CAN IT BE JAPAN’S SAVIOR?

Fumio Hayashi  
University of Tokyo

Koji Nomura  
Keio University

7 September 2005

Abstract

Journal of Economic Literature Classification Numbers: E2, E13, O4, O5,
Key words: IT, growth model, TFP, Japan
1. Introduction

- Recent productivity literature shows (1) TFP growth much higher for IT, (2) IT growth rate is accelerating.

- Seems a problem to Hayashi-Prescott that ignores sectoral heterogeneity. Question: the rapid IT productivity growth changes HP’s conclusion?

- Summarize results.

- Organization of the paper.

2. The Multi-Sector Accounting Framework

The theoretical model to be presented later in the paper is a multi-sector model with two market sectors (non-IT and IT goods-producing sectors) and two non-market sectors (the household and government sectors). This section describes how the model’s empirical counterpart, a multi-sector accounting system, is constructed. Its production account is derived from the 47-sector system developed in Jorgenson and Nomura (2005) (hereafter, JN).\(^1\) Its final demand components, too, are from the KEO Database.

\(^1\)It builds on the KEO Database, which is a comprehensive productivity database for the Japanese economy maintained at the Keio Economic Observatory (KEO), Keio University, Japan. It consists of a time-series of input-output tables and detailed inputs of capital and labor. See Kuroda, Shimpo, Nomura, and Kobayashi (1997) for a detailed documentation. From the 43 industries in the KEO database, JN separates out three IT producing industries — computers, communications equipment, and electronic components — to form 47 industries.
Output and Value Added

Table 1: Value Added at Factor Costs

<table>
<thead>
<tr>
<th></th>
<th>market production</th>
<th>non-market production</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>non-IT sector ($j=1$)</td>
<td>IT sector ($j=2$)</td>
</tr>
<tr>
<td>value added</td>
<td>$P_{jt}Y_{jt}$</td>
<td>$P_{jt}Y_{jt}$</td>
</tr>
<tr>
<td>non-IT capital cost</td>
<td>$P_{jt}(r_{j1}K_{jt})$</td>
<td>$P_{jt}(r_{j1}K_{jt})$</td>
</tr>
<tr>
<td>IT capital cost</td>
<td>$P_{jt}(r_{j2}K_{jt})$</td>
<td>$P_{jt}(r_{j2}K_{jt})$</td>
</tr>
<tr>
<td>labor cost</td>
<td>$W_{jt}L_{jt}$</td>
<td>$W_{jt}L_{jt}$</td>
</tr>
</tbody>
</table>

By way of establishing the notation, Table 1 shows value added (also called net output in the productivity analysis literature) at producer prices and their breakdown into factor costs for the four sectors. We define the IT sector ($j=2$) as consisting not only of the three IT industries in JN’s 47 industries (which are computers and peripherals, communications equipment, and electronic components), but also of computer software. The software sector is defined narrowly as computer programming and other software services: custom software, pre-packaged software, own-account software, games, and other software, excluding data processing and other related information services.\(^2\)

Real value added of the IT sector, $Y_{jt}$, is the translog index of value added of these four industries.\(^3\) Similarly, we define real value added of non-IT sector $Y_{jt}$ as the translog index

\(^2\)We estimate the output and inputs of computer software sector as follows. In the Japanese 2000 benchmark input-output table produced by Statistics Bureau, Ministry of Internal Affairs and Communications (MIC), production activity of software sector is not divided from information services (851201), although the commodity of software (8512011) is separated. We estimate production of the software sector using the activity in 851201. The own-account software is not included in the production of software (8512011) of the Japanese 2000 benchmark IO table and still is not capitalized in the Japanese National Accounts by Economic and Social Research Institute (ESRI), Cabinet Office. For output and inputs of own-account software, we use the estimates in Nomura (2004b).

\(^3\)In general, let $Y_{jt}$ and $P_{jt}$ be real value added and the associated price index of sector $j$ in period $t$. The translog quantity index $Y_t$ is defined as

$$\Delta \ln Y_t = \sum_j \beta_j \Delta \ln Y_{jt}$$
of value added of all the industries except the IT sector, household, and government sectors. The price indexes of value added at producer prices, \( P_{jt} \) \((j = 1, 2)\), are derived by dividing the aggregated nominal value added by real value added in each sector.

As in JN (Jorgenson and Nomura (2005)), the household sector produces rental service of owner-occupied housing and consumer durables. Those rental services are consumed by the household sector itself, by definition. Nominal production, \( P_{Ht}Y_{Ht} \), equals factor costs, which consist entirely of the user costs of owner-occupied housing and consumer durables. The government sector produces government service that is consumed by the government itself, by definition. The imputed nominal value of government services, \( P_{Gt}Y_{Gt} \), is defined as capital costs and the value of labor input, where capital costs include not only consumption of public capital but also total capital service cost of publicly owned capital. For both sectors, the imputed prices of value added \( P_{jt} \) \((j=H,G)\) are defined by dividing nominal value added by the quantities \( Y_{jt} \) \((j=H,G)\), which are defined in Equation (2.5) below.\(^4\)

\[ \Delta \ln Y_t \equiv \ln Y_t - \ln Y_{t-1} \] and \( \bar{v}_{jt} \) is the two-period average share of sector \( j \)'s nominal value added in total nominal value added.

\(^4\)In JN, the government and household sectors have intermediate inputs to produce their non-market services. In this paper, to simplify the production functions, we treat these intermediate inputs as government consumption and household consumption of non-IT good (\( i=1 \)) at the final demand, respectively, so that there is no intermediate inputs for these sectors.
Table 2: Value Added Shares

<table>
<thead>
<tr>
<th>Year</th>
<th>non-IT ( (v^Y_{1t}) )</th>
<th>IT ( (v^Y_{2t}) )</th>
<th>H ( (v^Y_{Ht}) )</th>
<th>G</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960</td>
<td>95.0</td>
<td>89.3</td>
<td>0.8 (0.8)</td>
<td>10.0</td>
<td>5.0</td>
</tr>
<tr>
<td>1973</td>
<td>94.9</td>
<td>87.5</td>
<td>1.1 (1.0)</td>
<td>11.4</td>
<td>5.1</td>
</tr>
<tr>
<td>1984</td>
<td>93.8</td>
<td>85.7</td>
<td>2.1 (1.6)</td>
<td>12.2</td>
<td>6.2</td>
</tr>
<tr>
<td>1990</td>
<td>94.1</td>
<td>84.7</td>
<td>2.7 (1.9)</td>
<td>12.5</td>
<td>5.9</td>
</tr>
<tr>
<td>1995</td>
<td>93.5</td>
<td>83.7</td>
<td>2.6 (1.8)</td>
<td>13.7</td>
<td>6.5</td>
</tr>
<tr>
<td>2000</td>
<td>93.2</td>
<td>82.0</td>
<td>3.3 (2.0)</td>
<td>14.7</td>
<td>6.8</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>∞</td>
<td>—</td>
<td>81.4</td>
<td>2.1</td>
<td>15.7</td>
<td>—</td>
</tr>
</tbody>
</table>

*Note: Shares in percents. Values in () represent shares of IT producing sector excluding software.*

Private GDP at factor costs is defined as \( P_{1t}Y_{1t} + P_{2t}Y_{2t} + P_{Ht}Y_{Ht} \) and total GDP at factor costs adds \( P_{Gt}Y_{Gt} \) to private GDP. Table 2 displays nominal value added shares of each sector in GDP at factor costs for selected years (for now, ignore the row for year ∞). As mentioned above, our GDP includes imputed service cost for capital that government and household own. Non-market services produced by government and household expand from 14.4 percent of the GDP in 1960 to 20.5 percent in 2000.

Various capital assets can be divided into two groups, depending on their sectoral origin. The non-IT capital or asset 1 is those assets produced in the non-IT sector, while the IT capital or asset 2 are produced in the IT sector. Since value added is at factor costs, it can be divided into payments to asset 1, asset 2, and labor. We use \( v^K_{ijt} \) for factor payment to asset \( i (i = 1, 2) \) in sector \( j (j = 1, 2, H, G) \) and \( v^L_{ijt} \) for labor cost in sector \( j \). Table 3 has factor cost shares \( (v^K_{1jt} + v^K_{2jt} + v^L_{ijt} = 100\%) \).

5In JN, the most detailed asset classification has 102 assets. Of those 102 assets, to reflect the definition of the IT sector mentioned above, the IT capital in this paper is composed of electronic computer and peripherals, wired communication equipment, wireless communication equipment, other communication equipment, custom software, pre-packaged software, and own-account software.
Table 3: Factor Cost Share

<table>
<thead>
<tr>
<th></th>
<th>non-IT</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$v^K_{1H}$</td>
<td>$v^K_{2H}$</td>
<td>$v^K_{1L}$</td>
<td>$v^K_{2L}$</td>
<td>$v^K_{1H}$</td>
<td>$v^K_{2H}$</td>
<td>$v^K_{1G}$</td>
</tr>
<tr>
<td>1960</td>
<td>43.7</td>
<td>0.4</td>
<td>55.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>41.2</td>
<td>1.4</td>
<td>57.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>34.8</td>
<td>1.8</td>
<td>63.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>36.6</td>
<td>3.2</td>
<td>60.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1995</td>
<td>31.6</td>
<td>3.2</td>
<td>65.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>29.9</td>
<td>4.3</td>
<td>65.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Shares in percents. $v^K_{ijt}$ and $v^K_{ijt}$ are the cost shares of capital and labor, respectively.

Finally, the rental rates, $r_{ijt}$ ($i = 1, 2; j = 1, 2, H, G$) in Table 1, are defined so that $P_i r_{ijt} K_{ijt}$ equals the capital cost of asset $i$. See below in this section for more details.

**Capital and Labor**

Each sector utilizes many different capital assets. At the most detailed level of asset classification, the real capital stock and capital services are identical. However, when assets are properly aggregated into broader classes, the two are not the same, as first pointed out by Jorgenson and Griliches (1967). The real capital stock is the simple sum (valued at some base year prices) of those assets that belong to the broader asset class in question, while capital services aggregated over those assets is an index (e.g., the translog index) constructed from the user costs and the real capital stocks for those assets.

For the 47 sectors, JN calculated the real capital stock and the translog index of capital services for non-IT and IT assets. Those quantities are aggregated into our four broader sectors ($j = 1, 2, H, G$) to obtain the real capital stock and the translog capital services index for the non-IT

---

6The user costs in JN incorporate the Japanese tax structure. The detailed formula is given in Nomura (2004a, ch.3), where he considers capital consumption allowance, income allowance and reserves, special depreciation, corporate income tax, business tax, property tax, acquisition taxes, debt/equity financing, and personal taxes on capital gain and dividend. Nomura (2004a, ch.3) measures effective tax rates and tax wedges, based on the endogenously estimated before-tax and after-tax rates of return. This estimate gives a so-called effective tax rate for capital income $\tau_{it}$ in our model.
and IT assets. For asset $i$ ($i = 1$ for non-IT or 2 for IT) in sector $j$ in period $t$, we use $K_{ijt}$ for the real capital stock and $K^*_ijt$ for the capital services index. By definition, the IT capital stock $K^*_2$ and IT capital services $K^*_2$ do not include land. We exclude land from non-IT capital; see below for our treatment of land. The ratio $Q^K_{ijt} ≡ K^*_ijt/K_{ijt}$, which converts real capital stock into capital services, is called the capital quality in the productivity literature. Going back to Table 1, the rental rate $r_{ijt}$ is defined to satisfy the relationship

$$P_{ijt}r_{ijt}K_{ijt} = \text{nominal value of capital services for asset } i \text{ in sector } j.$$  

As mentioned above, our definition of non-IT capital excludes land, but the rental rate $r_{1jt}$ of non-IT capital includes land rent.

We can do the same thing for labor. The labor equivalent of real capital stock is total hours worked. Labor input differs from hours worked because it is an index of labor inputs of various kinds, distinguished by worker characteristics such as education. JN calculated total hours worked and the translog index of labor input for the 47 industries. We can aggregate those quantities into our four broader sector classification ($j = 1, 2, H, G$). We use $L_{jt}$ for total hours worked in sector $j$ and $L^*_jt$ for the translog index of labor services. The labor quality $Q^{L}_jt (j=1,2,G)$ is $L^*_jt/L_{jt}$. Nominal hourly wage rate in sector $j$, $W_{jt}$, is defined to satisfy

$$W_{jt}L_{jt} = \text{nominal labor costs in sector } j.$$  

Allocation of the capital stock (excluding land) and total hours among sectors are reported in Table 4.

---

7JN use symbol $K$ for capital services and $Z$ for the real capital stock.

8JN utilizes the KEO Database, which classifies workers by sex, age (eleven classes), educational attainment (four classes for males, three classes for females), employment class (three types: employees, self-employed, and unpaid family workers), and industry (forty-three). See Kuroda, Shimpo, Nomura, Kobayashi (1997, ch.4) for more detail.
Table 4: Allocation of Capital and Labor

<table>
<thead>
<tr>
<th></th>
<th>non-IT capital stock: ( K_{1jt} )</th>
<th>IT capital stock: ( K_{2jt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>private breakdown</td>
<td>private breakdown</td>
</tr>
<tr>
<td></td>
<td>non-IT IT</td>
<td>non-IT IT H</td>
</tr>
<tr>
<td></td>
<td>private breakdown</td>
<td>private breakdown</td>
</tr>
<tr>
<td></td>
<td>non-IT IT H</td>
<td>non-IT IT H</td>
</tr>
<tr>
<td></td>
<td>hours worked: ( L_{jt} )</td>
<td>private breakdown</td>
</tr>
<tr>
<td></td>
<td></td>
<td>non-IT IT</td>
</tr>
<tr>
<td>1960</td>
<td>97.1</td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>99.1</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.9</td>
<td>2.9</td>
</tr>
<tr>
<td>1973</td>
<td>96.0</td>
<td>64.7</td>
</tr>
<tr>
<td></td>
<td>99.2</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
<td>4.0</td>
</tr>
<tr>
<td>1984</td>
<td>95.5</td>
<td>63.1</td>
</tr>
<tr>
<td></td>
<td>98.2</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>1.8</td>
<td>4.5</td>
</tr>
<tr>
<td>1990</td>
<td>95.7</td>
<td>62.9</td>
</tr>
<tr>
<td></td>
<td>97.5</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>4.3</td>
</tr>
<tr>
<td>1995</td>
<td>95.9</td>
<td>62.5</td>
</tr>
<tr>
<td></td>
<td>97.5</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>4.1</td>
</tr>
<tr>
<td>2000</td>
<td>96.3</td>
<td>62.0</td>
</tr>
<tr>
<td></td>
<td>97.2</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
<td></td>
</tr>
<tr>
<td>∞</td>
<td>96.3</td>
<td>51.5</td>
</tr>
<tr>
<td></td>
<td>97.2</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>2.8</td>
<td>7.8</td>
</tr>
<tr>
<td></td>
<td>3.7</td>
<td></td>
</tr>
</tbody>
</table>

Note: Shares in percents. \( K_{1jt} \) excludes land.

Productivity

To account for the role of land in the measurement of TFP (total factor productivity), we introduce a new variable, \( \varphi_{jt} \), which converts the translog capital services index without land, \( K^{*}_{1jt} \) into one with land for sector \( j \). For each sector, we can define two measures of TFP growth, one (the “pseudo” TFP growth) that takes neither the capital and labor quality nor land into account \( (\upsilon^{T}_{jt}) \) and the one (the “genuine” TFP growth) that does \( (\upsilon^{'T}_{jt}) \). The former is defined as

\[
\upsilon^{T}_{jt} = \Delta \ln Y_{jt} - \sum_{i=1,2} \bar{\upsilon}^{K}_{ijt} \Delta \ln K_{ijt} - \bar{\upsilon}^{L}_{jt} \Delta \ln L_{jt},
\]

where \( \bar{\upsilon}^{K}_{ijt} \) and \( \bar{\upsilon}^{L}_{jt} \) are the two-periods average cost shares of capital and labor in value added.

Clearly, the genuine TFP growth \( \upsilon^{'T}_{jt} \) is related to its pseudo version \( \upsilon^{T}_{jt} \) as

\[
\upsilon^{'T}_{jt} = \upsilon^{T}_{jt} + \sum_{i=1,2} \bar{\upsilon}^{K}_{ijt} \Delta \ln Q^{K}_{ijt} + \bar{\upsilon}^{L}_{jt} \Delta \ln Q^{L}_{jt} + \bar{\upsilon}^{K}_{1jt} \Delta \ln \varphi_{jt}.
\]

Quantities of services produced by the household and government sectors are defined as a translog index of factor inputs in sector \( j \) (\( j=H, G \)):

\[
\Delta \ln Y_{jt} = \sum_{i=1,2} \bar{\upsilon}^{K}_{ijt} \Delta \ln \left( Q^{K}_{ijt} K_{ijt} \right) + \bar{\upsilon}^{L}_{jt} \Delta \ln \left( Q^{L}_{jt} L_{jt} \right) + \bar{\upsilon}^{K}_{1jt} \Delta \varphi_{jt},
\]
where labor input in the household sector is zero: $L_{Ht} = 0$. The growth of the genuine TFP in these two sectors, which are engaged in non-market production, are zero by construction.

Table 5 reports two measures of TFP growth, $v^T_{jt}$ and $v^{T^*}_{jt}$, and aggregate TFP growth for the private sector. By construction, the genuine TFP growths for the household and government sectors are zero. In the other measure of TFP growth (the pseudo TFP growth), $v^T_{jt}$ $(j=H,G)$ are positive, thanks to the exclusion of quality change and land. Figure 1 displays the genuine TFP growth $v^{T^*}_{jt}$ for the non-IT producing and IT producing sectors. The rapid IT productivity growth is in stark contrast to the stagnation of the non-IT sector. The dotted line in the Figure is the TFP growth of the IT sector when software is not included. The inclusion of software, while raising the value-added share of the IT sector in the 1980s and after as shown in Table 2, substantially pulls down the TFP growth.

<table>
<thead>
<tr>
<th></th>
<th>pseudo TFP: $v^T_{jt}$</th>
<th>genuine TFP: $v^{T^*}_{jt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>by sector</td>
<td></td>
</tr>
<tr>
<td></td>
<td>non-IT</td>
<td>IT</td>
</tr>
<tr>
<td>1960-73</td>
<td>3.0</td>
<td>2.5</td>
</tr>
<tr>
<td>1973-84</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>1984-90</td>
<td>1.4</td>
<td>1.4</td>
</tr>
<tr>
<td>1990-95</td>
<td>-0.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>95-2000</td>
<td>0.2</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Note: Average annual growth rate in percents. Values in () represent TFP growth rates for IT producing sector excluding software. Land is excluded from non-IT capital in the definition of $v^T_{jt}$ and included in $v^{T^*}_{jt}$.

### Consumption and Investment

The components of domestic final demand are in Table 6. Domestic final demand consists of consumption and investment. Household consumes non-IT goods $P_{it}C_{it}$ and the own-produced rental services of consumer durables and owner-occupied housing, which equals
Figure 1: TFP Growth Rate: Non-IT and IT Producing Sectors

\( P_{iH}Y_{iH} \) in Table 1. \( C_{iH} \) includes intermediate inputs to own-produced household service \( Y_{iH} \). Similarly, \( C_{iG} \) includes intermediate inputs for government to produce government service. The output of the IT sector is not consumed.

<table>
<thead>
<tr>
<th>Table 6: Domestic Final Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
</tr>
<tr>
<td>( H )</td>
</tr>
<tr>
<td>non-IT goods</td>
</tr>
<tr>
<td>IT goods</td>
</tr>
<tr>
<td>Hou. service</td>
</tr>
<tr>
<td>Gov. service</td>
</tr>
</tbody>
</table>

Our estimate of consumption is based on the time-series of Input-Output tables in the KEO Database. To define the value at before-tax prices, we deducted net indirect tax from \( C_{iH} \). The consumption tax rate \( \tau_{ct} \) is calculated as the ratio of the total value of net indirect tax to \( P_{iH}C_{iH} \). Investments are calculated from the fixed capital formation matrix (FCFM) in Nomura (2004a).

Physical depreciation rates \( \delta_{ij} \) for asset \( i \) in sector \( j \) are computed from the depreciation rates for 95 produced assets and 16 consumer durables used in JN. They can differ across sectors and
over time because the asset composition within each sector varies. Nevertheless, the aggregated
depreciation rates reported in Table 7 are fairly uniform across sectors, except for the non-IT
depreciation rate for government. This is due to the inclusion of infrastructure in government
capital.

Table 7: Depreciation Rates

<table>
<thead>
<tr>
<th>Non-IT Capital: $\delta_{1t}$</th>
<th>IT Capital: $\delta_{2t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-IT</td>
<td>IT</td>
</tr>
<tr>
<td>1960</td>
<td>5.5</td>
</tr>
<tr>
<td>1973</td>
<td>6.8</td>
</tr>
<tr>
<td>1984</td>
<td>6.4</td>
</tr>
<tr>
<td>1990</td>
<td>6.8</td>
</tr>
<tr>
<td>1995</td>
<td>6.5</td>
</tr>
<tr>
<td>2000</td>
<td>6.4</td>
</tr>
</tbody>
</table>

Note: in percents.

3. Aggregation over Sectors and Assets

We have calculated, in Table 5, the aggregate TFP growth $v^f_t$, that takes into account the heterogeneity in the cost of capital between the two assets (non-IT and IT), and also the “genuine” TFP growth $v^g_t$ that takes into account the heterogeneity within each asset. In contrast, the growth accounting in the macro literature (see, e.g., Klenow and Rodriguez-Clare (2001) and Hayashi and Prescott (2002) (hereafter HP)) is so wedded to the one-sector paradigm that the aggregate capital input is simply defined as the value of nominal capital stock deflated by the output deflator. In this section, we apply this “macro” growth accounting to the multi-sector system described in the previous section. Our calculation excludes the government sector throughout, in order to be consistent with the model and the simulations to be shown later in the paper.

For real GDP (real aggregate value added) at factor costs, we use the translog index. That is, from data on $(P_j, Y_j)$ for $j = 1, 2, H$, we calculate aggregate real value added $Y_t$ as the translog
index over industry value added. The implicit GDP deflator is defined as the ratio of nominal aggregate value-added to $Y_t$.

For the capital stock, the macro growth accounting typically uses the GDP deflator to convert nominal into real. That is, the aggregate capital stock $K_t$ is defined as the ratio of the total nominal value of the capital stocks in sectors 1, 2, and H to the GDP deflator just defined above. So the capital-output ratio $K_t/Y_t$ equals the ratio of nominal capital stock to nominal output.

The black line in Figure 2 is real GDP per working-age population (the number of persons aged 20-69) thus calculated from our multi-sector system, detrended at 2% (which has been the long-run growth rate for the leader country (the U.S.) over the past century) and normalized to 100 for 1990. It shows that $Y_t$ grew much faster than 2% until 1991 but slower than 2% thereafter. The gray line in the figure is similarly detrended official real GDP per worker from the Japanese national accounts (on the SNA93 basis). There are a number of definitional differences between our GDP and the official SNA93 GDP: the government sector is included in the SNA93 GDP while our GDP doesn’t; our GDP is a (translog) chain index while the SNA93 is a fixed-weight Paache index; service flows from consumer durables are included in our GDP, and so forth. Compared to the official GDP, our measure shows slightly less growth in the 1980s and a slightly severer slump in the 1990s.

Figure 3 has the capital-output ratio, again from the same two sources. The black line is the ratio $K_t/Y_t$ constructed from the multi-sector system. It shows a 25% rise from 2.62 in 1990 to 3.28 in 2000. The official capital-output ratio, with the capital stock as well as nominal GDP from the SNA93, is the gray line. It is much lower and shows a much slower rise in the 1990s. The most important reason for the difference is that the depreciation rates in our multi-sector accounting system are lower.

The “macro” growth accounting as practiced in HP (Hayashi and Prescott (2002)) is based on

---

9We also calculated the Fisher chain index and found it to be virtually identical to the translog chain index.

10Currently, the official SNA93 chain index is available only since 1994.

11Another reason is that our measure of nominal capital stock, $P_{1t}K_{1t} + P_{2t}K_{2t}$ (where $K_{it} = \sum_{j=1,2,H} K_{ijt}$), does not equal the nominal capital stock valued at investment goods prices.
the following identity:

\[
\frac{Y_t}{N_t} = A_t^{1/(1-\theta)} \left( \frac{K_t}{Y_t} \right)^{\theta/(1-\theta)} \left( \frac{L_t}{N_t} \right),
\]

where \(N_t\) is the working-age population (persons aged 20-69) and \(A_t\) is the aggregate TFP defined to satisfy the aggregate Cobb-Douglas production function:

\[
Y_t = A_t K_t^\theta L_t^{1-\theta}.
\]

Our measure of aggregate labor \(L_t\) is aggregate hours worked in the three private sectors. If we feed \((Y_t, K_t, L_t)\) calculated from the multi-sector system as explained above to the above growth-accounting formula, the result is shown in Table 8 with \(\theta = 0.362\) (the capital share parameter used in HP). It is comforting to note that, despite the substantial differences in the definition, the overall picture is the same as in HP: both per capita output growth and the TFP growth slowed down to less than 1%, the capital-output ratio increased, and labor input declined in the 1990s. The only notable difference is that the growth in labor input for 1960-1990 is less in HP.

Total hours in HP is defined as employment times the average hours worked for the economy as a whole. The total hours in the multi-sector system, \(L_t\) here, is estimated from more detailed industry-level data.

<table>
<thead>
<tr>
<th>period</th>
<th>(\frac{Y_t}{N_t})</th>
<th>(A_t)</th>
<th>(A_t^{1/(1-\theta)})</th>
<th>(\frac{K_t}{Y_t})</th>
<th>(\left( \frac{K_t}{Y_t} \right)^{\theta/(1-\theta)})</th>
<th>(\frac{L_t}{N_t})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1960-73</td>
<td>6.8</td>
<td>4.2</td>
<td>6.7</td>
<td>-0.1</td>
<td>-0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>1973-84</td>
<td>2.5</td>
<td>1.1</td>
<td>1.8</td>
<td>2.2</td>
<td>1.2</td>
<td>-0.6</td>
</tr>
<tr>
<td>1984-90</td>
<td>3.4</td>
<td>1.9</td>
<td>3.0</td>
<td>0.4</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>1990-2000</td>
<td>0.5</td>
<td>0.4</td>
<td>0.6</td>
<td>2.3</td>
<td>1.3</td>
<td>-1.4</td>
</tr>
<tr>
<td>1990-95</td>
<td>0.1</td>
<td>0.1</td>
<td>0.2</td>
<td>3.3</td>
<td>1.9</td>
<td>-1.9</td>
</tr>
<tr>
<td>1995-2000</td>
<td>0.9</td>
<td>0.7</td>
<td>1.1</td>
<td>1.2</td>
<td>0.7</td>
<td>-0.8</td>
</tr>
</tbody>
</table>

Note: In annual percentages. \(N_t\) is the working-age population (number of persons aged 20-69). \(Y_t\) is chained real GDP, \(K_t\) is real capital stock, \(N_t\) is employment outside government, and \(A_t\) is aggregate TFP. See text for the precise definition of \(Y_t, K_t, L_t, A_t\). \(\theta = 0.362\).
4. The Two-Sector Growth Model with Consumer Durables

This section presents our theoretical model to be confronted with the data from the multi-sector accounting system. Main features of the model are the following.

- Consistent with the multi-sector accounting system, there are two market-provided goods (non-IT goods and IT goods) and two nonmarket goods (government services and household services). Government services do not enter the representative agent’s utility function.

- The two market goods (non-IT and IT goods) are internationally tradable. We assume that the country is small enough not to influence the relative price of IT goods in terms of non-IT goods. This means that the relative price is exogenous to the model.

- The non-IT good can be consumed or invested. If invested, it turns into a capital stock of good 1 (to be referred to as asset 1 or capital 1). The IT good can only be invested and will be referred to as asset 2 or capital 2.

- The model treats the government policy, which specifies the amounts of inputs to the production of government services as well as taxes, as exogenous. Production and consumption of the household good are endogenous in the model.

- Unlike in HP (Hayashi and Prescott (2002)), where the capital stock includes claims on the rest of the world, we separate external assets from domestic capital. However, the time path of external assets is treated as exogenous. This means that both net income from abroad and net exports are exogenous to the model.

- Unlike in HP, labor supply is exogenous. Furthermore, the sectoral allocation of total labor, too, is exogenous. We are forced to make this assumption because the market equilibrium is one of complete specialization under the observed relative price if labor is allowed to move freely between sectors.

We now turn to a more detailed description of the model.
Households

The stand-in household’s utility function is

\[
\sum_{t=0}^{\infty} \beta^t N_t u(c_{1t}, d_t), \quad \beta \in (0, 1),
\]  

(4.1)

where \(N_t\) is working-age population, \(c_{1t}\) is per-worker consumption of good 1 (the non-IT good), and \(d_t\) is the service flow (from consumer durables and owner-occupied housing) produced by the household. If \(K_i\) is the quantity of private capital stock \(i\) obtained from investment in good \(i\) \((i = 1, 2)\), the household sets aside \(\mu_i N_t d_t\) of it as capital input for producing \(N_t d_t\) units of household services. These capital input requirement coefficients \((\mu_{i1}, \mu_{21})\) depend on the rental rates of capital (see below). We allow two distorting taxes, the consumption tax (the tax rate: \(\tau_{ct}\)) and the tax on capital income (\(\tau_{it}\)). We will take labor supply as exogenous to the model, so the tax on labor is not distortionary and will be lumped into the lump-sum tax \(\tau_{lt}\). The household’s budget constraint is

\[
(1 + \tau_{ct})N_t c_{1t} + [K_{1t+1} - (1 - \delta_1)K_{1t}] + P_t[K_{2t+1} - (1 - \delta_2)K_{2t}] + (FA_{t+1} - FA_t)
\]

\[
\leq w_{1t} L_{1t} + w_{2t} L_{2t} + w_{3t} L_{3t} + [r_{1t}(K_{1t} - \mu_{11} N_t d_t) + P_t r_{2t}(K_{2t} - \mu_{21} N_t d_t)]
\]

\[
- \tau_{it}[(r_{1t} - \delta_1)(K_{1t} - \mu_{11} N_t d_t) + P_t (r_{2t} - \delta_2)(K_{2t} - \mu_{21} N_t d_t)] - \tau_{lt} + NI_t,
\]

where \(P_t\) is the relative price (the price of good 2 in terms of good 1), \(\delta_i\) is the depreciation rate of asset \(i\), \(w_{jt}\) is the wage rate measured in good 1 paid by sector \(j\), \(L_{jt}\) is labor supply to sector \(j\), \(r_{it}\) is the rental rate for asset \(i\) (so the rental price in terms of good 1 of asset 2 equals \(P_t r_{2t}\)), and \(NI_t\) is net income from abroad measured in good 1.\(^{12}\)

In terms of the notation of Tables 1 and 6, \(P_t = P_{2t}/P_{1t}, c_{1t} = C_{1ht}/N_t, d_t = Y_{1ht}/N_t, w_{jt} = W_{jt}/P_{1t}, \) and \(K_{jt+1} - (1 - \delta_i)K_{jt} = I_{jt+1} + I_{2jt} + I_{3ht} (i = 1, 2)\). In the data, the rental rate \(r_{ij}\) depends on sector \(j\) because of the asset heterogeneity within the broader asset classes of non-IT and IT assets, but in the model, which does not recognize this heterogeneity, the rental rate does not depend on \(j\).

\(^{12}\)This budget constraint implies that the household does not pay the consumption tax on purchases of non-IT and IT goods to be used for household production. To avoid this counterfactual assumption, we would have to treat those purchases separately from investments in the capital stock to be rented out to firms, but that will add another co-state variable to the system. Our numerical procedure for computing the perfect foresight equilibrium path cannot handle more than on co-state variable.
Let $\beta \Lambda_t^{-1}$ be the Lagrange multiplier. Being the reciprocal of the shadow price of the budget constraint, $\Lambda_t$ measures the wealth of the household. The FOCs (first-order conditions) for optimality are:

\[ u_c(c_t, d_t) = (1 + \tau) \Lambda_t^{-1}, \]

\[ u_d(c_t, d_t) = P_d \Lambda_t^{-1}, \]

\[ \beta [1 + (1 - \tau)(r_{1,t+1} - \delta_1)] = \frac{A_{t+1}}{A_t}, \]

\[ \beta [1 + (1 - \tau)(r_{2,t+1} - \delta_2)] = \frac{P_{t+1}}{P_t} \frac{A_{t+1}}{A_t}, \]

where

\[ P_d \equiv [(1 - \tau)\mu_1 + \tau \delta_1][\mu_1 + P_1(1 - \tau)\mu_2 + \tau \delta_2], \]

is the imputed price of household services in terms of sector 1 output.\(^{13}\) Equations (4.5) and (4.6) yield

(Euler equation) \[ \frac{A_{t+1}}{A_t} = \beta [1 + (1 - \tau)(r_{1,t+1} - \delta_1)], \]

(arbitrage) \[ \frac{P_{t+1}}{P_t} = \frac{1 + (1 - \tau)(r_{1,t+1} - \delta_1)}{1 + (1 - \tau)(r_{2,t+1} - \delta_2)} \]

The first of these two equations will be referred to as the Euler equation because it describes how the household wealth $\Lambda_t$ evolves over time. The second equation will be called the arbitrage condition governing the portfolio of the two assets $K_{1t}$ and $K_{2t}$.

We can solve (4.3) and (4.4) for $(c_t, d_t)$ as a function of the consumption tax rate $\tau$, the imputed price ($P_d$), and the shadow price $\Lambda_t$:

(Frisch demands) \[ c_t = c_1(\tau, P_d, \Lambda_t), \quad d_t = d(\tau, P_d, \Lambda_t). \]

This is a system of "Frisch demands". Use of these Frisch demand functions for $c_t$ and $d_t$ enforces the household FOCs (4.3) and (4.4) in the equilibrium conditions.

The input requirement coefficients ($\mu_{1t}, \mu_{2t}$) for producing household services can be made endogenous, as the solution to the cost minimizing problem:

\[ \min_{\mu_{1t}, \mu_{2t}} P_d \quad \text{s.t.} \quad \begin{array}{l}
F_d(\mu_{1t}, \mu_{2t}) = 1,
\end{array} \]

\(^{13}\)It is $P_{Ht}/P_{1t}$ in the notation of Section 2.
where $F_d$ is a linear homogeneous production function for household services. We write the solution as

$$
\mu_{1t} = \mu_1 \left( \frac{(1 - \tau_{1t})r_{1t} + \tau_{1t}\delta_1}{P_t[(1 - \tau_{1t})r_{2t} + \tau_{1t}\delta_1]} \right),
\mu_{2t} = \mu_2 \left( \frac{(1 - \tau_{1t})r_{1t} + \tau_{1t}\delta_1}{P_t[(1 - \tau_{1t})r_{2t} + \tau_{1t}\delta_1]} \right).
$$

(4.12)

**Firms**

The technology of the two market sectors are described by the constant-returns-to-scale production functions

$$
Y_{1t} = Y_1(\phi_{1t}Q_{11t}, K_{11t}, Q_{12t}, K_{21t}, A_{1t}^*),
$$

(4.13)

$$
Y_{2t} = Y_2(\phi_{2t}Q_{12t}, K_{12t}, Q_{22t}, K_{22t}, A_{2t}^*),
$$

(4.14)

where $K_{ijt}$ is the amount of privately-held asset $i$ rented by sector $j$ in date $t$ and $A_{jt}^*$ is the level of technology. As explained in Section 2, the multiplier $Q_{ijt}^*$ is the factor (called the capital quality index) that converts the capital stock into capital services, $Q_{jt}^*$ measures the quality of labor in sector $j$, and the factor $\phi_{jt}$ accounts for land. The FOCs are

$$
r_{1t} = \frac{\partial Y_{1t}}{\partial K_{11t}},
\quad p_{1t}r_{2t} = \frac{\partial Y_{1t}}{\partial K_{21t}},
\quad r_{1t} = P_t\frac{\partial Y_{2t}}{\partial K_{12t}},
\quad r_{2t} = \frac{\partial Y_{2t}}{\partial K_{22t}},
\quad w_{1t} = \frac{\partial Y_{1t}}{\partial L_{1t}},
\quad w_{2t} = P_t\frac{\partial Y_{2t}}{\partial L_{2t}}.
$$

(4.15)

**The Government**

The government collects taxes to finance government expenditure in goods and services (the sum of government consumption and investment) in two goods, $G_{1t}$ and $G_{2t}$, and payments to hire labor $L_{Gt}$ used for producing government services. The government’s production function is

$$
Y_{Gt} = F_G(\phi_{Gt}Q_{1Gt}^*, K_{1Gt}, Q_{2Gt}^*, K_{2Gt}, Q_{Gt}^*, L_{Gt}).
$$

(4.16)

The lump-sum tax $\tau_{ht}$ is adjusted to meet the government budget constraint. In terms of the notation of Table 6, $G_{1t} = C_{1Gt} + I_{1Gt}$ and $G_{2t} = I_{2Gt}$. 

17
Market Equilibrium

The market equilibrium conditions are:

\[
\begin{align*}
\text{(asset 1)} & \quad K_{11t} + K_{12t} + \mu_t N dt = K_{1t}, \\
\text{(asset 2)} & \quad K_{21t} + K_{22t} + \mu_t N dt = K_{2t}, \\
\text{(RC)} & \quad N_{1t} c_{1t} + [K_{1,t+1} - (1 - \delta_1) K_{11}] + P_t [K_{2,t+1} - (1 - \delta_2) K_{21}] \\
& \quad = (Y_{1t} - G_{1t}) + P_t (Y_{2t} - G_{2t}) - NX_t. 
\end{align*}
\]

Here, “RC” stands for resource constraint and \(NX_t\) is net exports in terms of good 1.

Equilibrium

We can now define competitive equilibrium. Take as given:

- a government policy \(\{K_{11t}, K_{22t}, L_{Gt}, G_{1t}, G_{2t}, \tau_c, \tau_k\}_{t=0}^{\infty}\)
- labor supply to each sector \(\{L_{1t}, L_{2t}\}_{t=0}^{\infty}\)
- external assets \(\{FA_t\}_{t=0}^{\infty}\) and net exports \(\{NX_t\}_{t=0}^{\infty}\)
- the relative price \(\{P_t\}_{t=0}^{\infty}\)
- the capital and labor quality and the land conversion factor \(\{Q^K_{ijt}, Q^L_{jt}, \phi_{jt}\}_{t=0}^{\infty} (j = 1, 2; i = 1, 2, H, G)\)
- the technology level \(\{A_{1t}^*, A_{2t}^*\}_{t=0}^{\infty}\).

A competitive equilibrium given an initial condition \((K_{10}, K_{20})\) is a sequence of factor prices, \(\{r_{1t}, r_{2t}, w_{1t}, w_{2t}\}_{t=0}^{\infty}\), the household wealth \(\{\Lambda_t\}_{t=0}^{\infty}\), and associated quantities, \(\{K_{1,t+1}, K_{2,t+1}, K_{11t}, K_{12t}, K_{21t}, K_{22t}\}_{t=0}^{\infty}\), such that the Euler equation (4.8), the arbitrage condition (4.9), the firm’s FOCs (4.15), the market equilibrium conditions ((4.17)-(4.19)) are satisfied, where \((Y_{1t}, Y_{2t})\) in those conditions are given by (4.13) and (4.14).

In this definition, the government budget constraint is not an equilibrium condition, because the lump-sum tax \(\tau_{lt}\) is assumed to meet the constraint. The household budget constraint is not included, because it is implied by the government budget constraint, the factor exhaustion condition that value added equals factor payments (an implication of the linear homogeneity
of the production function and the marginal productivity conditions), the market equilibrium conditions, and the identity that the increase in external assets, \( FA_{t+1} - FA_t \), equals net exports \( NX_t \) plus net income from abroad \( NI_t \).

**Implications of The Cobb-Douglas Technology**

In what follows, we assume the Cobb-Douglas form for the production functions. So (4.13) and (4.14) can be written as

\[
Y_{1t} = A'_{1t} \left( \varphi_{1t} Q_{11t}^K K_{11t} \right)^{\theta_{11}} \left( Q_{21t}^K K_{21t} \right)^{\theta_{21}} \left( Q_{12t}^L L_{12t} \right)^{1-\theta_{11}-\theta_{21}} = A_{1t} K_{11t}^{\theta_{11}} K_{21t}^{\theta_{21}} L_{12t}^{1-\theta_{11}-\theta_{21}},
\]

\[
Y_{2t} = A'_{2t} \left( \varphi_{2t} Q_{12t}^K K_{12t} \right)^{\theta_{12}} \left( Q_{22t}^K K_{22t} \right)^{\theta_{22}} \left( Q_{12t}^L L_{22t} \right)^{1-\theta_{12}-\theta_{22}} = A_{2t} K_{12t}^{\theta_{12}} K_{22t}^{\theta_{22}} L_{22t}^{1-\theta_{12}-\theta_{22}},
\]

where

\[ A_{j} \equiv A'_{j} \left( \varphi_{j} Q_{1j}^K \right)^{\theta_{1j}} \left( Q_{2j}^K \right)^{\theta_{2j}} \left( Q_{j}^L \right)^{1-\theta_{1j}-\theta_{2j}}, \quad j = 1, 2. \]

This shows that, for the Cobb-Douglas technology, the production function can be defined for the capital stocks, with an appropriate re-definition of the TFP term. The growth rate of \( A'_{j} \) is the “genuine” TFP growth rate \( \bar{v}_{j} \) of Section 2. The above expression of the production function makes it clear that what matters for equilibrium is the “pseudo” TFP growth rate, \( \bar{v}^T_{j} \), which equals the growth rate of \( A_{j} \).

We will also assume the Cobb-Douglas form for the household unit production function \( F_d \) in (4.11):

\[ F_d(\mu_{1t}, \mu_{2t}) = \mu_{1t}^{\gamma} \mu_{2t}^{1-\gamma}. \]

We also assume that the utility function \( u(c_{1i}, d_{i}) \) is linear logarithmic:

\[ u(c_{1i}, d_{i}) = \mu \log(c_{1i}) + (1 - \mu) \log(d_{i}). \]

With these functional-form assumptions for household, the Frisch demands (4.10) and the demand for capital inputs for household production (4.12) become

\[ c_{1t} = \frac{\mu \Lambda_{t}}{1 + \tau_{ct}}, \quad d_{t} = \frac{(1 - \mu) \Lambda_{t}}{P_{dt}}, \]

\[ P_{dt} = \gamma^{-\gamma} (1 - \gamma)^{(1-\gamma) \frac{1}{(1 - \tau_{ct}) \mu_{1t}} \left[ \frac{1}{(1 - \tau_{ct}) \mu_{2t} + \tau_{ct} \delta_{1t}} \right]^{1-\gamma}, \]

\[ \mu_{1t} N_{1t} d_{t} = \frac{\gamma}{1 - \tau_{ct} \mu_{1t} + \tau_{ct} \delta_{1t}} N_{1t} (1 - \mu) \Lambda_{t}, \quad \mu_{2t} N_{2t} d_{t} = \frac{1 - \gamma}{P_{1t} \left[ (1 - \tau_{ct}) \mu_{2t} + \tau_{ct} \delta_{2t} \right]} N_{1t} (1 - \mu) \Lambda_{t}. \]
Detrending

With the Cobb-Douglas technology and the linear-logarithmic preferences, it is possible to transform the system so that no variables in the system depend on the level of technology. To this end, define two trends:

\[
X_{it} \equiv A_{1i}^{\alpha_1} A_{2i}^{\alpha_2} N_t, \quad X_{it} \equiv A_{1i}^{\alpha_1} A_{2i}^{\alpha_2} N_t \quad \text{with} \quad \psi \equiv 1 - \theta_{11} - \theta_{22} + \theta_{12} \theta_{21}. \tag{4.28}
\]

Define also lower-case letters as ratios to these trends:

\[
k_{it} \equiv \frac{K_{it}}{X_{it}} \quad (i = 1, 2), \quad k_{ijt} \equiv \frac{K_{ijt}}{X_{it}} \quad (i, j = 1, 2), \quad \ell_{jt} \equiv \frac{L_{jt}}{N_t} \quad (j = 1, 2), \quad y_{jt} \equiv \frac{Y_{jt}}{X_{jt}} \quad (j = 1, 2),
\]

\[
p_t \equiv \frac{P_t}{(\frac{X_{kt}}{N_t})}, \quad \lambda_t \equiv \frac{\Lambda_t}{(\frac{X_{kt}}{N_t})}. \tag{4.29}
\]

A very tedious calculation shows that the equilibrium conditions in terms of these detrended variables can be reduced to the following set of equations.

**(Euler)**

\[
\frac{\lambda_{t+1} \left( \frac{X_{1t+1}}{X_{it}} \right)}{\lambda_t \left( \frac{X_{kt}}{N_t} \right)} = \beta \left[ 1 + (1 - \tau_{k,t+1})(r_{1,t+1} - \delta_t) \right], \tag{4.30}
\]

**(arbitrage)**

\[
p_{t+1} \left( \frac{X_{1t+1}}{X_{it}} \right) = \frac{1 + (1 - \tau_{k,t+1})(r_{1,t+1} - \delta_t)}{1 + (1 - \tau_{k,t+1})(r_{2,t+1} - \delta_t)}, \tag{4.31}
\]

**(production FOCs)***

\[
r_{1t} = \theta_{11} \frac{y_{1t}}{k_{11t}}, \quad p_t r_{2t} = \theta_{21} \frac{y_{1t}}{k_{21t}}; \quad r_{1t} = \theta_{12} p_t \frac{y_{2t}}{k_{12t}}, \quad r_{2t} = \theta_{22} \frac{y_{2t}}{k_{22t}},
\]

with\[
y_{1t} = k^{\beta_{11}}_{11t} k^{\beta_{12}}_{21t} r_{1t}^{\theta_{11} - \theta_{12}} (1 - \mu) \lambda_t = k_{11t}, \tag{4.32}
\]

\[(\text{asset 1}) \quad k_{11t} + k_{12t} + \frac{1 - \gamma}{p_t (1 - \tau_{k,t}) r_{2t} + \tau_{k,t} \delta_t} (1 - \mu) \lambda_t = k_{11t}, \tag{4.33}
\]

\[(\text{asset 2}) \quad k_{21t} + k_{22t} + \frac{1 - \gamma}{p_t (1 - \tau_{k,t}) r_{2t} + \tau_{k,t} \delta_t} (1 - \mu) \lambda_t = k_{22t}, \tag{4.34}
\]

\[(z \text{ defined}) \quad z_{t+1} \equiv \frac{1}{k_{1,t+1}} + \frac{p_t}{(\frac{X_{1t+1}}{X_{2t+1}})} k_{2,t+1}, \tag{4.35}
\]

**(RC)**

\[
\frac{X_{1t+1}}{X_{it}} z_{t+1} = (1 - \psi_{1t}) y_{1t} + p_t (1 - \psi_{2t}) y_{2t} - \frac{\mu}{1 + \tau_{k,t}} \lambda_t + (1 - \delta_t) k_{1t} + (1 - \delta_t) p_t k_{2t} - \nu_t (y_{1t} + p_t y_{2t}). \tag{4.36}
\]

Here, \((\psi_{1t}, \psi_{2t})\) is the government share of output for each good and \(\nu_t\) is net exports-to-gdp ratio:

\[
\psi_{it} \equiv \frac{G_{it}}{Y_{it}} \quad (i = 1, 2) \quad \text{and} \quad \nu_t = N X_t / (Y_{1t} + P_t Y_{2t}). \tag{4.37}
\]

From this set of equations, we can define a first-order dynamical system \((k_{1t}, k_{2t}, \lambda_t)\), namely a mapping from \((k_{1t}, k_{2t}, \lambda_t)\) to \((k_{1,t+1}, k_{2,t+1}, \lambda_{t+1})\), in the following fashion.

\[
20
\]
i) Given \((k_{1t}, k_{2t}, \lambda_1)\), use eight equations, (4.32)-(4.34) to solve for eight unknowns \((r_{1t}, r_{2t}, k_{11t}, k_{21t}, k_{12t}, k_{22t}, y_{1t}, y_{2t})\). This step also gives \((r_{1i}, r_{2i})\) as functions of \((k_{1i}, k_{2i}, \lambda_i)\). Write them as:

\[ r_{ii} = r_i(k_{1i}, k_{2i}, \lambda_i) \text{ (i = 1, 2)} \]

ii) Given \((k_{1t}, k_{2t}, \lambda_1, y_{1t}, y_{2t})\), use (4.36) to calculate \(z_{t+1}\).

iii) Substitute \(r_{i,t+1} = r_i(k_{1,i+1}, k_{2,i+1}, \lambda_{i+1})\) into (4.30) and (4.31). Then given the value of \(z_{t+1}\) obtained in the previous step, three equations — (4.30), (4.31), and (4.35) — can be solved for \((k_{1,t+1}, k_{2,t+1}, \lambda_{t+1})\).

**The Steady State**

This dynamical system becomes autonomous (i.e., the mapping from \((k_{1t}, k_{2t}, \lambda_t)\) to \((k_{1,t+1}, k_{2,t+1}, \lambda_{t+1})\) is stationary or time-invariant) if the forcing or exogenous variables — \(\ell_1t, \ell_2t, X_{1t+1}/X_{1t}, X_{2,t+1}/X_{2,t}, N_{1t+1}/N_{1t}, \tau_{zt}, \tau_{zt}, \psi_{zt}, \psi_{zt}, p_t\), and \(v_t\) — are constant over time. The steady state or the equilibrium of this autonomous system can be determined uniquely from the above equations by dropping the time subscript. Dropping the time subscript in (4.30) pins down \(r_1\), the steady-state value of \(r_1\). Given \(r_1\), use the steady-state version of (4.31) to obtain \(r_2\). Given \(r_1\) and \(r_2\), use (4.32) to calculate \((k_{11}, k_{21}, k_{12}, k_{22}, y_1, y_2)\). Use the rest of the equations, (4.33)-(4.36) to pin down \((k_1, k_2, z, \lambda)\).

It is straightforward to show that, in the steady state,

\[ Y_{1t} \propto X_{1t}, \ Y_{2t} \propto X_{2t}, \ Y_{1Ht} \propto X_{1Ht}^{1-\gamma}, \ P_1 (\equiv P_{2H} \equiv P_{2H}) \propto \frac{X_{1H}}{X_{2H}}, \ P_{ilt} (\equiv P_{ilt}) \propto \left(\frac{X_{1H}}{X_{2H}}\right)^{1-\gamma}. \]  (4.38)

Therefore, the sectoral nominal value-added shares in private GDP

\[ v'^{\gamma}_p = \frac{P_{p} Y_p}{P_{1t} Y_{1t} + P_{2t} Y_{2t} + P_{1Ht} Y_{1Ht}} \text{, } j = 1, 2, H, \]  (4.39)

are constant in the steady state. If, as is the case in the calibrated model below, \(X_{2t}\) grows much faster than \(X_{1t}\) thanks to the rapid IT productivity growth, the relative price of IT goods declines rapidly and the growth of relative output level \(Y_{2t}/Y_{1t}\) is as rapid as the decline in the relative price.

Also in the steady state, for each asset \(i\) \((i = 1, 2)\), the sectoral allocation of capital is constant because the trend in \(K_{ijt}\) does not depend on \(j\).
5. Calibration and Results

The detrended dynamical system described in the previous section has a set of parameters and exogenous variables. We specify them and solve the system from 1990 onwards. Given the solution of the detrended system, we can back out the equilibrium of the model by multiplying the solution by the time trends $X_1$ and $X_2$.

**Calibration**

The detrended dynamical system has nine parameters ($\beta, \mu, \gamma, \delta_1, \delta_2, \theta_{11}, \theta_{21}, \theta_{12}, \theta_{22}$). These parameters are calibrated as follows.

$\theta_{ij} (i, j = 1, 2)$: These are capital shares, whose values for selected years are reported in Table 3 for the non-IT and IT sectors. Given that $\beta$ is estimated from the 1984-1989 data, we should use the 1984-1999 averages. However, as seen from the Table, the factor shares of IT assets tend to increase in the 1990s. For this reason, we use the 1990-2000 averages.

$\gamma$: It is asset 1’s share in household production. Its values for selected years are reported in Table 3 for the household sector. We use the 1990-2000 average.

$\mu$: From the parameterized Frisch demands (4.25), $\mu$ equals $(1 + \tau_c)P_1Y_{hit}/((1 + \tau_c)P_1C_{1hit} + P_{1hit}C_{1hit})$. We use the 1990-2000 average; the 1984-89 average is similar.

$\delta_i (i = 1, 2)$: The depreciation rate for each asset differs slightly across sectors in data, as shown in Table 7. For each asset and for each year, we take the weighted average of the depreciation rates over three sectors, $j = 1, 2, H$, with the capital stocks of that asset in three sectors as weights. We then use their 1990-2000 averages for $\delta_i$ ($i = 1, 2$). Again, their 1984-89 averages are similar.

$\beta$: From the Euler equation (4.8), the parameterized Frisch demands (4.25), and the marginal productivity condition for asset 1 in sector 1 under the Cobb-Douglas production function, we can derive

$$\frac{\Lambda_{t+1}}{\Lambda_t} = \beta[(1 - \tau_{k,t+1})(\theta_{11} \frac{Y_{1hit}}{K_{1hit}} - \delta_1)] \text{ with } \Lambda_t = \frac{(1 + \tau_c)P_{1hit}C_{1hit} + P_{1hit}Y_{1hit}}{P_{1hit}N_t}.$$  

(5.1)
Following Hayashi and Prescott (2002), we take the sample average over 1984-1989 of both sides of this Euler equation and then solve for $\beta$. In taking the sample averages, $\theta_{11}$ and $\delta_1$ are the year-dependent values, not the calibrated values of them.

There are eleven exogenous variables in the detrended system. Their values from 1990 to 2000 are set to their actual values. Their paths beyond 2000 are projected into the future as follows.

Most importantly, the share of total hours allocated to sectors 1 and 2, $\ell_1t$ and $\ell_2t$, are set at their 2000 values, so the allocation of labor remains the same as in year 2000 shown in Table 4. We are assuming no emigration from the non-IT sector to the IT sector. This may be too extreme. We will also examine the case with mass emigration of labor into the IT sector later.

$N_t$ (working-age population): We assume zero growth, so $N_t = N(2000)$ for $t = 2001, 2002, ...$.

$X_{1t}$ and $X_{2t}$: They are the time trends defined in (4.28) and depend on the “pseudo” TFP levels, $A_{1t}$ and $A_{2t}$ defined in (4.22). For the non-IT sector ($j = 1$), we assume no productivity growth, so $A_{1t} = A_{1,2000}$ for $t = 2001, 2002, ...$. For the IT sector, we set the growth rate of $A_{2t}$ after 2000 to the average growth rate in 1995-2000. As Dale Jorgenson forcefully argues in his recent writings (see, e.g., Jorgenson and Motohashi (2005), Jorgenson and Nomura (2005)), the TFP growth in the IT sector has accelerated after 1995. We are thus assuming that the enhanced IT growth will continue into the future.

$\tau_{kt}$ (tax rate on capital): Its conceptually appropriate definition is the effective tax rate on income from capital, which includes taxes on capital at both the corporate and personal levels and incorporates various other features of the tax code such as accelerated depreciation, tax-free reserves. As mentioned in footnote 6, the tax rate was calculated in Nomura (2004a). The value beyond 2000 is set equal to its 2000 value of 61.8%. It is substantially higher than 48% used in Hayashi and Prescott (2002).

$\tau_{ct}$ (consumption tax rate): Its measurement is in Section 2. The value beyond 2000 is its 2000 value (9.8%).
ψ_{it} (government share of good i): The values beyond 2000 are the 2000 values. The tax rate is higher than the statutory rate of 5% because the tax base is assumed to be \( P_i C_{it} \).

\( p_t \) (the relative price): The value beyond 2000 is set at its 2000 value.

\( \nu_t \) (net exports/GDP ratio): The value beyond 2000 is set as: \( \nu_t = 0.95^{t-2000} \times \nu_{2000} \) for \( t = 2001, 2002, \ldots \). Thus, the trade balance is assumed to gradually decline to zero.

The calibrated parameter values and projected growth rates are shown in Table 9.

<table>
<thead>
<tr>
<th>Table 9: Calibrated Parameter Values and Projected Growth Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_{ij} ) (asset i's share in sector j)</td>
</tr>
<tr>
<td>( \gamma ) (asset 1's share in household production)</td>
</tr>
<tr>
<td>( \mu ) (share of non-IT goods in consumption)</td>
</tr>
<tr>
<td>( \delta_i ) (depreciation rate for asset i)</td>
</tr>
<tr>
<td>( \beta ) (discounting factor)</td>
</tr>
<tr>
<td>( \tau_{kt} ) (capital income tax rate) for ( t &gt; 2000 )</td>
</tr>
<tr>
<td>( \tau_{ct} ) (consumption tax rate) for ( t &gt; 2000 )</td>
</tr>
<tr>
<td>growth rate of ( N_t ) (working-age population) for ( t &gt; 2000 )</td>
</tr>
<tr>
<td>growth rate of ( A_{1t} ) (pseudo TFP in non-IT sector) for ( t &gt; 2000 )</td>
</tr>
<tr>
<td>growth rate of ( A_{2t} ) (pseudo TFP in IT sector) for ( t &gt; 2000 )</td>
</tr>
<tr>
<td>implied growth rate of ( X_{1t} ) for ( t &gt; 2000 )</td>
</tr>
<tr>
<td>implied growth rate of ( X_{2t} ) for ( t &gt; 2000 )</td>
</tr>
</tbody>
</table>

Note: The pseudo TFPs, \( A_{1t} \) and \( A_{2t} \), are defined in (4.22). The two trends, \( X_{1t} \) and \( X_{2t} \), are defined in (4.28).

**Results**

For the calibrated parameter values and the projected exogenous variables described in the previous section, we first solved for the unique steady state for the detrended dynamical system in \((k_{1t}, k_{2t}, \lambda_t)\) and the implied steady-state values for \( y_{1t}, y_{2t}, p_t, c_{it}, d_t, r_{1t}, r_{2t} \), and so forth are calculated. From this, we can determine the steady-state values of the sectoral value added shares in private GDP, \( \psi_{jt}^{\gamma} \) (\( j = 1, 2, G \)), defined in (4.39). They are reported in the last row of Table 2 (the row labeled “\( \infty \)”). The steady-state allocation of non-IT capital stock \( K_{1t} \) and IT capital stock \( K_{2t} \) between the three private sectors is shown in the last row of Table 4.
From the steady-state value added shares, we can easily calculate the steady-state growth rate of real GDP defined as the translog chain index. If \( v_Y \) is the steady-state nominal value-added share of sector \( j (=1,2,H) \), we can easily see from (4.38) that the steady-state real GDP growth rate is given by

\[
\sum_{j=1,2} v_Y \times (\text{long-run growth rate of } X_{jt}) + v_H \times (\text{long-run growth rate of } X_{lt}^{1-\gamma})
\]

\[
= (v_Y + \gamma v_H) \times (\text{long-run growth rate of } X_{1t}) + (v_Y + (1 - \gamma) v_H) \times (\text{long-run growth rate of } X_{2t})
\]

From Table 2, we have: \( v_Y = 81.4\% \), \( v_Y = 2.1\% \), \( v_H = 15.7\% \). From Table 9, we have: \( \gamma = 0.985 \), the long-run growth rates of \( X_{1t} \) and \( X_{2t} \) are 0.7\% and 14.5\%. So the steady-state real GDP growth rate (which is also the per-worker growth rate because the working-age population is assumed to remain constant) is 1.0\%.

To see if this pessimistic growth prediction is due to the assumption that the labor allocation remains the same as in year 2000 (which, as shown in Table 4, is 97.2\% in non-IT and 2.8\% in IT), we also calculated the steady state under the assumption that the IT sector’s labor share is much larger, 12.7\% rather than 2.8\%. The value added shares are now \( v_Y = 70.4\% \), \( v_Y = 13.8\% \), and \( v_H = 15.8\% \). The implied steady-state GDP growth rate is 2.0\%. Table 10 summarizes these findings.

<table>
<thead>
<tr>
<th>Allocation of Hours Worked</th>
<th>Sectoral Value-added Shares</th>
<th>Real GDP growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>97.2</td>
<td>2.8</td>
<td>81.1</td>
</tr>
<tr>
<td>87.3</td>
<td>12.7</td>
<td>70.4</td>
</tr>
<tr>
<td>81.1</td>
<td>3.2</td>
<td>15.7</td>
</tr>
<tr>
<td>13.8</td>
<td>15.8</td>
<td>1.0</td>
</tr>
<tr>
<td>15.8</td>
<td></td>
<td>2.0</td>
</tr>
</tbody>
</table>

Note: Shares and growth rates in percents. In both scenarios, 96.3\% of total hours is in the private sector.

We have considered two scenarios about the steady-state allocation of labor between non-IT and IT sectors (one in which the allocation between non-IT and IT remains (97.2\%, 2.8\%) and the other with (87.3\%, 12.7\%)). To examine transitional dynamics, we solved the detrended
dynamical system in \((k_{1t}, k_{2t}, \lambda_{t})\) from year 1990 on, with the initial condition that the initial values of the detrended capital stocks are their actual 1990 values. For the second scenario, where the steady-state labor allocation is different from that for year 2000, we need to specify the path \(\{\ell_{1t}, \ell_{2t}\}_{t=2001}^{\infty}\) from their 2000 value to the steady-state values implied by the labor allocation \((87.3\%, 12.7\%)\). We assume that every year 10% of the gap is filled.

Under either scenario, the shooting algorithm finds that there is a unique initial value for the co-state, \(\lambda_{1990}\), such that the system converges to the steady state. The transition paths under the two scenarios are graphed in Figure 4 for the detrended real GDP per worker and in Figure 5 for the “macro” capital-output ratio. Consistent with the different steady-state growth rates under the two scenarios, the detrended path of real GDP diverges eventually, as shown in Figure 4, but the difference is not apparent until around 2006. For either scenario, the model’s ability track GDP is not impressive. The model does far better for the capital-output ratio, as seen in Figure 5.

6. Conclusion

The answer to the question posed in the title is, depends on labor mobility.
References


