Commercial Policy under Cross-Border Ownership and Control*

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Abstract

It is often observed that in order to serve the domestic market, foreign firms not only export but also control domestic firms through foreign direct investment (FDI). This paper examines the effects of tariffs, production subsidies, and foreign ownership regulation on prices, outputs, profits, and welfare when both exports and FDI coexist. Cross-border ownership on the basis of both financial interests and corporate control leads to horizontal market-linkages through which tariffs and production subsidies may not benefit locally-owned firms, because the foreign firm shifts production across borders to evade the burden or even take advantage of commercial policies. The effects of ownership regulation depend on both the initial ownership share and the substitutability between goods.

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1 Introduction

Cross-border ownership (CBO) is widespread in this age of globalization. According to Kang and Sakai (2000), cross-border strategic alliances worldwide increased from 860 in 1989 to 4400 in 1999. From an index compiled by Morgan Stanley Capital International, Wojcik (2002, table 1) documents that 711 companies had foreign ownership in 16 northern and western European countries. The share of foreign ownership varies with an average of 61 percent. The highest is Norway at 91 percent and the lowest is Switzerland at 23 percent.

Although there are various CBO arrangements, an interesting fact is that foreign direct investment (FDI) often coexists with exports. A typical example is the automobile industry. General Motors (GM) is the 100 percent shareholder of Opel in Germany and Saab in Sweden and a heavy shareholder of Suzuki and Isuzu in Japan and Daewoo in Korea.\(^1\) To serve the Japanese market, GM directly exports large and luxury cars such as Cadillac and Corvette and supplies compact cars through Suzuki and Isuzu.\(^2\) Moreover, Shanghai GM is a 50-50 joint venture (JV) between GM and Shanghai Automotive Industry Corporation. Since the Chinese government does not allow foreign auto makers to have their own subsidiaries in China, world leading makers have been forming JVs with Chinese auto makers as well as exporting to China.\(^3\)

Several authors have analyzed the relationship between collusion/competition and partial ownership within a country. For instance, Reynolds and Snapp (1986), Farrell and Shapiro (1990), Malueg (1992), and Reitman (1994) model horizontal partial ownership; Morita (2001) investigates the Japanese manufacturer-supplier relationship; and Alley (1997) finds empirically that Japanese firms form partial ownership to collude in the domestic market, but not in the export market.

When partial ownership schemes are across country borders, they have important consequences on trade and foreign investment. For instance, by forming CBO schemes, firms can not only share profits, but also shift production to meet local demands and to avoid high cost regions. Those involved in CBO may be able to prey on independent rival firms through production shifting.

The present paper examines the effects of import tariffs, production subsidies, and foreign ownership regulation when both exports and FDI coexist. We are interested

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\(^1\) GM owns respectively 20% of Suzuki, 8.4% of Isuzu, and 50.1% of Daewoo.

\(^2\) Similar strategies can be seen between Ford and Mazda.

\(^3\) The upper limit of foreign ownership imposed by the Chinese government is 50 percent.
in how outputs, profits, consumer prices and national welfare change when a certain commercial policy is adopted, with particular emphasis on the impacts on independent rival firms.

Without any ownership relationships, firms are independent to each other. On the other hand, under full ownership the parent firm has complete control power. Therefore, in the case of partial ownership, it is inferred that the partial owner has some control power under certain cases.\(^4\) In fact, it is widely observed that the principal shareholder sends executives such as the chief executive officer and chief operating officer to the partially owned company.\(^5\) Thus, we assume that the foreign firm has some corporate control over a domestic firm by undertaking FDI. Then, the domestic firm under foreign ownership cares about the profits of the foreign firm (i.e., the headquarters) as well as its own; And the higher the foreign ownership, the more the domestic firm takes into account the foreign firm’s profit.

We show that CBO on the basis of both financial interests and corporate control leads to horizontal market-linkages through which import tariffs and production subsidies may not benefit firms that are 100% locally-owned. Further, regulating CBO may hurt local firms in terms of market share and profits when foreign ownership is low. However, the opposite is true when foreign ownership is sufficiently high. These arise because CBO and corporate control enable the foreign firm to shift production so as to evade the burden or even take advantage of commercial policies such as import tariffs and subsidies to local production. Thus, our analysis and result lead to important policy implications for countries intending to develop local industries. In particular, this paper presents results that are different to Markusen and Venables (1999), who establish circumstances under which FDI is complementary to local industries in developing countries.

Although there are several papers which analyze commercial policies under CBO in the framework of international oligopoly (see, for example, Lee, 1990; Weltzel, 1995; and Long and Soubeyran, 2001), our analysis is distinguished from these studies. We explicitly incorporate the fact that foreign firms control domestic ones through FDI, following Krugman and Obstfeld (2003, p.171): “The distinctive feature of direct foreign investment is that it involves not only a transfer of resources but also the acquisition of

\(^4\)Krugman and Obstfeld (2003, p.171) says “In U.S. statics, a U.S. company is considered foreign-controlled, ..., if 10 percent or more stock is held by a foreign company; the idea is that 10 is enough to convey effective control”.

\(^5\)For example, Ford, which has Mazda’s 33% stocks, and Renault, which has Nissan’s 44% stocks, have sent presidents to Mazda and Nissan, respectively.
control. That is, the subsidiary does not simply have a financial obligation to the parent company; it is part of the same organization structure."\textsuperscript{6} In contrast, the literature so far has ignored the control problem, i.e., subsidiaries only maximize their own profits but ignore how well the headquarters is doing and how its interest is related to those of the subsidiaries.

Also, there is a large literature focusing on transfer pricing, through which the multinational headquarters influences its foreign subsidiaries. However, in such settings the headquarters usually supplies intermediate inputs to the subsidiaries (e.g., Rugman and Eden, 1985). By changing their prices, the multinational can shift profits across country borders to reduce tax burdens. Again the control issue is not examined.

The rest of the paper is organized as follows. Section 2 sets up the basic model. Section 3 investigates the effects of import tariffs and production subsidy under foreign ownership and control. Section 4 introduces regulation on foreign ownership. Section 5 looks into the impact on national welfare. And section 6 concludes the paper.

## 2 Model Setup

### 2.1 Basic Structure

Consider two goods $X$ (say, large cars) and $Y$ (say, small cars), which are imperfect substitutes. Good $X$ is made by a foreign firm $f$, who exports to the domestic market for sales. There are also two domestic firms $d$ and $h$, that produce and sell good $Y$ locally. Let us denote the marginal cost of firm $i$ as $c^i (i=f,d,h)$, which is constant. We wish to model the fact that firm $f$ has financial interest in firm $d$. Specifically, it holds firm $d$’s stocks, by a share $k (0 \leq k \leq 1)$.

We assume that the domestic government imposes a specific tariff $t$ on the imported good $X$ and provides a specific subsidy $s$ to the locally produced good $Y$. Based on the tariff and subsidy, the firms compete in a Cournot fashion.

The inverse demands for the imperfectly substitutable goods $X$ and $Y$ are given respectively as

$$\begin{align*}
p_x &= a - x - \gamma (y^d + y^h), \\
p_y &= b - (y^d + y^h) - \gamma x,
\end{align*}$$

\textsuperscript{6}In fact, there are few studies that explicitly consider corporate control when analyzing partial ownership. An exception is Bresnahan and Salop (1986) which examines the relationship between corporate control and the Herfindahl index.
where \( p_x \) and \( p_y \) are the prices of goods \( X \) and \( Y \), \( 0 < \gamma < 1 \) is a parameter indicating the degree of substitutability between the two goods, \( a \) and \( b \) are parameters, and \( x, y^d \) and \( y^h \) are, respectively, the outputs of firms \( f, d \) and \( h \). We define \( Y \equiv y^d + y^h \).\(^7\)

Given the above structure, the profit functions of firms \( f, d \) and \( h \) can be written respectively as

\[
\pi^f = (p_x - c^f - t)x + k\pi^d = \pi^x + k\pi^d, \quad (2a)
\]

\[
\pi^d = (p_y - c^d + s)y^d, \quad (2b)
\]

\[
\pi^h = (p_y - c^h + s)y^h. \quad (2c)
\]

where \( \pi^x \) is the profit earned by selling good \( X \), i.e., \( \pi^x \equiv (p_x - c^f - t)x \).

2.2 Foreign firm’s control over the domestic firm

In this subsection, we model the relationship between partial ownership and corporate control in detail. The Industrial Organization literature and the Antitrust literature distinguish between financial interest and corporate control (e.g., O’brien and Salop, 2000). Financial interest refers to the right to receive the stream of profits generated by the firm from its operations and investments. Corporate control refers to the right to make the decisions that affect the firm. In a sole proprietorship, a single individual has the right to 100 percent of the profit of the firm. The same individual also has complete control over the company, making the decisions about levels of prices, outputs, investments and where to purchase inputs and locate plants, etc. In the case of a partial ownership, nobody has 100 percent ownership. However, a principal shareholder may have 100 percent corporate control and the others have none. Generally, higher ownership share brings greater corporate control.

In our model, since firm \( f \) holds firm \( d \)’s stocks, the former may also affect the latter’s corporate control. For instance, it may be able to constrain firm \( d \) from taking any action which might be harmful to firm \( f \). Specifically, we assume that the objective function of firm \( d, \tilde{\pi}^d \), is the weighted average of firm \( d \)’s and firm \( f \)'s profit functions:\(^8\)

\[
\tilde{\pi}^d \equiv (1 - v)(1 - k)\pi^d + v\pi^f, \quad 0 \leq v \leq 1. \quad (3)
\]

\(^7\)If \(-1 < \gamma < 0\) (i.e., the goods are complements), then most of the following results are simply reversed.

\(^8\)For example, Carlos Ghosn, whom Renault has sent to Nissan as President and Chief Executive Officer, would consider Renault’s profit as well as Nissan’s in his management. Nissan says “Both companies share a single joint strategy of profitable growth and a community of interests” (http://www.nissan-global.com/EN/HOME/0,1305,SI9-LO3-MC92-IFN-CH120,00.html).
The parameter $v$ represents the degree of firm $f$’s control over firm $d$’s decision. In other words, parameters $k$ and $v$ respectively represent firm $f$’s financial interest (“ownership share” in our terminology) and corporate control (“control power” in our terminology) of firm $d$. The interests of firm $f$ depend on its own profits (i.e., $\pi^f$), while those of the other shareholders depend on the profits they get (i.e., $(1-k)\pi^d$). The objective of firm $d$ is to maximize its own profit when $v = 0$ (i.e., without any control power) and firm $f$’s profit when $v = 1$ (i.e., with full control). Firm $d$ takes into account both firms’ profits when $v$ is in between.

We next formulate the relationship between firm $f$’s ownership share $k$ and control power $v$. It seems reasonable that control power $v$ is weakly increasing in ownership share $k$. For simplicity, we assume that $v$ is determined by the following continuous function of $k$:

\begin{equation}
\text{Assumption 1}
\end{equation}

\[ v(k) = \begin{cases} 
  v(k), & v' \geq 0 \quad \text{if} \quad 0 \leq k < \bar{k} \\
  1, & \text{if} \quad \bar{k} \leq k \leq 1
\end{cases} \]

where $v(0) = 0$ and $v(\bar{k}) = 1$.

Note that when firm $f$ holds more than a critical share $\bar{k}$, it fully controls firm $d$. As pointed out by O’Brien and Salop (2000), $v = 1$ could hold even if $k < 1/2$. Figure 1 illustrates the relationship between firm $f$’s ownership share $k$ and its control power $v$. $v$ increases as $k$ does until $\bar{k}$ when $v$ reaches 1. Note that $v(k)$ is drawn linear for the purpose of illustration, but it needs not to be.

**Figure 1 around here**

From (2b) and (3), firm $d$ maximizes

\[ \tilde{\pi}^d = \lambda(k)[\pi^d + \eta(k)\pi^x], \tag{4} \]

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9 An alternative formulation of firm $d$’s objective function may be $\tilde{\pi}^d = (1-v)\pi^d + v\pi^f$. In both formulations, we have obtained similar results (see Ishikawa et al. 2004).

10 What is crucial for our results is that $v$ is weakly increasing in $k$. Note that $v(k) = 0$ may arise when $k$ is small, which is called “silent interests”. Also, $v(k)$ could be a step function such as $v(k) = 0$ for $0 \leq k < \bar{k}$ and $v(k) = 1$ for $\bar{k} \leq k \leq 1$. Even if these cases arose, the essence of our results would not change.

11 Some firms may have very fractioned ownership and a shareholder with, say, 10% of the stocks may be able to fully control the firm. Moreover, firms often issues stocks without any control right. See also footnote 4.
where \( \lambda(k) \equiv 1 - k - v(k) + 2v(k)k > 0 \) and \( \eta(k) \equiv v(k)/\lambda(k) \). We can regard \( \eta \) as firm \( d \)'s weight attached to the profit from good \( X \).

An increase in the ownership share changes the weight \( \eta \) as follows

\[
\theta \equiv \frac{d\eta(k)}{dk} = \frac{v'(1 - k) + v(1 - 2v)}{\lambda^2}. \tag{5}
\]

From Assumption 1 and (5), the weight \( \eta \) is increasing in the share \( k \) when \( k \) is smaller than \( k_1 \) which is defined by \( v(k_1) = 1/2 \), while \( \eta \) is decreasing in \( k \) after \( v \) reaches 1 and \( v' = 0 \). If \( k \) is small, an increase induces firm \( d \) to attach more weight to the profit of its foreign owner. But when \( k \) is sufficiently large, firm \( f \) finds it best to “tell” firm \( d \) to care about its own good, as this results in profit for \( f \) too. However, it increases the financial interest from firm \( d \), so that firm \( f \) controls firm \( d \) to take into account of its own profit, i.e., \( \eta \) decreases. These two effects make the sign of \( \theta \) ambiguous when \( k \) lies between \( k_1 \) and \( \tilde{k} \). Thus, we assume that it is positive for simplicity as well as for the uniqueness of thresholds \( k_2 \) and \( k_4 \), which will be defined later.

**Assumption 2**  If \( k_1 \leq k < \tilde{k} \), then \( \theta > 0 \), where \( k_1 \) is defined as \( v(k_1) = 1/2 \).\(^{12}\)

Under Assumptions 1 and 2, \( \eta \) has the following properties:

**Lemma 1** (i) \( \eta \) is increasing in \( k \in [0, \tilde{k}] \) and decreasing in \( k \in [\tilde{k}, 1] \); (ii) \( \eta > 1 \) if and only if \( k_1 < k < 1 \); and (iii) \( \eta - 1/k = -(1 - v)(1 - k)/\lambda k \geq 0 \).

The first claim in Lemma 1 implies that \( \eta \) reaches its maximum value, \( 1/\tilde{k} \), at \( \tilde{k} \). Once \( k = \tilde{k} \) (i.e., \( v = 1 \)) holds, the objective function of firm \( d \), \( \tilde{\pi}^d \), becomes identical with that of firm \( f \). In view of (2a), firm \( d \)'s weight (as well as firm \( f \)'s) on \( \pi^x \) becomes smaller as \( k \) further increases. The second and third claims in Lemma 1 show that firm \( d \) cares about \( \pi^x \) more than \( \pi^d \) but no more than firm \( f \) cares about \( \pi^x \).

Finally, firm \( d \) maximizes (4), and firms \( f \) and \( h \) maximize their own profits simultaneously and independently, giving rise to the following first order conditions respectively:

\[
\frac{d\pi^f}{dx} = -x + p_x - c^f - l - k\gamma y^d = 0, \tag{6a}
\]

\[
\frac{d\tilde{\pi}^d}{dy^d} = \lambda(-y^d + p_y - c^d + s) - v\gamma x = 0, \tag{6b}
\]

\[
\frac{d\pi^h}{dy^h} = -y^h + p_y - c^h + s = 0. \tag{6c}
\]

\(^{12}\)For example, \( v(k) = (k/\tilde{k})^3 \), \( \beta \geq 1 \) satisfies Assumption 1 and 2. Note that as \( \beta \) goes to \( +\infty \), \( v(k) \) converges to a step function where \( v = 0 \) if \( k < \tilde{k} \) and \( v = 1 \) if \( k \geq \tilde{k} \).
The necessary and sufficient conditions for interior solutions are given in the appendix.

Before going on to the analysis of commercial policies, we establish a lemma on the changes of profits. Totally differentiating \( \pi^d \), \( \pi^x \) and \( \pi^f \) and combining them with the first order conditions above, we obtain

\[
\begin{align*}
    d\pi^d &= y^d(dp_y + ds) + (y^d + \gamma x \eta)dy^d, \\
    d\pi^x &= x(dp_x - dt) + (x + k \gamma y^d)dx, \\
    d\pi^f &= d\pi^x + kd\pi^d.
\end{align*}
\]

Differentiating firm \( h \)'s first order condition and the demand functions, we obtain \( dp_y + ds = dy^h \), \( dp_x = -dx - \gamma dY \), and \( dp_y = -dY - \gamma dx \). Thus, the following lemma is straightforward:

**Lemma 2** The changes of firm profits are decomposed as:

\[
\begin{align*}
    d\pi^d &= y^d dY + \gamma x \eta dy^d, \quad (7a) \\
    d\pi^x &= k \gamma y^d dx - x(\gamma dY + dt), \quad (7b) \\
    d\pi^f &= -\gamma x \left( \frac{(1 - v)(1 - k)}{\lambda} \right) dy^d - (\gamma x + ky^d)dy^h + ky^d ds - xd. \quad (7c)
\end{align*}
\]

3 Trade policies under foreign ownership and control

In this section, we analyze the effects of the import tariff imposed on good \( X \) and the production subsidy to good \( Y \). Differentiating the first order conditions (6a), (6b) and (6c) to derive:

\[
\begin{pmatrix}
    2 & \gamma(1+k) & \gamma \\
    \gamma(v + \lambda) & 2\lambda & \lambda \\
    \gamma & 1 & 2
\end{pmatrix}
\begin{pmatrix}
    dx \\
    dy^d \\
    dy^h
\end{pmatrix} =
\begin{pmatrix}
    -1 \\
    0 \\
    0
\end{pmatrix} dt +
\begin{pmatrix}
    \lambda \\
    0 \\
    1
\end{pmatrix} ds,
\]

where \( \Delta \equiv \lambda(6 - 2\gamma^2 - k\gamma^2) - \nu \gamma^2(1 + 2k) > 0 \) is required for stability.
3.1 Import Tariffs

The tariff has the following effects on outputs.

\[
\begin{align*}
\frac{dx}{dt} &= -\frac{3\lambda}{\Delta} < 0, \quad (8a) \\
\frac{dy^d}{dt} &= \frac{\gamma(2v + \lambda)}{\Delta} > 0, \quad (8b) \\
\frac{dy^h}{dt} &= \frac{\gamma(1-k)(1-2v)}{\Delta}, \quad (8c) \\
\frac{dY}{dt} &= \frac{dy^d + dy^h}{\Delta} = \frac{\gamma(2\lambda + v)}{\Delta} > 0. \quad (8d)
\end{align*}
\]

Conditions (8a) and (8b) say respectively that an increase in the tariff reduces the output of the foreign firm but increases that of domestic firm \(d\), which are as expected. However, we find a surprising result: \(y^h/dt\) in (8c) is negative if and only if \(k_1 < k < 1\). Recall \(v(k_1) = 1/2\). Note that when \(k_1 < k < 1\), firm \(d\)'s weight attached to \(\pi^x\) (i.e., \(\eta\)) is greater than 1 and thus is greater than that attached to \(\pi^d\) (Lemma 1). Thus, we obtain

**Proposition 1** An increase in the import tariff on good \(X\) reduces firm \(h\)'s output if and only if \(k_1 < k < 1\).

While the original purpose of the tariff is to help domestic firms, Proposition 1 says that if the foreign firm is tied up with a domestic firm, the other domestic firm could lose market share from the tariff, contrary to conventional wisdom. The intuition lies in the production shifting from \(x\) to \(y^d\) due to the control power \(v\) and the initial ownership share \(k\).

To see this more clearly, let us derive the reaction functions, using the FOCs:

\[
\begin{align*}
x &= r^f(y^d, y^h) = \frac{(a - c^f - t)}{2} - \frac{\gamma(1+k)}{2}y^d - \frac{\gamma}{2}y^h, \quad (9a) \\
y^d &= r^d(x, y^h) = \frac{(b - c^d + s)}{2} - \frac{y^h}{2} - \frac{\gamma x}{2}(1 + \eta), \quad (9b) \\
y^h &= r^h(x, y^d) = \frac{(b - c^d + s) - \gamma x - y^d}{2}, \quad (9c)
\end{align*}
\]

**Figure 2 around here**

Figure 2 depicts the reaction curves of firms \(f\) and \(d\) for given \(y^h\). From (9b) and Lemma 1, a larger \(k\) leads to a steeper reaction curve for firm \(d\) when \(k < \bar{k}\); whereas it
leads to a flatter reaction curve when \( k > \bar{k} \). Thus depending on \( k \), two reaction curves of firm \( d \), \( r^{d'} \) and \( r^{d} \), are drawn in the figure.

Figure 2 also shows that the production shifting from \( x \) to \( y^{d} \) becomes larger as the reaction curve becomes steeper. Suppose that the tariff on good \( X \) increases. Then the reaction curve \( r^{f} \) shifts downward. In turn \( x \) falls and \( y^{d} \) rises, because they are strategic substitutes. In Figure 2, since curve \( r^{d'} \) is steeper than curve \( r^{d} \), \( y^{d} \) increases more on the former curve than on the latter one for a given \( y^{h} \).

Now, whether or not \( y^{h} \) increases depends on the scale of the production shifting from \( x \) to \( y^{d} \). Proposition 1 implies that the increase in \( y^{d} \) dominates the decrease in \( x \) if \( k \) is between \( k_{1} \) and 1. As a consequence, the increase in \( y^{d} \) squeezes the production of firm \( h \), \( y^{h} \), giving rise to Proposition 1.

**Figure 3 around here**

Next, we investigate the effects of the tariff on prices.

\[
\frac{dp_{x}}{dt} = \frac{(1-k)(3-2\gamma^{2}) - v\{(3 - \gamma^{2}) - k(6 - 4\gamma^{2})\}}{\Delta}, \quad (10)
\]

From (10), \( dp_{x}/dt \) is negative if and only if \( v > (1-k)(3-2\gamma^{2})/\{(3 - \gamma^{2}) - 2k(3 - 2\gamma^{2})\} (\equiv f(k)) \) and \( k < (3 - \gamma^{2})/(6 - 4\gamma^{2}) \). And (11) says that \( dp_{y}/dt \) becomes negative if and only if \( k_{1} < k < 1 \). In Figure 3, \( v(k) \) intersects with \( f(k) \) at \( k = k_{2} \) and \( k = \gamma^{2}/(3 - 2\gamma^{2}) < (3 - \gamma^{2})/(6 - 4\gamma^{2}) \). Thus, we have

**Proposition 2** An increase in the tariff, (i) reduces the price of good \( Y \) if and only if \( k_{1} < k < 1 \); and (ii) also reduces the price of good \( X \) if and only if \( k_{2} < k < \gamma^{2}/(3 - 2\gamma^{2}) \).

Proposition 2 is again surprising. Normally when the tariff rises, imports decrease while import prices rise, and the prices of substitutes also rise. However, Proposition 2 says that both prices can fall following an increase in the import tariff. The intuition can be understood as follows. For (i), conditions (8a) and (8d) state that \( dx/dt < 0 \) and \( dY/dt > 0 \). But due to the production shifting of firm \( f \), if \( k \) is within the satisfied range, the effect of \( dY/dt \) dominates \( dx/dt \) in affecting the price of good \( Y \) through equation

\(^{13}\)From Lemma 1, the reaction curve under \( k \geq k_{1} \) is always steeper than that under \( k < k_{1} \).

\(^{14}\)In view of Figure 3, the parameters in which the price of good \( X \) falls exist if and only if \( \bar{k} < \gamma^{2}/(3 - 2\gamma^{2}) \). If \( k_{2} \) exists, it is unique from Assumption 2.
(1b), lowering $p_y$. For (ii), since the two goods are substitutes, a large decrease in $p_y$ also lowers $p_x$.

Finally, we turn to the effects of the tariff on profits. With the aid of Lemma 2, we can derive

$$\frac{d\pi^h}{dt} = 2y^h\frac{dy^h}{dt},$$
$$\frac{d\pi^d}{dt} = y^d\frac{dY}{dt} + \gamma x\eta\frac{dy^d}{dt} > 0,$$
$$\frac{d\pi^x}{dt} = k\gamma y\frac{dx}{dt} - \gamma x\frac{dY}{dt} - x < 0.$$

Firm $h$’s profit rises as its output rises. Invoking Proposition 1, the effect of the tariff on firm $h$’s profit is obvious. Since the tariff increases firm $d$’s profit but reduces the profit from selling good $X$, the change in firm $f$’s total profit is generally ambiguous. The tariff may benefit the foreign firm $f$, because the output of the locally-owned firm $h$ is reduced. As shown in the Appendix, the following can be obtained:

**Proposition 3** Firm $h$ loses from an increase in the import tariff if and only if $k_1 < k < 1$. An increase in the tariff (i) raises firm $f$’s profit if $k \leq k \leq k_3$, where $k_3$ is defined by $\gamma^2(k_3 + 1)(k_3 + 2) - 6k_3 = 0$; (ii) but reduces its profit if $k \leq k_1$ or $k = 1$.

### 3.2 Production Subsidy

Now, we turn to the impact of the production subsidy. First, on outputs we obtain

$$\frac{dx}{ds} = -\frac{\gamma\lambda(2 + k)}{\Delta} < 0,$$  \hspace{1cm} (12a)
$$\frac{dy^d}{ds} = \frac{2\lambda + v\gamma^2}{\Delta} > 0,$$  \hspace{1cm} (12b)
$$\frac{dy^h}{ds} = \frac{2(1 - k) - v\{2 + \gamma^2 - k(4 - \gamma^2)\}}{\Delta},$$  \hspace{1cm} (12c)
$$\frac{dY}{ds} = \frac{4(1 - v)(1 - k) + kv(4 - \gamma^2)}{\Delta} > 0.$$  \hspace{1cm} (12d)

**Figure 4 around here**

A counter-intuitive result is that $dy^h/ds$ in (12c) is negative if and only if $v > 2(1 - k)/(2 + \gamma^2 - k(4 - \gamma^2))$ (equiv $g(k)$) and $k < (2 + \gamma^2)/(4 - \gamma^2)$. Figure 4 illustrates the relationship between $g(k)$ and $v(k)$. Curve $v(k)$ intersects with $g(k)$ at $k = k_4$ and $k = \gamma^2/(2 - \gamma^2) (< (2 + \gamma^2)/(4 - \gamma^2))$. Assumption 2 assures that $v(k)$ intersects with
$g(k)$ at most once in $[0, \bar{k}]$. This implies that there exist parameters $(k, v)$ in which firm $h$ decreases its output if and only if $\bar{k} < \gamma^2/(2 - \gamma^2)$. Therefore, the following proposition can be established.

**Proposition 4** An increase in the production subsidy to good $Y$ reduces the output of firm $h$ if and only if $k_4 < k < \gamma^2/(2 - \gamma^2)$.

This interesting result again stems from the production shifting from $x$ to $y^d$ due to the control power $v$. Figure 5 is similar to Figure 2, but shows the production shifting under the subsidy. When the reaction curve $r^d$ becomes steeper (from $r^d$ to $r^d_0$), the effect of a change in the production subsidy on $y^d$ becomes larger.

**Figure 5 around here**

In Figure 6, we compare the effects of the tariff and the subsidy. Because firm $h$’s reaction curve also shifts upward, the range of $k$ in which firm $h$ reduces its output becomes smaller than in the tariff case. This is because the subsidy affects domestic production directly, while the tariff does it indirectly by first reducing imports.

As expected, the subsidy lowers the prices of both goods as follows:

\[
\frac{dp_x}{ds} = -\gamma(1-k)(1-\lambda)(2-k) + \gamma kv (2-k - \gamma^2) < 0, \quad (13a)
\]

\[
\frac{dp_y}{ds} = -\frac{(1-v)(1-k)\{4-\gamma^2(2+k)\} + kv\{4-\gamma^2(3+k)\}}{\Delta} < 0. \quad (13b)
\]

These arise because the subsidy gives domestic firms incentives to increase outputs.

As to the profit of firm $h$, substitutions yield

\[
\frac{d\pi^h}{ds} = 2y^h\frac{dy^h}{ds}.
\]

That is, when the output of firm $h$ decreases, its profit also falls. From Lemma 2, the production subsidy increases the profit of firm $d$, but decreases the profit from selling good $X$ as follows

\[
\frac{d\pi^d}{ds} = y^d\frac{dY}{ds} + \gamma x\eta\frac{dy^d}{ds} > 0,
\]

\[
\frac{d\pi^X}{ds} = k\gamma y^d\frac{dx}{ds} - \gamma x\frac{dY}{ds} < 0.
\]
The change in firm $f$’s total profit is generally ambiguous. However, the production subsidy may benefit the foreign firm through two channels. One is firm $f$’s financial interest in firm $d$ and the other is the reduction of firm $h$’s output. Specifically, we can state the following proposition, the proof of which is given in the Appendix:

**Proposition 5** An increase in the production subsidy to good $Y$ reduces the profit of firm $h$ if and only if $k_4 < k < \gamma^2/(2-\gamma^2)$. Firm $f$ (i) gains from an increase in the production subsidy to good $Y$ if $\bar{k} \leq k \leq \gamma^2/(2-\gamma^2)$; (ii) but loses if $k$ is sufficiently small.

4 Regulated foreign ownership

In many developing countries, there exist legal limits on foreign ownership (e.g., China, see footnote 3). Our model can be used to analyze such a policy. We focus on the effects on the outside agents, who are not directly involved in the partial ownership, i.e., the consumer prices and the profit of firm $h$.

Differentiating the first order conditions (6a), (6b) and (6c), we obtain

$$\begin{pmatrix}
\frac{2}{\gamma(1 + k)} & \gamma \\
\gamma(\lambda + v) & 2\lambda & \lambda \\
\gamma & 1 & 2
\end{pmatrix}
\begin{pmatrix}
\frac{dx}{dy} \\
\frac{dy}{dy} \\
\frac{dy}{dy}
\end{pmatrix}
= \begin{pmatrix}
-\gamma y^d \\
-\theta \lambda \gamma x \\
0
\end{pmatrix} \frac{dk}{dk}.

(14)

Recall from Assumption 2 and Lemma 1 that $\theta$ captures the change in firm $d$’s weight on the profit from good $X$, i.e., the change in firm $d$’s incentives to reduce its own output for the sales of good $X$. There are two cases: (i) if $0 \leq k < \bar{k}$, because a higher ownership share leads to a greater control power, the reduction of firm $d$’s output for the sales of good $X$ increases as firm $f$’s share rises; (ii) if $\bar{k} < k \leq 1$, firm $f$ acquires full control of firm $d$, and an increase in its share raises the cost to reduce the output and profit of the latter firm (see Lemma 1). Thus, in the latter case, the higher share mitigates firm $d$’s output reduction.

4.1 The effects on the outside agents

First, we look into firm $h$. From the FOCs, foreign ownership changes firm $h$’s profit as follows

$$\frac{d\pi^h}{dk} = 2y^h \frac{dy^h}{dk},$$
which depends on the change of firm h’s output
\[
\frac{dp_h}{dk} = \frac{\gamma^2 y^d(1 - k)(1 - 2v) + \theta \lambda \gamma x (2 - \gamma^2 - k \gamma^2)}{\Delta}. \tag{15}
\]
As can be seen in Figure 3, \( v < 1/2 \) if \( k < k_1 \), so that the sign is ambiguous when \( k_1 < k < \bar{k} \). In the other cases, from (14), we can state:

**Proposition 6** Suppose that firm f’s ownership share rises. Then the output and profit of firm h increase if \( k < k_1 \) but decrease if \( k \geq \bar{k} \).

Proposition 6 says that when firm f owns a small share of firm d, an increase in foreign ownership benefits the rival firm h, because firms f and d take into account the intra-marginal effects of their own output expansion on each other, leading them to reduce outputs. That is, the higher control power leads firms f and d to internalize their over-production by reducing their outputs. And when the control power is not high, the room for output reduction is large. However, if firm f owns a large enough share of firm d, then it has full control of firm d, and further increase in foreign ownership enables the two to become closer into one entity, thus hurting the rival firm h.

Proposition 6 provides interesting policy implications. If foreign ownership is sufficiently large, then regulating it helps the locally-owned firm in terms of market share and profits; on the other hand, if foreign ownership is small, then regulation will hurt the locally-owned firm.

Next we investigate the consumer prices. From the FOC for firm h, we derive
\[
\frac{dp_y}{dk} = \frac{dy_h}{dk}.
\]
Using Proposition 6, we establish:

**Lemma 3** Suppose that firm f’s ownership share increases. Then the price of good Y rises if \( k < k_1 \), but falls if \( k \geq \bar{k} \).

The price of good X changes as follows:
\[
\frac{dp_x}{dk} = \frac{\gamma y^d(f(k) - v)((3 - \gamma^2) - k(6 - 4\gamma^2)) + \theta \lambda \gamma^2 x (1 - k(2 - \gamma^2))}{\Delta}. \tag{16}
\]
It can be seen from Figure 3 that the sign is ambiguous if either \( k_2 < k < \bar{k} \) or \( \gamma^2/(3 - 2\gamma^2) < k < 1/(2 - \gamma^2) \) holds, and that \( v > f(k) \) if and only if \( k_2 < k < \gamma^2/(3 - 2\gamma^2) \). Since \( \gamma^2/(3 - 2\gamma^2) < 1/(2 - \gamma^2) \), we have:

---

15The changes of outputs \( x, y^d \) and \( y^h \) all depend on the ratio \( x/y^d \) as can be seen from (14). However, the ratio takes any positive value \([0, +\infty)\), because either \( x \) or \( y^d \) can be zero. See the Appendix for such corner solutions.
Lemma 4 Suppose that firm f’s ownership share rises. Then the price of good X increases if either $k \leq k_2$ or $k \geq 1/(2 - \gamma^2)$ holds, but decreases if $\bar{k} < k < \frac{\gamma^2}{(3 - 2\gamma^2)}$.

Furthermore, in view of Lemmas 3 and 4, the following Proposition is straightforward:

Proposition 7 Suppose that firm f’s ownership share increases. Then the prices of both goods X and Y rise if $k < k_1$, while they both fall if $\bar{k} < k < \frac{\gamma^2}{(3 - 2\gamma^2)}$.

The intuition for Proposition 7 follows from Proposition 6. If $k$ is small, an increase in foreign ownership reduces the joint outputs of firms f and d, and the reduction dominates the expansion of firm h’s output, leading to a lower industry output, which in turn results in a higher price. On the other hand, if $k$ is large enough, firm f gains control of firm d, and further increase in foreign ownership strengthens the two firms as a single entity, enabling it to compete with firm h by expanding output, thus lowering the price.

5 Welfare

In this section, we look into the welfare effects of commercial policies under CBO. For computational simplicity, we assume the following on the ownership of the domestic firms.

Assumption 3 The residual share $(1 - k)$ of firm d’s stocks and all of firm h’s stocks are owned by domestic residents.

We define the domestic welfare $W$ as the sum of the consumer surplus, the domestic firms’ profit and the government revenue:

$$W \equiv U(x, Y) - p_xx - p_yY + \pi^h + (1 - k)\pi^d + tx - sY,$$

where $\partial U/\partial x = p_x$ and $\partial U/\partial Y = p_y$. Totally differentiating $W$ yields:

$$dW = -\{xdp_x + ky^d dp_y\} + \{(p_y - c^h)dy^h + (1-k)(p_y - c^d)dy^d\}$$

$$+ \{tdx + xdt - k(sdy^d + y^d ds)\}. \quad (17)$$

The first three brackets respectively express the terms of trade effect, the resource allocation effect, and the tariff revenue effect.
5.1 Tariff and production subsidy

We are now in a position to state the following proposition, the proof of which is given in the Appendix:

**Proposition 8** Suppose that \( s = 0 \) and \( t = 0 \) hold initially. Then, (i) a small tariff on good \( X \) raises domestic welfare if \((k - k_1)(c^h - c^d) \geq 0\); and (ii) a small production subsidy to good \( Y \) enhances domestic welfare if \((k - k_4)(k - \gamma^2 / (2 - \gamma^2))(c^h - c^d) \leq 0\).

Even though foreign ownership and control cause distortions to outputs, prices and profits, a small tariff or a small production subsidy can shift rents and benefit the domestic country. If \( c^d = c^h \), both the tariff and the production subsidy increase domestic welfare, a la Brander and Spencer (1984). If \( c^d \neq c^h \), on the other hand, it is not a simple rent-shifting argument. Lahiri and Ono (1988) show in a closed economy that an increase in the output of the more efficient firm and a decrease in the output of the less efficient firm enhance welfare, and vice versa. Also Neary (1994) demonstrates that when subsidies are justified, they should be given to the more efficient rather than less efficient firms. In our model, this effect also exists, in addition to the effect of rent-shifting. For example, the tariff raises firm \( d \)'s output and reduces firm \( h \)'s output when \( k > k_1 \). In this case, if firm \( d \) is more efficient than firm \( h \) (i.e., \( c^h > c^d \)), then the tariff improves welfare.

5.2 Foreign ownership regulation

We next examine the effect of the foreign ownership regulation on the host country’s welfare. To this end, we need to consider the stock market explicitly. Following Grossman and Hart (1980) and Flath (1991), however, we simply assume a competitive stock price, \( \rho = \pi^d \) (where \( \rho \) is the price of firm \( d \)'s stock), under which the domestic stockholders are indifferent to sell or buy the stock. Thus, the domestic surplus from the sales of firm \( d \)'s stock to firm \( f \) (i.e., \((\rho - \pi^d)dk\)) becomes zero.

Propositions 6 and 7 suggest that the welfare change is generally ambiguous because the consumers and the locally-owned firm are affected in opposite ways from an increase in foreign ownership. When the prices of both goods fall, however, we find that the gain in the consumer surplus dominates the loss in the profit of the local firm. Therefore, we establish the following proposition whose proof is given in the Appendix:

**Proposition 9** Suppose that \( s = t = 0 \) and \( \rho = \pi^d \) hold. An increase in the foreign ownership improves the domestic welfare if \( k \leq k \leq \gamma^2 / (3 - 2\gamma^2) \) and \( c^h \geq c^d \).
5.3 Foreign control regulation

When the foreign firm $f$ holds a large control power, the acquired firm $d$ ignores the interests of the minority shareholders. However, in many countries, antitrust law or corporate law obliges an acquiring firm to serve the interests of the minority shareholders. The domestic government may want to protect the interests of domestic shareholders by strengthening the enforcement of the laws (see O’Brien and Salop 2000, for example). Our model can analyze such a practice by treating it as a reduction in the control power $v$ for a given share $k$.

Under $dk = 0$, we have

$$\begin{pmatrix}
2 & \gamma(1 + k) & \gamma \\
\gamma(\lambda + v) & 2\lambda & \lambda \\
\gamma & 1 & 2
\end{pmatrix}
\begin{pmatrix}
\frac{dx}{dv} \\
\frac{dy^d}{dv} \\
\frac{dy^h}{dv}
\end{pmatrix} =
\begin{pmatrix}
0 \\
-(1 - k)\gamma x / \lambda \\
0
\end{pmatrix}
dv.
$$

Comparative statics yields

$$\frac{dx}{dv} = \frac{\gamma^2 x(1 - k)(2k + 1)}{\lambda \Delta} > 0, \quad \frac{dy^d}{dv} = -\frac{\gamma x(1 - k)(4 - \gamma^2)}{\lambda \Delta} < 0,$$

$$\frac{dy^h}{dv} = \frac{dp_y}{dv} = \frac{\gamma x(1 - k)(2 - k\gamma^2 - \gamma^2)}{\lambda \Delta} > 0, \quad \frac{dY}{dv} = -\frac{\gamma x(1 - k)(2 + k\gamma^2)}{\lambda \Delta} < 0.$$

Moreover, we obtain

$$\frac{dp_x}{dv} = \frac{\gamma^2 x(1 - k)(k\gamma^2 - 2k + 1)}{\lambda \Delta} > 0 \text{ if and only if } k < \frac{1}{2 - \gamma^2}.$$

The reduction in foreign control leads firm $d$ to expand its output, while the other firms to reduce their outputs because all products are substitutes. Since firm $d$’s output and hence $\pi^d$ increases, the minority shareholders benefit from foreign control regulation. The total output of good $Y$ increases and the prices of both goods fall as long as $k$ is small. However, if firm $f$ owns a large share, the price of good $X$ rises because a large production-shifting from goods $X$ to $Y$ is generated.

Under $dk = 0$, the welfare change (A8) in the Appendix is expressed as

$$dW = -x(1 + 2\eta)dp_x + (1 - k)y^d dY - (c^h - c^d)dy^h.$$

The gain in the consumer surplus and the profit of firm $d$ can dominate the loss of firm $h$ as long as $k$ is small. On the other hand, if $k$ is large, a small domestic share in firm $d$’s stock cannot compensate for the loss of firm $h$ and a rise in the price of good $X$. These are stated as
**Proposition 10** Suppose that \( s = t = 0 \) holds. A decrease in the foreign firm’s control power improves domestic welfare, i.e. \( dW/dv < 0 \) for given \( k \), if \( k < 1/(2 - \gamma^2) \) and \( c^d \leq c^h \) hold. However, it lowers domestic welfare if \( k \) is sufficiently large and \( c^d \geq c^h \) holds.

In practice, domestic pressure for the regulation of foreign control power is more likely to be high when \( k \) is high, and thus the results of this section suggest that such regulation could well be counterproductive in a wide range of empirically relevant cases.

### 6 Concluding Remarks

In a model of cross-border partial ownership, we have investigated the effects of commercial policy such as import tariffs, production subsidies and regulation on foreign ownership. In particular, we have explicitly incorporated the aspect of the foreign firm’s control over the domestic firm through CBO. Specifically, control is modeled as the maximization of a weighted sum of the two parties’ profits. It might seem somewhat crude, but our study is the first attempt to take into account partial corporate control in the FDI literature. And it is certainly worthwhile to research into the micro-foundation of such an important phenomenon.

We have found that due to foreign ownership and control, commercial policies may not benefit the 100 percent locally-owned firm, because the foreign firm with corporate control can shift production across borders and thus evade the burden or even take advantage of commercial policy. Many developing countries adopt tariffs, tax holidays and special economic zones to attract FDI. As our results imply, if cross-border control is not properly taken into account, domestic firms could lose profits and the government loses revenue. We hope that the findings in the present paper can shed light on host countries of FDI. Certainly more work needs to be done before more detailed conclusions can be drawn.

CBO is sometimes accompanied by technology transfer. We have not dealt with this issue explicitly, because our focus is rather on the horizontal market-linkages through CBO. One might think, however, that technology transfer is already reflected implicitly in the marginal costs. That is, because of CBO, we have \( c^d < c^h \). As has been shown, the difference becomes crucial when evaluating welfare.

We have assumed that the share of foreign ownership \( k \) and the degree of control \( v \) are exogenously given. It would be interesting to analyze how commercial policies
affect them when they are endogenously determined, especially when many developing
countries impose legal limits on foreign ownership.

We have also assumed that goods $X$ and $Y$ are produced in the two countries sepa-
arrowly. One could allow either country to produce both goods, but the mechanism of
production shifting under foreign ownership and control remains the same, and most of
our qualitative results should carry through.

Finally, the present paper focused only on horizontally related firms. In the tradition
of Markusen (2002) and Qiu and Spencer (2002), it is also interesting to investigate
vertically related firms. Our setup of cross-border ownership and control can be applied.
These remain fruitful avenues for future research.

Appendix

**Interior solution.** We provide the necessary and sufficient conditions for $x$ and $y_d$
to have interior solutions. The FOCs and the demand functions yield the equilibrium
outputs as

\[
\begin{align*}
x &= \frac{\lambda \{3(A - \gamma \delta) - \gamma(B - \delta)(2 + k)\}}{\Delta}, \\
y_d &= \frac{(B - \delta)(2\lambda + v\gamma^2) - \gamma(2v + \lambda)(A - \gamma \delta)}{\Delta}
\end{align*}
\]

where $A \equiv a - cf - t$, $B \equiv b - cd + s$, and $\delta \equiv c^d - c^h$.

First, for a given $k$, (A1) and (A2) give rise to $x > 0$ and $y_d > 0$ if and only if $(A - \gamma \delta) \geq 0$
and

\[
u(k) \equiv \frac{3}{(k + 2)} > \frac{\gamma(B - \delta)}{(A - \gamma \delta)} > \frac{\gamma^2(2v + \lambda)}{(2\lambda + v\gamma^2)} \equiv l(k).
\]

Because $u(k) - l(k) = \Delta / (2\lambda + v\gamma^2) (k + 2) > 0$, there exist parameters $(A, B, \delta)$$\dollar \text{for an interior solution when } k \text{ is given.}^{16}$

Next, we examine the conditions for all $k \in [0, 1]$. $u(k)$ is obviously decreasing in $k$
and, from Assumption 2, $l(k)$ is maximized at $k = \bar{k}$ because

\[
l'(k) = \frac{\gamma^2(4 - \gamma^2)}{(2 + \eta\gamma^2)^2} \frac{d\eta(k)}{dk} \begin{cases} 
\geq 0 \text{ if } k < \bar{k} \\
< 0 \text{ if } \bar{k} < k
\end{cases}
\]

\footnote{A little manipulation brings $x = 0$ if $u(k) < \gamma(B - \delta)/(A - \gamma \delta)$ and $y^d = 0$ if $l(k) > \gamma(B - \delta)/(A - \gamma \delta)$. From the FOC for firm $h$, a non-negative $\delta$ assures $y^h > 0$ if $y^d > 0$.}
Thus, the parameters which make \( x > 0 \) and \( y^d > 0 \) for all \( k \in [0, 1] \) exist if and only if
\[
u(1) - l(\bar{k}) = \frac{(2 - \gamma^2)\bar{k} - \gamma^2}{2k + \gamma^2} > 0 \iff \bar{k} > \frac{\gamma^2}{2 - \gamma^2}.
\]

**Proof of Proposition 3.** From Lemma 2, the change of firm \( f \)'s profit is
\[
\frac{d\pi_f}{dt} = -\left(\gamma x + k y^d\right) \frac{dy^h}{dt} - \gamma x \left(\frac{(1 - v)(1 - k)}{\lambda}\right) \frac{dy^d}{dt} - x.
\]

(A4)

Because the sum of the last two terms is always negative, equation (A4) becomes negative when \( \frac{dy^h}{dt} \geq 0 \), i.e., \( k \leq k_1 \) or \( k = 1 \) (see Proposition 1).

Now if \( k \geq \bar{k} \), (A4) becomes,
\[
\frac{d\pi_f}{dt} \bigg|_{v=1} = -k y^d \frac{dy^h}{dt} - x \left(\gamma \frac{dy^h}{dt} + 1\right).
\]

(A5)

Proposition 1 states that the first term in (A5) is positive. Using (8c), the second term becomes
\[
-\Delta \left(\gamma \frac{dy^h}{dt} + 1\right) \bigg|_{v=1} = \gamma^2 (k + 1)(k + 2) - 6k.
\]

(A6)

The right hand side of (A6) is decreasing in \( k \).

**Proof of Proposition 5.** Using firm \( h \)'s first order condition, \( dp_y = dy^h - ds \), equation (7c) in Lemma 2 can be rewritten as
\[
\frac{d\pi_f}{ds} = -\gamma x \frac{dy^h}{ds} - ky^d \frac{dp_y}{ds} - \gamma x \left(\frac{(1 - k)(1 - v)}{\lambda}\right) \frac{dy^d}{ds}.
\]

The second term is always positive for \( k > 0 \). The last term is negative if \( k < \bar{k} \) and zero if \( \bar{k} \leq k \). Proposition 4 implies that the first term is positive if and only if \( k_4 \leq k \leq \gamma^2/(2 - \gamma^2) \).

**Proof of Proposition 8.** Expression (17) can be rewritten as
\[
dW = x(dt - dp_x) + (tdx - ksdy^d) + d\omega,
\]
where \( d\omega = -ky^d(dp_y + ds) + (p_y - c^h)dy^h + (1 - k)(p_y - c^d)dy^d \). Given \( s = 0 \) and \( t = 0 \) initially, the second term becomes \( (tdx - ksdy^d) = 0 \). It is thus sufficient to show that \( (dt - dp_x) \) and \( d\omega \) are both positive.

First, recall that \( dp_x/ds < 0 \) from (13a). And from (10) we have
\[
1 - \frac{dp_x}{dt} = \frac{(1 - v)(3 - k\gamma^2) + kv(3 - k\gamma^2 - 2\gamma^2)}{\Delta} > 0.
\]

Therefore, \( (dt - dp_x) > 0 \); that is, the producer price of good \( X \) always falls.
From the FOC for firm $h$, we derive $dp_y + ds - dy^h = 0$, and from that for firm $d$, we have $p_y - c^d = y^d + \eta \gamma x$. Thus $d\omega$ can be simplified as
\[
d\omega = -ky^d dy^h + (p_y - c^d)(dY - kdy^d) + (c^d - c^h)dy^h
\]
\[
= (1 - k)y^d dY + \eta \gamma x (dY - kdy^d) + (c^d - c^h)dy^h.
\] (A7)

Because $dY/dt > 0$ in (8d) and $dY/ds > 0$ in (12d), the first term in (18) is positive. The second term is also positive because
\[
dY/ds - kdy^d/ds = \frac{(1 - k)(1 - v)(4 - 2k) + \gamma k(4 - 2k - 2\gamma^2)}{\Delta} > 0,
\]
\[
dY/dt - kdy^d/dt = \frac{\gamma (1 - v)(1 - k)(2 - k) + \gamma v (1 - k^2)}{\Delta} > 0.
\]

Using Proposition 4, the last term in (18), $(c^d - c^h)(dy^h/ds) \geq 0$ if and only if $(k - k_4)(k - \gamma^2/(2 - \gamma^2))(c^h - c^d) \leq 0$. Similarly, from Proposition 1, $(c^d - c^h)(dy^h/dt) \geq 0$ if and only if $(k - k_4)(c^h - c^d) \geq 0$.

**Proof of Proposition 9.** The FOC of firm $f$, $dp_x = dx + \gamma kdy^d + \gamma y^d dk$, and the inverse demand, $dp_x = -dx - \gamma dY$, can be used to simplify the welfare decomposition (17) as
\[
dW = -x(1 + 2\eta)dp_x + \gamma \eta x dk + (1 - k)y^d dY - (c^h - c^d)dy^h.
\] (A8)

In view of Lemma 4, the price of good $X$ decreases if $\bar{k} < k < \gamma^2/(3 - 2\gamma^2)$. Comparative statics yields $dY/dk = [\gamma^2 y^d(2\lambda + v) - \theta \lambda x (2 + k\gamma^2)]/\Delta$, which is positive if $k \geq \bar{k}$. Moreover, $dy^h/dk < 0$ if $k \geq \bar{k}$ (recall Proposition 8). Therefore, we obtain $dW/dk > 0$ if $\bar{k} \leq k \leq \gamma^2/(3 - 2\gamma^2)$. ■
References


Figure 1: The schedule of ownership $k$ and control $v$. 
Figure 2: Production shifting from \( x \) to \( y^d \) (Tariff).

Figure 3: Tariff on good \( X \).
Figure 4: Production Subsidy for good $Y$.

Figure 5: Production shifting from $x$ to $y^d$ (Production Subsidy).
Figure 6: The range of the counter-intuitive case.