Arms Race and Economic Growth

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Abstract
This paper investigates dynamic implications of an arms race between two rival countries. By incorporating defense spending into the conventional overlapping generations growth model, this paper explores long-run consequences for national security and economic growth. The security spending - GDP ratio increases with economic growth if defense technology has a fixed benefit. Although the steady-state defense spending is too much in terms of the static efficiency (or compared with private consumption), it may be too little if private saving is too little in terms of the dynamic efficiency. We also explore the unstable nature of an arms race when defense technology needs a fixed cost in the cases of ‘open war’ and ‘closed war’ or it is efficient and the initial capital stock is low in the case of ‘open war’. In such cases both countries could not grow in the long run due to the arms race effect.

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1. INTRODUCTION

Although the Cold War ended, the interest in transnational conflicts has grown in importance. As Sandler and Hartley (2001) explain, the superpower confrontation has given way to small, vicious wars driven by territorial disputes, international power struggles, resource claims, and ethnic conflicts. Recently, Murdoch and Sandler (2002) investigate empirically long-run and short-run effects of civil wars on income per-capita growth in the host country and its neighbors. They find evidence of significant collateral damage on economic growth in neighboring nations.

It is important to investigate the long-run consequence of an arms race analytically since international conflicts may well be related to arms race behavior. Some of the previous papers investigated the welfare effects of arms races by including adversaries’ optimal reactions explicitly [see Bruce (1990) and Ihori (2000)]. Others developed dynamic property of arms race [see Richardson (1960), Brito (1972), Intriligator (1975), and Sandler and Hartley (1985) among others]. But there are few papers that investigate the relation between arms race and economic growth. These studies do not give any clear indication about the long-run consequence of defense spending and capital accumulation or how defense spending should be in relation to economic growth. By incorporating defense spending into the standard overlapping generations growth model of two competing countries, we investigate theoretically first, the normative implication of defense spending and then the dynamic relation between an arms race and economic growth. It is useful to consider two cases with respect to the benefit of defense spending, ‘open war’ and ‘closed war’ models, separately. In the open war model competing countries play a non-zero sum game, while in the closed war model they play a zero-sum game.

As to the normative issue, it is true that defense spending-GDP ratio at the non-cooperative Nash equilibrium is too much due to the arms-race effect. Namely, each country disregards a negative externality of defense
spending which spills over into the rival country in choosing its own defense spending [see, among others, McGuire (1974,1990)]. This is true for the closed war model even if we allow for over accumulation of capital. However, in the case of open war, the steady-state level of defense spending could be under-provided on the long-run growth path if the arms race effect is dominated by the growth effect. In a situation where private savings are too little in the long run, lifetime income becomes too little as well in the steady state. This negative growth effect would depress provision of defense spending. We show that if the negative growth effect dominates the positive arms-race effect, defense spending may be too little in terms of the dynamic efficiency (or as the steady state level). In such a seemingly paradoxical case it is desirable to promote both capital accumulation and defense spending.

In a symmetry world both countries normally grow at the same time. As to the nature of growth, it is shown that physical capital and defense spending normally grow at the same time. We also show that the national security spending-GDP ratio increases with economic growth if defense technology has a fixed benefit, which is consistent with the exploitation hypothesis. Economic growth in one country would have a negative impact on economic growth in another country due to the arms race effect. Moreover, we also show that arms race could produce an unstable outcome when defense technology has a fixed cost in the cases of open war and closed war or it is very efficient and the initial capital stock is low in the case of open war. In these cases an arms race would not normally be compatible with world-wide economic growth.

Section 2 presents the analytical model of defense spending with overlapping generations in two cases: open war and closed war models. Section 3 investigates long-run properties of the open war model and compares them with the first best solution. Section 4 explores the long-run nature of the arms-race behavior on economic growth when defense technology is used in the closed war model. Finally, section 5 concludes the
paper.

2. MODEL
2.1 Open War

We develop a standard model of two-period overlapping generations of two competing identical countries $\alpha$ and $\beta$. For simplicity, we do not consider any economic interaction between two countries. Either goods or capital cannot move between the two countries.

An agent $i$ of generation $t$ born at time $t$, considers itself young in period $t$, old in period $t+1$, and dies at time $t+2$. When young an agent of generation $t$ supplies one unit of labor inelastically and receives wages $w_t$ out of which the agent provides defense spending $g_{t+1}$ (if $g_{t+1} > 0$), and private spending $s_{t+1}$. These expenditures are used for obtaining a capital stock at the beginning of the next period, $K_{t+1}$ by doing a ‘war’. An agent receives capital income $(1+r_{t+1})K_{t+1}$ when old, which the agent then spends entirely on consumption $c_{t+1}$. We do not incorporate consumption by the younger generation for simplicity. There are no bequests. $r_t$ is the rate of interest in period $t$. There is no population growth and each generation has $n$ identical individuals. $n$ is normalized to 1.

Let us first formulate optimizing behavior of country $\alpha$ in the open war model. A member $i$ of country $\alpha$ faces the following budget constraints:

$$g_{t+1} + s_{t+1} = w_t,$$  \hspace{1cm} (1-1)

$$c_{t+1} = (1 + r_{t+1})K_{t+1}.$$  \hspace{1cm} (1-2)

In the case of ‘open war’, two competing countries play a non-zero sum game, so that both countries could gain positive profits in an open space or ‘colony’. Namely, capital stock available in the next period $K_{t+1}$ could be larger than private spending, $S_{t+1}$. Capital stock at the beginning of the next period depends positively on private spending $s$ and the security index, $G^\theta$, which increases with national security $G$ and $\theta (>0)$ is an exogenous parameter. $\theta$
reflects productivity of national security or defense spending. We assume for simplicity the following functional relation:

$$K_{t+1} = s_{t+1} G_{t+1}^0.$$  

(2)

National security of each country is formulated respectively as follows:

$$G_t^\alpha = g_t^\alpha - \varepsilon g_t^\beta + A,$$  

(3-1)

$$G_t^\beta = g_t^\beta - \varepsilon g_t^\alpha + B,$$  

(3-2)

where superscript $\alpha$ (or $\beta$) means country $\alpha$ (or $\beta$) and $\varepsilon$ denotes a degree of negative spillover effect from the rival country’s defense spending ($0 < \varepsilon \leq 1$). $A$ (or $B$) means the initial given level of national security for country $\alpha$ (or $\beta$) if it is positive. If it is negative ($A, B < 0$), defense technology needs a fixed cost. We only consider the case where $G > 0$. We will drop superscript $\alpha$ (or $\beta$) unless necessary.

From (1)(2) and (3-1), consumption of country $\alpha$ is given by:

$$c_{t+1}^\alpha = (1 + r_{t+1})(w_t^\alpha - g_t^\alpha)(g_t^\alpha - \varepsilon g_t^\beta + A)^0.$$  

(4)

As in the static model of an arms race, we will exclude cooperative behavior between the two countries and will explore the outcome of non-cooperative Nash behavior.

Let us then formulate the aggregate production function in each country. The firms are perfectly competitive profit maximizers who produce output using the production function:

$$Y_t = F(K_t, n_t) = K_t^\gamma.$$  

(5)

$F( )$ exhibits constant returns to scale. For simplicity we assume the Cobb-Douglas technology ($0 < \gamma < 1$). As for the standard first-order conditions from the firm’s maximization problem in period $t$, we have:

$$r_t = r(K_t) = \gamma K_t^{\gamma-1},$$  

(6)

$$w_t = w(K_t) = (1-\gamma)K_t^{\gamma}.$$  

(7)
2.2 Closed War

In the case of ‘closed war’, two countries play a zero-sum game in the sense that total private spending \( s_{t+1}^\alpha + s_{t+1}^\beta \) is redistributed between these countries depending on the relative size of defense spending. Thus, in place of (2) we have:

\[
K_{t+1}^\alpha = (s_{t+1}^\alpha + s_{t+1}^\beta) \frac{g_{t+1}^\alpha + A}{g_{t+1}^\alpha + A + g_{t+1}^\beta + B}.
\]  
(8)

Then, in place of (4) consumption of country \( \alpha \) is given by:

\[
c_{t+1}^\alpha = (1 + r_{t+1})(w_t^\alpha - g_{t+1}^\alpha + w_t^\beta - g_{t+1}^\beta) \frac{g_{t+1}^\alpha + A}{g_{t+1}^\alpha + A + g_{t+1}^\beta + B}.
\]  
(9)

3 LONG-RUN PROPERTY IN OPEN WAR MODEL

3.1 Dynamics

In the case of open war the agent maximizes consumption given as (4) by choosing defense spending. The first order condition is given as:

\[
g_{t+1}^\alpha - \vartheta g_{t+1}^\beta + A = (w_t^\alpha - g_{t+1}^\alpha) \partial.
\]

Or

\[
g_{t+1}^\alpha = \frac{1}{1 + \vartheta} (w_t^\alpha - \vartheta g_{t+1}^\beta - A).
\]  
(10)

Equation (10) means that an increase in defense spending by the rival country induces an increase in defense spending of the home country. This is the arms race effect. An increase in \( A \), a fixed benefit of defense technology, reduces defense spending. An increase in \( \vartheta \), the negative spillover effect of the rival country’s defense spending, raises defense spending. An increase in wage income, \( w \), stimulates defense spending. These results are intuitively plausible. Moreover, an increase in \( \vartheta \), defense productivity, on defense spending is positive. From (10), \( \partial g_{t+1}^\alpha / \partial \vartheta > 0 \) if and only if \( w_t^\alpha - \vartheta g_{t+1}^\beta + A > 0 \).

Considering (1-1) and (3-1), \( w_t^\alpha > g_{t+1}^\alpha \) and \( g_{t+1}^\alpha - \vartheta g_{t+1}^\beta + A > 0 \). Hence, it is easy
to see that condition $w_i^\alpha - \varepsilon g_i^\beta + A > 0$ holds in the present model. Thus, an increase in $\theta$ stimulates defense spending from the substitution effect.

When both countries are identical, we have at any time $g^\alpha = g^\beta = g$ and hence:

$$G^\alpha = G^\beta = G = (1-\varepsilon)g + A.$$  

(11)

An increase in one country's defense spending always raises its national security even if it induces negative spillovers from the rival country. Then, (10) reduces to:

$$g_{t+1} = \frac{\theta}{1+\theta - \varepsilon} w_t - \frac{1}{1+\theta - \varepsilon} A.$$  

(10a)

From equation (10a) spending on national security $g$ and capital accumulation $K$ always move in the same direction, which implies a positive relation between $g$ and $K$. An increase in defense spending is consistent with economic growth. Intuition is as follows. An increase in capital stock raises real income, stimulating the demand for national security. In order to have a larger amount of national security, the country is willing to pay more contributions to defense spending. So long as we assume the Cobb-Douglas technology, $w$ and $Y$ vary proportionally over time. In such a case, from (10a), the national security spending - GDP ratio, $g_{t+1}/Y_t$, increases with capital accumulation if and only if $A>0$, which is consistent with the exploitation hypothesis [see Sandler and Hartley (2001)].

Considering (1-1) and (7), (2) may be rewritten as:

$$K_{t+1} = [w(K_t) - g_{t+1}]G_{t+1}^\theta.$$  

(2a)

Substituting (10a) into (2a), we have

$$K_{t+1} = [Zw(K_t) + X][\bar{Z}w(K_t) + \bar{X}]^\theta \equiv \Phi(K_t)$$  

(12)

where $Z \equiv \frac{1-\varepsilon}{1+\theta - \varepsilon}$, $\bar{Z} \equiv \frac{(1-\varepsilon)\theta}{1+\theta - \varepsilon}$ and $X \equiv \frac{1}{1+\theta - \varepsilon} A$, $\bar{X} \equiv \frac{\theta}{1+\theta - \varepsilon} A$. Equation (12) is the fundamental dynamic equation in the open war model.

3.2 Stable Case
Suppose for simplicity $A=0$. Then, we have:
$$
\Phi(K) = Jw(K)^{\theta+1} = J\theta^{(\theta+1)} K^\theta K^{\gamma(\theta+1)},
$$
where $J \equiv \theta Z^2$ and $J \equiv \theta(1-\gamma)^{\theta+1}$. Since we assume the Cobb-Douglas technology, we have $w'(0) = \infty > 1$, $w''<0$ and $w'(\infty) = 0 < 1$. We also have:
$$
\Phi'(K) = J\gamma(\theta+1)K^\theta K^{\gamma(\theta+1)-1},
$$
$$
\Phi''(K) = J\gamma(\theta+1)[\gamma(\theta+1)-1]K^{\gamma(\theta+1)-2}
$$
Hence, it is easy to see that if $\theta < \frac{1-\gamma}{\gamma}$, then $\Phi'(0) = \infty > 1$, $\Phi'(<\infty) = 0 < 1$, and $\Phi''<0$. In such a case, the long-run equilibrium is unique and stable (see Azariadis 1993). In other words, when $\theta$ is low enough, the system is globally stable, as shown in Figure 1.

$K$ and $G$ move to the steady state levels, respectively, in the same direction due to the income effect. Capital accumulation enhances the overall national security level by stimulating its own defense spending despite the arms race effect. Since the two countries are identical, both countries $\alpha$ and $\beta$ grow in the same way. Capital accumulation in one country stimulates its defense spending, which has a negative spillover effect on the rival country. Since $\varepsilon <1$, the arms race effect does not work strongly. The rival country also accumulates capital and national security increases. When the income effect dominates the arms race effect, an arms race is consistent with economic growth for both countries.

The steady state level of $r, r^*$, is low when $\varepsilon$ is small. Namely, an increase in $\varepsilon$ stimulates each country's defense spending due to the arms race effect, which has a negative impact on capital accumulation. Although an increase in $\varepsilon$ will depress capital accumulation and hence demand for national security, it will raise $g$ at the given level of $G$.

We could also consider a change in $A=B$. An increase in the initial level of national security, $A=B$, results in enhancing capital accumulation and welfare.
3.3. Welfare Analysis

In order to investigate the normative aspect of the arms race behavior, it is useful to derive the first best solution. From (1)(2) and (4), the feasibility condition in each country is given as:

\[ g_{t+1} = Y(K_t) + K_t - K_{t+1}^\theta - c_t. \]  

We analyze the optimal path which would be chosen by a world central planner, who maximizes social welfare of each country expressed as the sum of generational utilities discounted by the social discount factor on future generations, \( \rho \), which is between 0 and 1, by choosing the time paths of defense spending and capital accumulation: \( g_{t+1} \) and \( K_{t+1} \). Since both countries are identical, we only consider one country’s welfare:

\[ \text{Max} \sum_{t=0}^\infty \rho^t c_t \quad \text{subject to (13)}. \]

In other words, the first best problem is to maximize the following function:

\[ W = \sum_{t=0}^\infty \rho^t \{ Y(K_t) + K_t - K_{t+1}^\theta \} \cdot \]  

Then, the first order conditions with respect to \( g_{t+1} \) and \( K_{t+1} \) are as follows:

\[ K_{t+1}^\theta (1 - \varepsilon) = G_{t+1}^\theta, \]  

(15-1)

\[ G_{t+1}^\theta [1 + r(K_{t+1})] = 1. \]  

(15-2)

Note that in the non-cooperative Nash equilibrium from (1-1)(2)(3-1) and (10) we always have:

\[ K_{t+1}^\theta = G_{t+1}^\theta. \]

Considering (15-1) and the above equation, we have:

\[ G^F < G^N \]

if and only if

\[ K^F (1 - \varepsilon) < K^N, \]

(16)
where superscript $F$ denotes the first best solution and superscript $N$ means the non-cooperative solution.

If $\varepsilon$ is very large and hence close to 1, then $K^F(1-\varepsilon) < K^N$. In such a case inequality (16) means that $G^N$ in the competitive economy ($G^N$) is larger than in the first best economy ($G^F$). This is because each country does not fully recognize the negative spillover effect of national defense. Defense spending (and hence national security) provided by each country is too much and private consumption is too little in the non-cooperative economy in terms of the static efficiency. This is the conventional result at the non-cooperative Nash solution due to the arms-race behavior.

In terms of the dynamic efficiency from (15-2) we have as the modified golden rule:

$$\rho(1+r) = G^{-\theta}. \quad (15-2a)$$

In the standard overlapping generations growth model it is well known that capital may be either over-supplied or under-supplied in the competitive equilibrium. Capital accumulation is too little in the long run in this model as well when the competitive steady state economy is on the efficient path

$$((1+r)\rho < G^{-\theta}).$$

An interesting implication from (16) is that if $K^F > K^N$ and $\varepsilon$ is relatively small, we may have the seemingly paradoxical case of $G^N < G^F$. Defense spending (and hence national security level) at the non-cooperative Nash solution could be under-provided. When capital is under-accumulated at the non-cooperative Nash solution, it produces the negative growth effect. When $\varepsilon$ is small, the arms race effect is not large. Then, the negative growth effect likely dominates the positive arms-race effect, and hence $G^N$ could be smaller than $G^F$. The lower the discount rate (the higher $\rho$), it is more likely to have such a seemingly paradoxical case. In such a case it is desirable to promote both capital accumulation and defense spending.
3.4 High Productivity of Security Technology

We then consider the case where security technology parameter $\theta$ is very high. When $\theta$ is very high ($\theta > \frac{1-\gamma}{\gamma}$), $\Phi'(0) = 0 < 1$, $\Phi'(\infty) = \infty > 1$, and the second derivative of $\Phi(K)$ is positive. In such a case, as shown in Figure 2, increasing returns affect the qualitative properties of growth path in a substantive way. There are two steady states, an asymptotically stable one at $K=0$ and an unstable one at some point $E$, where $KE>0$. Furthermore, the asymptotic behavior of equilibrium trajectories depends on the initial capital of the economy. If $K(0)<KE$, the economy starts with an insufficient stock of capital and converges to $K=0$, which we may interpret as a form of poverty trap. Given a large enough stock of capital, that is, $K(0)>KE$, the economy will experience unbounded growth. It is easy to see that an increase in $\epsilon$ will raise $KE$. Therefore, the larger the arm-race effect, it is more likely that the country will experience a poverty trap.

Figure 2 Here

3.5 Fixed Cost of Defense Technology

We now consider the case where $A <0$ and hence defense technology needs a fixed cost. In this case if $\theta$ is low ($\theta < \frac{1-\gamma}{\gamma}$), we may have two equilibrium points, as shown in Figure 3. One is a stable point $H$, while the other is an unstable point $L$. If the initial level of capital is smaller than $K_L$, the country cannot grow. Both capital and national security decline forever. The larger the absolute value of $A$, it is more likely to have such an unstable case.

Figure 3 Here
If, in addition, \( A (<0) \) is low enough, we have the case where there is no long-run equilibrium. See Figure 4. In such a case, the economy will decline, irrespective of the initial level of capital. The larger the absolute value of \( A \), it is more likely to have such an unstable case.

Figure 4 Here

4. LONG-RUN PROPERTY IN CLOSED WAR

In the case of closed war, each country maximizes consumption given by (9) by choosing its defense spending. The first order condition is given as:

\[
(g^a_{t+1} + A)(g^a_{t+1} + A + g^\beta_{t+1} + B) = (w^\alpha_t - g^a_{t+1} + w^\beta_t - g^\beta_{t+1})(g^\beta_{t+1} + B)
\]

(17)

In the symmetry situation (17) reduces to:

\[
g_{t+1} = \frac{1}{2}(w_t - A)
\]

(17a)

As in the case of open war, the national security spending - GDP ratio, \( g/Y \), increases with capital accumulation if and only if \( A>0 \).

Then the dynamic system will be summarized as:

\[
K_{t+1} = \frac{1}{2}[w(K_t) + A].
\]

(18)

When \( A=0 \), the long-run properties are qualitatively the same as in section 3.2 where \( \theta < \frac{1-\gamma}{\gamma} \). The long-run equilibrium point is unique and stable, as drawn in Figure 1.

In the closed war model, it is interesting to note that we have \( g=0 \) at the cooperative solution. In other words, since \( g^a = g^\beta \) implies:

\[
K_{t+1} = s_{t+1},
\]

the optimal level of \( g \) at the cooperative solution is given by \( g=0 \). It follows that defense spending at the non-cooperative solution is always greater than the level at the first best solution.
If $A < 0$ and the defense technology needs a fixed cost, we may have two equilibrium points as in Figure 3. Also, if $A$ is low enough, we may have a case like Figure 4. The larger the absolute value of $A$, it is more likely to have such an unstable case. In this sense, the qualitative results are almost the same as in section 3.4, where $\theta$ is low in the open war model.

5. CONCLUSION

This paper has developed a general equilibrium model of overlapping generations of two rival countries which provide defense spending in an adversarial situation of open war and closed war models.

It is useful to investigate the long-run consequence of arms race using two terms: the arms race effect and the growth effect. Both national security and capital move in the same direction due to the optimizing behavior of each country. An increase in capital stock raises real income, stimulating the demand for national security. In order to have a larger amount of national security, the country is willing to have more spending on national defense. We have shown that the security spending - GDP ratio increases with GDP if defense technology has a fixed benefit, which is consistent with the exploitation hypothesis.

As to the normative implication, we have then explored the seemingly paradoxical possibility that the steady-state level of national defense may be too little in the competitive economy of open war. It is well known that capital accumulation may be too little in the long run when the competitive steady state economy is on the efficient path. This is the negative growth effect. If the negative growth effect dominates the positive arms-race effect, the long-run level of national defense at the non-cooperative solution could be smaller than the first best level. The lower the discount rate and the spillover effect, it is more likely to have such a seemingly paradoxical case in the open war model.
As to the dynamic nature, when defense technology needs a large amount of fixed cost and the efficiency parameter is relatively low in the case of open war or defense technology needs a large amount of fixed cost in the case of closed war, the arms race effect would dominate the growth effect, making it impossible for these countries to grow at the same time in the long run. For example, when the home country accumulates capital gradually and also accumulates its national security level, it would hurt the rival country. Hence, the rival country would react to raise its national defense by eating capital. When this reaction could not completely offset the negative spillover effect, capital and national security gradually decline for both countries. The larger the negative spillover effect and the fixed cost of defense spending, it is more likely to have such an unstable case.

Finally, in the open war model, if defense technology is very efficient and the initial capital stock is low, we may have another type of instability. Namely, if the economy starts with an insufficient stock of capital, it will experience a poverty trap.

We have thus shown that the degree of spillover of arms races and the efficiency of defense technology are crucial for dynamic properties of an arms race between two rival countries in relation to economic growth.
References


Figure 1
Figure 2
Figure 3
Figure 4