_2 \) even if \( H_1 \) is not equal to \( H_2 \), we can see that productivity difference between two countries does not change our basic results on international business cycles at all.

However, unless \( \mu_1 = \mu_2 \), the dynamic property of (28a, b) differs from that of (16a,b). To see this, recall that a one-dimensional map \( x_t = I(x_{t+1}) \) has sunspot equilibria around its steady state \( x^* \) if and only if \( |I(x^*)| > 1 \) where \( x^* = I(x^*) \). Then, we can derive the following proposition.

**Proposition** (in case of asymmetric economic structure): There exist stationary sunspot equilibria of \( \log y_1 t + f_1 \log y_2 t \) if and only if \( |\lambda_1| > 1 \). On the other hand, there exist stationary sunspot equilibria of \( \log y_1 t - f_2 \log y_2 t \) if and only if \( |\lambda_2| > 1 \).

Because \( |\lambda_1| > \alpha \gamma / \mu_1 \geq \alpha \gamma / \mu_2 > |\lambda_2| \), \( |\lambda_1| \) is always greater than one if \( |\lambda_2| \) is greater than one, but \( |\lambda_2| \) is not necessarily greater than one even if \( |\lambda_1| \) is greater than one. Therefore, stationary sunspot equilibria are more likely for \( \log y_1 t + f_1 \log y_2 t \), than for \( f_2 \log y_1 t - \log y_2 t \). In particular, when \( |\lambda_1| > 1 > |\lambda_2| \) (for example, when \( \mu_1 > \alpha \gamma > \mu_2 \)), extraneous uncertainty has an impact only on \( \log y_1 t + f_1 \log y_2 t \).

When \( |\lambda_1| > 1 > |\lambda_2| \), the general solution of (28a,b) is written as  

\[ \log y_1 t + f_1 \log y_2 t = \text{constant} + s \zeta_t, \]  
(29a) 
\[ f_2 \log y_1 t - \log y_2 t = \text{constant}, \]  
(29b) 

\[ \text{It holds that } |\lambda_1| > 1 > |\lambda_2| \text{ if and only if } g(1) < 0 \text{ when } \gamma > 0 \text{ and } g(-1) < 0 \text{ when } \gamma < 0. \]
or equivalently,

\[
\log y^1_t = \text{constant} + \frac{1}{1 + f_1 f_2} s \zeta_t, \quad (30a)
\]

\[
\log y^2_t = \text{constant} + \frac{f_2}{1 + f_1 f_2} s \zeta_t. \quad (30b)
\]

It is easy to see that the extraneous stochastic shock \(\zeta_t\) follows a stationary AR(1) process:

\[
\zeta_t = \frac{1}{\lambda_1} \zeta_{t-1} + \omega_t, \quad (31)
\]

where \(\omega_t\) follows \(N(0, 1)\) and is independent over time. In addition, because \(|\lambda_1| > 1\), a parameter \(s\) can be chosen arbitrarily.

Because \(1/\mu_1 \geq \lambda_1/\gamma > \alpha/\mu_1\) when \(\mu_1 \neq \mu_2\), it holds that \(0 < f_2 < 1\) when the production functions have different concavities. Thus, equations (30a,b) indicate that \(y^1_t\) can be more volatile than \(y^2_t\) when country 1 has a less concave production function than country 2. However, they also imply that even when \(f_2\) is not close to one, an extraneous shock \(\zeta_t\) moves both \(y^1_t\) and \(y^2_t\) in the same direction. That is, as long as \(|\lambda_1| > 1 > |\lambda_2|\), extraneous shocks can still cause synchronization of business cycles even when two countries do not have the same production functions. Therefore, as long as there exists some international linkage between two countries, extraneous uncertainty is more likely to cause synchronization of business cycles even in the case where production functions are not symmetric between two countries.

7. Concluding Remarks

We have raised the question of whether extraneous shocks matter for world business cycles. The answer to this question is “yes”, even in a standard two-country monetary model. As we have shown, as long as there exists some small international linkage between two countries, extraneous uncertainty can cause large synchronization of business cycles. Therefore, even if the fundamental value shows very small cross-country output correlations, sunspots can be one important source of world business cycles.

In previous literature, Spear, Srivastava, and Woodford (1990, p.281) argued, “it is clear that there is no qualitative distinction to be made between the kind of equilibrium fluctuations in price and allocations that occur in stationary sunspot equilibria, when endowments are non-stochastic, and those that occur in non-sunspot stationary equilibria, when endowments vary only a small amount. Hence, a sharp distinction between ‘sunspot equilibria’ and ‘non-sunspot equilibria’ is
of little interest in the case of economies subject to stochastic shocks to fundamentals.” Our result implies that the argument is not necessarily true when discussing cross-country output correlations because sunspots can cause world business cycles even if the fundamental drives negligible cross-country output correlations.

Needless to say, our monetary model is too simple a framework to explain all characteristics of international business cycles. We may extend our model framework to several directions, first introducing capital accumulation in our model. Not a few empirical studies report that correlations of output across countries are larger than those of consumption; The extension may explain this evidence in our monetary model. The second is to allow different cash-in-advance constraints, leading to different types of equilibria. However, to the extent that the cash-in-advance constraints cause sunspot equilibria, our basic result will probably hold in two country monetary models with the different types of cash-in-advance constraints. Third, since the existence of non-traded goods causes the deviation from the purchasing power parity condition, by introducing non-traded goods in the model, we may be able to allow the endogenous fluctuations of real exchange rates. The fourth would incorporate more general extraneous shocks. A sunspot equilibrium depends on extraneous uncertainty only because agents believe it to be so. The introduction of an extraneous stochastic shock that follows an AR(1) process is a simple way to incorporate this mechanism. In a more general model framework, we could, however, introduce the extraneous shock in alternative ways. For example, Guo and Sturzenegger (p.129) pointed out that the extraneous disturbances could also be caused by demand side shocks such as those related to fiscal and monetary policies. The introduction of such extraneous disturbances as well as those mentioned earlier will make our model implications richer.
Appendix 1: Derivations of (8a), (8b), (9a), and (9b) in section 2

(i) Derivations of (8a) and (9a)
Equations (7a) and (7b) lead to

\[ \frac{c^{A_1}}{c^{B_1}} = \left[ \frac{\alpha}{1-\alpha} \right] \frac{(p^B_t/p^A_t)}{(\lambda \epsilon_t + \chi_t)}/(\lambda_t + \phi_t). \]  
(A1)

Rearranging (A1) and taking the conditional expectations based on \( t-1 \) information, we obtain

\[ E_{t-1}[p^A_t c^{A_1}] = E_{t-1}[p^B_t c^{B_1}]. \]  
(A2)

Since the cash-in-advance constraints are binding, \( p^A_t c^{A_1} = M^{11}_t \) and \( p^B_t c^{B_1} = M^{21}_t \) are predetermined variables at time \( t \). We can thus rewrite (A2) as

\[ p^A_t c^{A_1} E_{t-1}[(\lambda + \phi)_t] = \left[ \frac{\alpha}{1-\alpha} \right] \frac{(p^B_t c^{B_1})}{E_{t-1}[(\lambda \epsilon_t + \chi_t)].} \]  
(A3)

Because (7d) and (7e) lead to

\[ e_{t-1} E_{t-1}[(\lambda + \phi)_t] = E_{t-1}[(\epsilon_t \lambda + \chi_t)], \]  
(A4)

we can derive (8a) from (A3) and (A4). We can also derive (9a) by symmetry.

(ii) Derivations of (8b) and (9b)
When \( n^1_t = N^1_t \), (4) and (7c) are respectively written as

\[ (n^1_t)^{1+\eta} = \frac{y^1_t}{[H \exp(w^1)]}, \]  
(A5)

\[ (n^1_t)^{1+\epsilon} = \lambda_d p^A \exp(w^1) \epsilon (n^1_t)^{1+\eta}. \]  
(A6)

Rearranging (A5) and (A6), we obtain

\[ (y^1_t)^{1+\epsilon} = \lambda_d p^A \epsilon H^\mu \exp(\mu w^1)_t, \]  
(A7)

where \( \mu = (1+\delta)/(\epsilon + \eta) \).
Since (7a) and (7d) lead to
\[
\lambda_t = \beta E_t \left[ \alpha \left( \frac{c^{B_1}}{c^{A_1}} \right)^{1-\alpha} \left( \frac{c^{A_1}}{c^{B_1}} \right)^{\alpha} (1/\rho^{A_1}) \right], \tag{A8}
\]
we can derive (8b) from (A7) and (A8). We can also derive (9b) by symmetry.

**Appendix 2:** A table of the definitions of the parameters used in the model

(i) Parameters in the utility function and the production function
α: the weight on consumption of home country product in the utility function (2a),
β: the discount factor,
γ: relative risk aversion in the utility function (2a),
δ: the elasticity of disutility to labor input in (2b),
ε: the elasticity of output to individual labor input in production function (4),
η: the elasticity of output to average labor input in production function (4),
H: the constant coefficient in production function (4),
ρ: the degree of serial correlation of the productivity shock \( w_1 \) in (5),
\( \sigma^2 \): variance of the innovation in the productivity shock \( w_1 \) in (5),
\( \sigma_{12} \): covariance between \( \xi_1 \) and \( \xi_2 \).

(ii) The other parameters
\[
\mu = \frac{(1+\delta)}{(1-\rho \alpha \gamma)}, \quad J = \beta \alpha \gamma (1-\alpha) e H^\mu, \\
h = \alpha \gamma \ a + (1-\alpha) \gamma b, \quad k = \alpha \gamma b + (1-\alpha) \gamma a, \\
a = (1 - \rho \alpha \gamma)/\{1 - \rho (2\alpha - 1) \gamma (1- \rho \gamma)\}, \\
b = (1-\alpha) \rho \gamma/\{1-\rho (2\alpha - 1) \gamma (1- \rho \gamma)\}, \\
\Gamma = J^{(1/\gamma\mu)} \exp \left\{ \frac{(h^2+k^2)}{2} \sigma^2 + 2hk\sigma_{12} \right\} \mu^2/[2(\mu-\gamma)].
\]

**Appendix 3:** Derivation of (27a, b) in section 6.

In a matrix form, equations (25a,b) are written as
\[
\begin{pmatrix}
\log y_t^1 \\
\log y_t^2
\end{pmatrix} = 
\begin{pmatrix}
\text{constant} \\
\text{constant}
\end{pmatrix} + 
A \begin{pmatrix}
\log y_{t+1}^1 \\
\log y_{t+1}^2
\end{pmatrix}
\]  
(A9)

where

\[
A \equiv \begin{pmatrix}
\frac{\alpha \gamma}{\mu_1} & \frac{(1-\alpha) \gamma}{\mu_1} \\
\frac{(1-\alpha) \gamma}{\mu_2} & \frac{\alpha \gamma}{\mu_2}
\end{pmatrix}
\]  
(A10)

Define

\[
P \equiv \begin{pmatrix}
(1-\alpha) \gamma & \mu_2 \lambda_2 - \alpha \gamma \\
\mu_1 \lambda_1 - \alpha \gamma & (1-\alpha) \gamma
\end{pmatrix}
\]  
(A11)

Then, it holds that

\[
P^{-1} AP = \begin{pmatrix}
\lambda_1 & 0 \\
0 & \lambda_2
\end{pmatrix}
\]  
(A12)

where

\[
P^{-1} = \frac{1}{|P|} \begin{pmatrix}
(1-\alpha) \gamma & \alpha \gamma - \mu_2 \lambda_2 \\
\alpha \gamma - \mu_1 \lambda_1 & (1-\alpha) \gamma
\end{pmatrix}
\]  
(A13)

Thus, (A9) can be transformed into

\[
P^{-1} \begin{pmatrix}
\log y_t^1 \\
\log y_t^2
\end{pmatrix} = 
\begin{pmatrix}
\text{constant} \\
\text{constant}
\end{pmatrix} + (P^{-1} AP) P^{-1} \begin{pmatrix}
\log y_{t+1}^1 \\
\log y_{t+1}^2
\end{pmatrix}
\]

Therefore, noting that
\( P^{-1} = \frac{(1 - \alpha)\gamma}{|P|} \begin{pmatrix} 1 & f_1 \\ f_2 & -1 \end{pmatrix} \) \tag{A15}

(A14) leads to (27a,b) in the text.
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Figure 1-a  Impacts of a Productivity Shock: $\gamma < 0$

Figure 1-b  Impacts of a Productivity Shock: $\gamma > 0$
Figure 2a-1. Simulation when $q = 0$ and $\gamma < 0$.

Figure 2a-2. Simulation when $q = 2$ and $\gamma < 0$. 
Figure 2b-1. Simulation when $q = 0$ and $\gamma > 0$.

Figure 2b-2. Simulation when $q = 2$ and $\gamma > 0$. 

- Domestic Output
- Foreign Output