STRATEGIC EXPORT POLICY UNDER INTERNATIONAL CROSS OWNERSHIP VS. CROSS SHAREHOLDING†

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ABSTRACT. This paper compares optimal export policies within a Cournot duopoly framework between international cross ownership and cross shareholding structure. We show that compared with cross ownership, cross shareholding structure facilitates collusion between firms and is likely to strengthen each country’s incentive for export subsidy. Moreover cross shareholding also lowers the world welfare in general. Therefore, cross shareholding structure should be regulated or banned.

1. INTRODUCTION

We have accelerated globalization of economic activities over the last century. Rapid growth in commodity trade has not only given rise to new trade in service but also creation of global production-marketing network supported by direct investments and cross licensing of technologies and know-how. This makes the ownership structure of a firm more complicated than we have experienced before. Any firm financing by equities located in a country is never totally owned by the nation herself. Her equities are also partially owned by the other nations. They are even sometimes held by the firms with the same or different nationalities through the mutual shareholding.

Such internationalized ownership of firms should alter the standard welfare implications of trade and industrial policies. This is because the traditional policy analysis is based on an explicit nationality of the firms involved. Any domestic firms are assumed to be 100% “domestic” in the sense that her equities are all held by the domestic residents including the

Date: March, 2004.

Key words and phrases. strategic trade policy; international cross ownership; international cross shareholding.

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“domestic” firms. The well-known argument of strategic trade policy towards international oligopoly shares the same assumption.

Thus we face a question. Does internationalized ownership of firms affect the decision of an individual country’s government seeking for her own national interest or welfare? If it does, how? Bhagwati and Brecher[1], Brecher and Bhagwati[3] and Bretcher and Findlay [4] have studied the foreign ownership under perfect competition using the classical approaches. More recent papers such as Lee[12], Dick[5], Welzel[22] and Long and Soubeyran[15] have discussed some detailed questions. However since internationalized ownership may also take the form of mutual shareholding among the firms themselves, it is also of a great interest to see whether the mutual shareholding only among the firms yield any different implications compared with the one only among the residents of the countries resided by those firms. To distinguish between these two types of mutual shareholding, we call the former only among the firms **cross shareholding** and the latter only among the residents **cross ownership**.

This set of questions motivates the analysis in the present paper. More specifically, we construct a familiar third-market export competition model in a Cournot duopoly, and clarify the differences in the market performance and the government’s strategic incentive for export subsidies between cross ownership and cross shareholding among the exporting countries. We will find that compared with cross ownership, cross shareholding structure is more likely to strengthen each country’s incentive for export subsidy and to facilitate collusion between the firms as in Krishna and Itoh[10] and Krishna[9]. In view of these dual types of distortions, our analysis implies that cross shareholding should be regulated or banned.

The remainder of the paper is organized as follows. In section 2, the basic third-market model as a general framework is introduced. Section 3 studies the effects under international cross ownership on optimal subsidy and employs some specific cases to work out the results in more details. In the same way, Section 4 extends the same examination under cross shareholding and compares the optimal subsidy between two structures. The world joint welfare for the optimal export policy is discussed in Section 5. Finally, the conclusion is summed up in Section 6.

2. **Basic Model**

There are three countries, domestic, foreign and the third country trading a certain good in the world. The third country has no domestic production and imports the good
in question from the other two countries. Each exporting country has a single firm, the home firm (with subscript $H$) in the home country and the foreign firm (with subscript $F$) in the foreign country. Here the firm is named either “home” or “foreign” in view of the country of her location. Assume that the marginal cost of production, $c^i_i (i = H, F)$ for firm $i$, is constant for each firm and that the markets of the countries are segmented. Then the assumption of constant marginal cost allows us to discuss the third country market independent of the domestic market in each exporting country. Let $x_H$ (or $x_F$) denote the output (=export) of the home (or foreign) firm to the third country, $X (= x_H + x_F)$ the total output (=export), $p$ the market price in the third country, and $p = P(X)$ the inverse market demand function in the third country.

Following the framework of Brander and Spencer[2], international trade is modelled as a two-stage game involving governments and firms as follows. In the first stage, given the ownership structure of the firms, each government determines its country-specific export subsidy $s_i (i = H, F)$ only to the firm located in the own country simultaneously. In the second stage, after observing the subsidy profile $s = (s_H, s_F)$, the firms engage in quantity competition in the third market. The equilibrium is solved by backward induction from the second stage.

Before inquiring into the equilibrium properties, let us first explain the difference between two structures: cross ownership and cross shareholding. When the equities of a firm are international owned by the residents of both countries, we call it cross ownership. When the equities are international owned by the shareholders of both firms, it will be called as cross shareholding. We depict these two structures into the following figure.

![Cross-Ownership vs. Cross-Shareholding](image)

**Figure 1.** cross ownership vs. cross-shareholding
Given the subsidy profile \( s \), the profit earned by firm \( i \) through her own export is expressed by

\[
\pi^i(x_i, x_j, c_i) = \{P(x_H + x_F) - c_i\} x_i,
\]

where \( c_i = c^0_i - s_i \) (\( i = H, F \)). Let \( \sigma_{ij} \in [0, 1] \) denote the percentage share of firm \( j \)'s equities owned by country \( i \) where \( i, j = H, F \) and \( \sigma_{ii} + \sigma_{ij} = 1 \). Values of \( \sigma_{ij} \) are assumed to be exogenously given. Then national welfare of country \( i \) is expressed by

\[
\tilde{W}^i(x, s, \sigma) = \sigma_{ii} \pi^i(x_i, x_j, c^0_i - s_i) + (1 - \sigma_{jj}) \pi^j(x_j, x_i, c^0_j - s_j) - s_i x_i,
\]

where \( \sigma = (\sigma_{HH}, \sigma_{FF}) \) represents what we call the **bilateral ownership structure** of the firms. Without loss of generality, we assume

**Assumption 1.** \( \sigma_{ii} \in (\frac{1}{2}, 1) \) for \( i = H, F \).

Without this assumption, there is no essential meaning to refer to the home firm as the “home” firm.

Although each government cares her own country’s welfare regardless of her ownership structure, the objective of each firm critically depends on different structure. When there is cross ownership between the exporting countries, then each firm maximizes her own profit. But when there is cross shareholding, each firm now maximizes not her own profit but the value of the firm defined as the sum of the dividends from each firm’s exporting activity. That is, the value of firm \( i \) is given by

\[
\tilde{V}^i(x, c, \sigma) = \sigma_{ii} \pi^i(x_i, x_j, c_i) + (1 - \sigma_{jj}) \pi^j(x_j, x_i, c_j).
\]

Such value maximization is incomplete, for each firm can choose on the own export only and she has no explicit measures for coordinating their exports and production.

Keep in mind this difference in the objective and the constrained set of actions imposed on cross shareholding, and let us first explore the properties of the cross ownership equilibrium as our benchmark for analysis.
3. cross ownership Equilibrium

3.1. Second-Stage Equilibrium. As mentioned earlier, we solve the equilibrium by backward induction from the second-stage. To make our analysis meaningful, we impose a standard set of assumptions for Cournot duopoly.¹

Throughout the rest of the paper, we denote the elasticity of the slope of its inverse demand curve as $E(X) = -\frac{XP'(X)}{P(X)}$ and the market share of each firm as $\theta_i = \frac{x_i}{X} (i = H, F)$. In a Cournot duopoly framework, $\theta_H + \theta_F = 1$. Using these notations, we may express additional assumptions usually required for comparative statics in a Cournot duopoly as below.

Assumption 2. Each firm’s profit function $\tilde{\pi}^i(x_i, x_j, c_i)$ is strictly concave in its own output, i.e.,²

$$E(X) < \min \left( \frac{2}{\theta_H}, \frac{2}{\theta_F} \right).$$

Given Assumption 2, firm $i$’s reaction function denoted by $R_i^O(x_j, c_i)$ is defined as a solution to the following first-order condition for maximizing (1) with respect to its own output and superscript $O$ shows the variables associated with the equilibrium value under cross ownership:

$$0 = \frac{\partial \tilde{\pi}^i(R_i^O(x_j, c_i), x_j, c_i)}{\partial x_i} \bigg|_{x_i = R_i^O(x_j, c_i)} = P \left( R_i^O(x_j, c_i) + x_j \right) - c_i + R_i^O(x_j, c_i)P' \left( R_i^O(x_j, c_i) + x_j \right) .$$

(2)

Let us denote $x_i^O(c_i, c_j)$ ($i, j = H, F; \ j \neq i$) firm $i$’s equilibrium output which should satisfy:

¹More specifically, we assume:

Assumption B-1: $P(X)$ is strictly decreasing, continuously differentiable, and $P(0) > c_i > P(+\infty)$ for $i = H, F$, where $P(0) = \lim_{X \to +0} P(X)$ and $P(+\infty) = \lim_{X \to +\infty} P(X)$.

Assumption B-2: $c_i < p_m = P(x_m^0(c_i))$ where $x_m^0(c_i) = \arg_{x_i} \tilde{\pi}^i(x_i, 0, c_i)$.

Assumption B-1 implies that each firm, when it is a monopolist in the third country market, has an incentive to produce a strictly positive output. And Assumption B-2 insures that neither firm can become a monopolist as the rival has an incentive to enter the market given the monopoly output and price.

²The condition below is obtained once we note

$$\frac{\partial^2 \tilde{\pi}^i(x_i, x_j, c_i)}{\partial x_i^2} = 2P'(x_i + x_j) + x_i P''(x_i + x_j) = P'(x_i + x_j) \cdot (2 - \theta_i E(x_i + x_j)) < 0.$$
Accordingly, \( X^O(c_H, c_F) \) denote the equilibrium total output, \( P^O(c_H, c_F) \) the associated equilibrium price, and \( \tilde{\pi}^O(c_i, c_j) \) the equilibrium profit of firm \( i \) under international cross ownership.

The following properties of the reaction function should already be familiar by using the implicit function theorem in (2).

\[
R^O_i(x_j, c_i) \overset{\text{def}}{=} \frac{\partial R^O_i}{\partial x_j} = \begin{cases} 
-1 - \theta^O_i \frac{E^O}{2 - \theta^O_i E^O} & \text{strategic substitution} \\
\frac{1}{P'(X^O)(2 - \theta^O_i E^O)} & \text{strategic complementarity}
\end{cases}
\]

where \( E^O(c_H, c_F) = -\frac{X^O(c_H, c_F) P''(X^O(c_H, c_F))}{P'(X^O(c_H, c_F))} \) and \( \theta^O_i(c_i, c_j) = \frac{x^O_i(c_i, c_j)}{X^O(c_H, c_F)} \).

(4) represents that the firm’s optimal response output is decreasing in the rival’s (by definition) if and only if there holds a strategic substitute relationship between the two outputs. In view of Assumption 2 (strict concavity of the profit function with respect to the own output), strategic substitution holds (i) for concave inverse demand functions (i.e., \( E(X) \leq 0 \)) or (ii) for a sufficiently small market share for convex demand functions. (5) implies that an increase in the own unit cost always lowers the optimal response output.

Hereafter to sharpen the results for the later discussion, we often resort to the assumption

**Assumption 3.** Each firm’s output is a strategic substitute to the other’s, i.e.,

\[
E \left( X^O(c_H, c_F) \right) < \min \left\{ \frac{1}{\theta^O_H(c_H, c_F)}, \frac{1}{\theta^O_F(c_H, c_F)} \right\}.
\]

Comparative statics on this second-stage equilibrium requires a certain condition for equilibrium stability. When we assume a standard Cournot adjustment process at disequilibria, the condition for local stability is given by

\[\dot{x}_i = \alpha_i \{ R^O_i(x_j, c_i) - x_i \} \text{ for } i, j = H, F (j \neq i) \]

where \( \alpha_i \) is a strictly positive constant showing the adjustment speed.
\begin{align}
\Delta_R^O & \overset{\text{def}}{=} 1 - R^H_x R^F_x = \frac{3 - E^O}{(2 - \theta^O_H E^O)(2 - \theta^O_F E^O)} > 0, \\
\end{align}

Thus we assume

**Assumption 4.** For all the relevant equilibria, there holds \( E^O(c_H, c_F) < 3 \).

This assumption is satisfied, for example, when the demand function is linear. And the assumption is less stringent than the standard assumption that the slope of each firm’s reaction curve is strictly less than unity in the absolute value.

### 3.2. Summary of Comparative Statics.

Under the cross ownership structure given by \( \sigma^O = (\sigma^O_{HH}, \sigma^O_{FF}) \), there hold the same comparative statics results as in Brander and Spencer[2]. This is because no changes in the bilateral ownership structure affect either firm’s output decision. We list the results only for the convenience of the later analysis.

\begin{align}
\frac{\partial x^O_i(c_i, c_j)}{\partial c_i} &= \frac{R^O_i(x^O_j(c_j, c_i), c_i)}{\Delta_R^O} = \frac{2 - \theta^O_j E^O}{P'(X^O)(3 - E^O)} < 0, \\
\frac{\partial x^O_i(c_j, c_i)}{\partial c_i} &= R^O_i(x^O_i(c_i, c_j), c_j) \frac{\partial x^O_i}{\partial c_i} = -\frac{1 - \theta^O_j E^O}{P'(X^O)(3 - E^O)} > 0.
\end{align}

where use was made of (4) and Assumption 2-4. Thus an increase in a firm’s unit cost (for example by virtue of a decrease in the government subsidy) lowers the own equilibrium output and raises the rival’s output when there holds strategic substitution.

An increase in a firm’s unit cost, however, necessarily decreases the equilibrium total output and raises the market price, as demonstrated by

\begin{align}
\frac{\partial X^O(c_H, c_F)}{\partial c_i} &= \left\{ 1 + R^O_x(x^O_i, c_j) \right\} \frac{\partial x^O_i(c_H, c_F)}{\partial c_i} = \frac{1}{P'(X^O)(3 - E^O)} < 0, \\
\frac{\partial P^O(c_H, c_F)}{\partial c_i} &= P'\left(X^O(c_H, c_F)\right) \frac{\partial P^O}{\partial c_i} = \frac{1}{3 - E^O} = \frac{\partial P^O(c_H, c_F)}{\partial c_j} > 0.
\end{align}

where use was made of (7) and Assumption 4. The results are straightforward from the assumption of the perfect substitutes produced by the firms and the constant marginal costs.\(^4\)

The equilibrium profit of each firm should then change as expressed by:

\(^4\)In fact, summation of the first-order conditions for profit maximization over the firms gives rise to
\[ \frac{\partial \hat{\pi}_i^O(c_i, c_j)}{\partial c_i} = x_i^O P'(X^O) \frac{\partial x_j^O}{\partial c_i} - x_i^O = \frac{-x_i^O[(3 - E^O) + (1 - \theta_j^O E^O)]}{3 - E^O} < 0 \]

\[ \frac{\partial \hat{\pi}_j^O(c_i, c_j)}{\partial c_i} = x_j^O P'(X^O) \frac{\partial x_i^O}{\partial c_i} = \frac{x_j^O(2 - \theta_j^O E^O)}{3 - E^O} > 0 \]

where use was made of (2), (7), (8) and Assumption 3.

3.3. **Optimal subsidy for welfare maximization.** At the first stage, the governments can predict the resulting second-stage equilibrium given the own choices over the subsidies. Country \( i \)'s welfare is now given by

\[ W_i^O(\sigma^O, s) = \tilde{W}_i(x_i^O(c_i^0 - s_i, c_j^0 - s_j), x_j^O(c_j^0 - s_j, c_i^0 - s_i), s_i, s_j, \sigma^O) = \sigma_{ii}^O \hat{\pi}_i^O(c_i^0 - s_i, c_j^0 - s_j) + (1 - \sigma_{ii}^O) \hat{\pi}_j^O(c_j^0 - s_j, c_i^0 - s_i) - s_i x_i^O(c_i^0 - s_i, c_j^0 - s_j). \]

We further assume\(^5\)

**Assumption 5.** The welfare function of each country under cross ownership is strictly concave in the own export subsidy, i.e.,

\[ \frac{\partial^2 W_i^O(\sigma^O, s)}{\partial s_i^2} < 0 \quad (i = H, F). \]

Each government maximizes her national welfare by choosing the optimal export subsidy, taking into account the responses of both firms. The Nash solution for the first-order condition should satisfy:

\[ 0 = \frac{\partial W_i^O(\sigma^O, s)}{\partial s_i} = \sigma_{ii}^O x_i^O P'(X^O) \left( -\frac{\partial x_j^O}{\partial c_i} \right) - (1 - \sigma_{ii}^O) x_i^O - s_i \left( -\frac{\partial x_i^O}{\partial c_i} \right) - (1 - \sigma_{jj}^O) \frac{\partial \hat{\pi}_j^O}{\partial c_i}. \]

which implies that the equilibrium total output depends only on the sum of the unit costs over the industry. See Varian et al.?\(^5\)

\(^5\)The assumption below holds, for example, when the import demand function is linear.
The terms on the right-hand side of the second equation show the decomposition of strategic export subsidization under cross ownership. The subsidy incentive structure is easy to understand once we compare the incentive under cross ownership ($\sigma_{kk} \neq 1$ for $k = H, F$) with the one in the standard strategic export subsidy model ($\sigma_{kk} = 1$ for $k = H, F$). For this purpose, we may rewrite (11) as below.

$$\frac{\partial W^O_{iO}(\sigma^O, s)}{\partial s_i} = x^O_i P'(X^O) \left( -\frac{\partial x^O_j}{\partial c_i} \right) - s_i \left( -\frac{\partial x^O_i}{\partial c_i} \right),$$

and thought

$$-(1 - \sigma^O_{ii}) x^O_i P'(X^O) \left( -\frac{\partial x^O_j}{\partial c_i} \right) - (1 - \sigma^O_{ii}) x^O_i - (1 - \sigma^O_{jj}) \frac{\partial \hat{\pi}^j_{O}}{\partial c_i}.$$

The terms on the first line show the subsidy incentive found in the standard model, while those on the second line show the new sources of subsidy incentive specific to cross ownership. For the later reference, we may call the terms on the second line the additional incentive under cross ownership and denote it by $I^O_i(\sigma^O, s)$ for country $i$, i.e.,

$$(12) I^O_i(\sigma^O, s) = -(1 - \sigma^O_{ii}) x^O_i P'(X^O) \left( -\frac{\partial x^O_j}{\partial c_i} \right) - (1 - \sigma^O_{ii}) x^O_i - (1 - \sigma^O_{jj}) \frac{\partial \hat{\pi}^j_{O}}{\partial c_i}.$$

The above three terms on the right-hand side are all strictly negative in view of Assumptions 1 and 3. The first term $-(1 - \sigma^O_{ii}) x^O_i P'(X^O) \left( -\frac{\partial x^O_j}{\partial c_i} \right)$ represents what we may call the cross rent-shifting effect. Export subsidy to the home firm, through the standard rent-shifting effect, increases her profit, but it leads to an increase in the dividend given to the foreign firm. This effect becomes smaller in the absolute value as the own share of the home firm gets larger. In other words, with the greater own share of the home firm, the government cares less about the outflow of the home firm’s rent through the cross rent-shifting effect.

The second positive term $-(1 - \sigma^O_{ii}) x^O_i$ shows the subsidy outflow effect, for it shows the portion of the subsidy expense going abroad as an increased dividend to the foreign residents. Also its absolute value gets smaller as the own share of the home firms becomes bigger.

The last term $-(1 - \sigma^O_{jj}) \frac{\partial \hat{\pi}^j_{O}}{\partial c_i}$ shows the dividend suppression effect, for it shows the portion of a decrease in the dividend from the shared foreign firm.
These three effects weakens the subsidy incentives in the presence of cross ownership, which we call the **cross ownership augmented subsidy incentive effect**. Note that no changes in the cross ownership structures affect the comparative statics results for the market outcomes given the export subsidy profile \( s = (s_H, s_F) \). And thus we find that an increase in the own share of the home firm increases the cross ownership augmented subsidy incentive, leading to an increase in the optimal export subsidy.\(^6\)

**Lemma 1.** An increase in the domestic residents’ ownership share over the domestic firm induces the government to raise the strategic export subsidy rate.

Similarly, an increase in the the foreign ownership over the foreign firm augments the dividend suppression effect, leading to a rise in the optimal export subsidy.\(^7\)

**Lemma 2.** An increase in the foreign residents’ ownership share over the foreign firm induces the home country’s government to raise the strategic export subsidy rate.

When there is a decrease in the domestic residents’ claim against the foreign firm’s profits, the home country’s government need care less the decrease in the foreign firm’s profits, which gives rise to stronger export subsidy incentive a la Brander-Spencer.

Let \( \gamma_i(s_j, \sigma) \) be a solution to (11), which represents country \( i \)'s reaction function. Then the full-game Nash equilibrium subsidy profile is thus defined as a solution to

\[
s_i^O = \gamma_i(s_j^O, \sigma) \quad \text{for} \quad i = H, F.
\]

The equilibrium subsidy profile depends on the cross ownership structure \( \sigma \), the relation of which we express by \( s_i^O(\sigma) \). In view of the two Lemmas above, it is straightforward to establish

\(^6\)More specifically, we can prove the result as follows. Let \( \gamma_i(s_j, \sigma) \) be a solution to (11), which represents country \( i \)'s reaction function. It is straightforward to see the effect of a change in the cross ownership structure on the optimal response subsidy, for the market outcome is independent on the cross ownership structure. In view of Assumption 5, application of the implicit function theorem yields

\[
\frac{\partial \gamma_i(s_j, \sigma)}{\partial \sigma_{ii}} = -\frac{\partial^2 W_{iO}/\partial \sigma_{ii} \partial s_i}{\partial^2 W_{iO}/\partial s_i^2} \propto \frac{\partial^2 W_{iO}/\partial \sigma_{ii} \partial s_i}{\partial^2 W_{iO}/\partial s_i^2} = -\frac{\partial^2 \hat{\pi}_{iO}}{\partial c_i} > 0.
\]

An increase in the own residents’ share of the domestic firm makes the government care more the profit of the domestic firm, which strengthens the strategic export subsidy incentive.

\(^7\)The result follows from

\[
\frac{\partial \gamma_i(s_j, \sigma)}{\partial \sigma_{jj}} = -\frac{\partial^2 W_{iO}/\partial \sigma_{jj} \partial s_i}{\partial^2 W_{iO}/\partial s_i^2} \propto \frac{\partial^2 W_{iO}/\partial \sigma_{jj} \partial s_i}{\partial^2 W_{iO}/\partial s_i^2} = \frac{\partial^2 \hat{\pi}_{iO}}{\partial c_i} > 0.
\]
Proposition 1. The equilibrium export subsidy rate under the cross ownership of firms is strictly lower than in its absence, i.e., \( s^O_i(\sigma^O_{ii}, \sigma^O_{jj}) < s^O_i(1, 1) \).

More specifically, solving for the optimal subsidy by using (9) and (10), we find that the equilibrium subsidy rate chosen by country \( i \)’s government should satisfy:

\[
\begin{align*}
\sigma_{ii} \frac{\partial \tilde{\pi}^O_i}{\partial c_i} + (1 - \sigma_{jj}) \frac{\partial \tilde{\pi}^O_j}{\partial c_i} + x_i^O \left( \sigma_{ii} \frac{\partial x_i^O}{\partial c_i} \right) \\
= \sigma_{ii} x_i^O P'(X^O) R^O_{ij} (1 - \sigma_{jj}) \frac{\partial x_j^O}{\partial c_i} + (1 - \sigma_{jj}) x_j^O P'(X^O).
\end{align*}
\]

The right-hand side shows again the three effects governing the optimal export subsidy, i.e., (i) the direct rent-shifting effect, (ii) the subsidy outflow effect and (iii) the dividend effect. The first effect is strictly positive by virtue of Assumption 3, while the other two effects are negative. Thus the sign of the equilibrium subsidy rate is generally ambiguous. However, there are several special cases with determinate signs as the following lemma shows.

Lemma 3. For \( i, j = H, F; j \neq i \)

(i) \( s^O_i(0, 0) < 0 \)  
(ii) \( s^O_i(1, 1) > 0 \)  
(iii) \( s^O_i(\sigma^O_{ii}, \sigma^O_{jj}) < s^O_i(1, 0) \) for all \( \sigma_{kk} < 1 \).

To clarify how the cross ownership structure governs the equilibrium export subsidy profile, let us focus our attention to linear demand cases and see what factors induce the government to impose export subsidies. At the marginal subsidy rate when \( s_i = 0 \), the first-order condition of (11) can be written as

\[
\begin{align*}
\left. \frac{\partial W^O_i(\sigma^O_{ii}, \sigma^O_{jj})}{\partial s_i} \right|_{s_i=0} &= 4\theta_i^O \sigma_{ii} + 2\theta_j^O \sigma_{jj} - (3 - \theta_j^O) \\
&= 4\theta_i^O \sigma_{ii} + 2\theta_j^O \sigma_{jj} - (3 - \theta_j^O)
\end{align*}
\]

where use was made of the assumption of a linear demand as we all as (7), (8), (9) and (10). The government has an incentive to subsidize the domestic firm if and only if \( 4\theta_i^O \sigma_{ii} + 2\theta_j^O \sigma_{jj} - (3 - \theta_j^O) \geq 0 \). (17) also gives rise to

Lemma 4. Under cross ownership, the government’s subsidy incentive is dependent on the market share \( \theta_i^O \) in the case of linear demand function.

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8 Welzel[22] and Dick[5] also explicitly identified the three effects on optimal export subsidization.

9 When each firm’s output is a strategic complementarity to the rival’s, then the equilibrium export subsidy rate is definitely negative, i.e., each country’s government imposes a strictly positive rate of export tax.

10 See Appendix A for the detail.
Figure 2 illustrates the pairs of the shares letting the government of country $i$ find zero subsidy optimal (14) for both countries, curve HH for the home country and curve FF for the foreign country, when the unit costs are the same between the two firms. The unit square $[0, 1] \times [0, 1]$ is divided into four areas, each representing both governments’ incentives for subsidization or taxation. Figure 2 implies that both governments have an incentive to implement a subsidy only when the domestic ownership share over the domestic firm is sufficiently high as shown in the shadow area. The standard export subsidy model a la Brander and Spencer corresponds to only the top corner $(1, 1)$. But in the presence of cross ownership, each government may rather tax the own firm. In fact, when the own equity share is half for both firms, they decide to tax the exports.\textsuperscript{11} This is because the case is exactly the same as when there is a single exporting country holding two firms. When the two domestic firms compete over the export market, the competition becomes excessive from the view-point of joint profit maximization which is equivalent to maximization of the exporting country’s welfare. The government thus has an incentive to suppress, rather than promote, their competition so as to maximize the national welfare.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Optimal Subsidy under cross ownership in the Linear Demand Case}
\end{figure}

\textsuperscript{11}See Appendix A for the detail.
4. cross shareholding Equilibrium

Next consider the cross shareholding case in which the equities of a firm are international owned by the shareholders of both firms. We also assume the heterogeneous mutual shareholding structures of the firms denoted by $\sigma^S = (\sigma^S_H, \sigma^S_F)$ and superscript $S$ associates with the equilibrium value under cross shareholding case. Define $\mu_i(\sigma^S) \overset{\text{def}}{=} \frac{\sigma^S_{ji}}{\sigma^S_{ii}} = \frac{1 - \sigma^S_{jj}}{\sigma^S_{ii}}$ as the relative cross shareholding ratio of each firm. Note that Assumption 1 assures $\mu_i \in (0, 1)$. For the later analysis, we further define

$\Delta_\mu \overset{\text{def}}{=} 1 - \mu_i \mu_F = \frac{\sigma^S_{HH} + \sigma^S_{FF} - 1}{\sigma^S_{HH} \sigma^S_{FF}},$ (15)

which should be strictly positive by virtue of Assumption 1. Since we often resort to the above results, we summarize it as in the following lemma.

**Lemma 5.** Under Assumption 1, there holds (i) $\sigma^S_{HH} + \sigma^S_{FF} > 1$, (ii) $\mu_i \in (0, 1)$, and (iii) $\Delta_\mu > 0$.

4.1. Second-Stage Equilibrium. Similarly, we solve the equilibrium by backward induction from the second-stage. As mentioned earlier, the value function of each firm is defined as the sum of fractional profit of both firms accruing to the domestic shareholders expressed as:

$V^i = \tilde{V}^i(\mathbf{x}, \mathbf{c}, \sigma^S) = \sigma^S_{ii} \tilde{\pi}^i(x_i, x_j, c_i) + (1 - \sigma^S_{jj}) \tilde{\pi}^j(x_j, x_i, c_j) \quad (i, j = H, F; \ j \neq i).$ (16)

Compared with the cross ownership case, each firm now aims to maximize not her own profit, but the firm value defined in (16). To make our analysis meaningful, we also assume the second-order condition is satisfied as the preceding section:

**Assumption 6.** The value function of each firm is strictly concave in its own output, i.e.,

\[ 0 > \frac{\partial^2 \tilde{V}^i(\mathbf{x}, \mathbf{c}, \sigma^S)}{\partial x_i^2} = \sigma^S_{ii} [2P'(x_i + x_j) + x_i P''(x_i + x_j)] + (1 - \sigma^S_{jj}) x_j P''(x_i + x_j) \]

\[ = \sigma^S_{ii} P'(x_i + x_j) \cdot [2 - (\theta_i + \mu_i \theta_j) E]. \]

\[ \text{max} \quad \sigma^S_{ii} P'(x_i + x_j) \cdot [2 - (\theta_i + \mu_i \theta_j) E]. \]
\[ E(X) < \min \left( \frac{2}{\theta_H + \mu_H \theta_F}, \frac{2}{\theta_F + \mu_F \theta_H} \right) \]

Under cross shareholding, each firm’s reaction function is denoted as \( R^i_S(x_j, \sigma^S, c_i) \) corresponding with the \( R^i_O(x_j, c_i) \) under cross ownership. Given assumption 6, it is a solution to the first-order condition for maximizing (16) with respect to its own output. It should satisfy:

\[
0 = \left. \frac{\partial \tilde{V}^i(R^i_S(x_j, \sigma^S, c_i), x_j, c, \sigma^S)}{\partial x_i} \right|_{x_i=R^i_S(x_j, \sigma^S, c_i)}
= \sigma^S_i \frac{\partial \tilde{\pi}^i(R^i_S(x_j, \sigma^S, c_i), x_j, c_i)}{\partial x_i}
+ (1 - \sigma^S_j) \frac{\partial \tilde{\pi}^j(x_j, R^i_S(x_j, \sigma^S, c_i), c_j)}{\partial x_i}
= \sigma^S_i [P(R^i_S(. + x_j) - c_i] + [\sigma^S_i R^i_S(. + (1 - \sigma^S_j)x_j] P'(R^i_S(. + x_j)
\]

Solving the above equation by using the definition of \( \mu_i \) yields

\[
0 = P(R^i_S(. + x_j) - c_i] + R^i_S(. + x_j) P'(R^i_S(. + x_j) + \mu_i x_j P'(R^i_S(. + x_j).
\]

To compare the reaction curves under both structures, we reexamine the first-order condition of (17) at the the equilibrium point under cross ownership. Substituting (2) into (17) gives

\[
\left. \frac{\partial \tilde{V}^i(R^i_O(x_j, c_i), x_j, c, \sigma^S)}{\partial x_i} \right|_{x_i=R^i_O(x_j, c_i)} = (1 - \sigma^S_j) \frac{\partial \tilde{\pi}^j(x_j, R^i_O(x_j, c_i), c_j)}{\partial x_i} < 0.
\]

Hence the relevant properties of value function in Assumption 6 implies \( R^i_S(x_j, \sigma^S, c_i) < R^i_O(x_j, c_i) \) given the rival firm’s output \( x_j > 0 \). This is because under cross shareholding each firm takes into account the effect of the own output increase to suppress the shared firm’s profit. Her output decision coincides that under cross ownership either when the shared firm produces none or when she fully owns the domestic firm’s shares. Thus we find that cross shareholding facilitates collusion between the shared firms.

Summarize the above results into the following corollary:

**Corollary 1.** The best-response output of each firm is no larger under cross shareholding than under cross ownership. More specifically, for the equilibrium output under cross ownership vs. cross shareholding,
\[
\begin{align*}
R^S(x_j, \sigma_{ii}^S, \sigma_{jj}^S, c_i) &< R^O(x_j, c_i) \quad \text{when} \quad \sigma_{ii}^S, \sigma_{jj}^S < 1 \quad \text{and} \quad x_j > 0 \\
R^S(x_j, 1, 1, c_i) &= R^O(x_j, c_i) \quad \text{or} \quad R^S(0, \sigma_{ii}^S, \sigma_{jj}^S, c_i) = R^O(0, c_i).
\end{align*}
\]

Figure 3 illustrates the related results in Corollary 1. $R^O$ and $R^S$ denote the reaction curves under cross ownership and cross shareholding respectively. The two reaction curves start from the same output when monopolizing the market. Given the same output by the rival firm, the optimal response outputs under cross ownership are always larger than those under cross shareholding. Point $E^O$ and $E^S$ denote the corresponding equilibrium outputs.

Figure 3. Reaction Curves under cross ownership vs. Cross-shareholding

Denote $x^S_i(\sigma^S, c) \ (i = H, F)$ as firm $i$’s equilibrium output under cross shareholding. It should satisfy:

\[(19) \quad x^S_i(\sigma^S, c) = R^S(x^S_j(\sigma^S, c), \sigma^S, c_i) \quad (i, j = H, F; \ j \neq i).\]

Accordingly, $X^S(\sigma^S, c)$ denotes the equilibrium total output, $P^S(\sigma^S, c)$ the associated equilibrium price, and $\hat{\pi}^S(\sigma^S, c)$ the associated equilibrium profit of Firm $i$ under cross shareholding in the same way.

For the convenience of later analysis, we can also show the following properties of each firm’s reaction function by using the implicit function theorem in (18).
\[
R_i^S(x_j, \sigma^S, c_i) \overset{\text{def}}{=} \frac{\partial R_i^S}{\partial x_j} = \frac{(1 - \theta^S_i E^S) + \mu_i (1 - \theta^S_j E^S)}{2 - (\theta_i + \mu_i \theta_j) E^S}
\]

\[
\begin{cases}
< 0 & \text{Strategic Substitution} \\
> 0 & \text{Strategic Complementarity}
\end{cases}
\]

(20)

\[
R_c^S(x_j, \sigma^S, c_i) \overset{\text{def}}{=} \frac{\partial R_i^S}{\partial c_i} = 1 - \frac{P'(X^S)}{P(X^S)} \left[ 2 - (\theta^S_i + \mu_i \theta^S_j) E^S \right] > 0
\]

(21)

\[
R_{\sigma_{ii}}^S(x_j, \sigma^S, c_i) \overset{\text{def}}{=} \frac{\partial R_i^S}{\partial \sigma_{ii}} = \frac{P(X^S) - c_i + x^S_i P'(X^S)}{\sigma^S_{ii} P(X^S) \left[ 2 - (\theta^S_i + \mu_i \theta^S_j) E^S \right]} > 0
\]

(22)

\[
R_{\sigma_{jj}}^S(x_j, \sigma^S, c_i) \overset{\text{def}}{=} \frac{\partial R_i^S}{\partial \sigma_{jj}} = \frac{x^S_j}{\sigma^S_{jj} \left[ 2 - (\theta^S_i + \mu_i \theta^S_j) E^S \right]} > 0
\]

(23)

where \( E^S(\sigma^S, c) = \frac{X^S(\sigma^S, c) P'(X^S(\sigma^S, c))}{P(X^S(\sigma^S, c))} \), \( \theta^S_i(\sigma^S, c) = \frac{x^S_i(\sigma^S, c)}{X^S(\sigma^S, c)} \).

**Lemma 6.** The reaction function of each firm under cross shareholding satisfies:

(i) Each firm’s output is a strategic substitute to the other’s under cross shareholding if both firm’s outputs are mutually strategic substitutes under cross ownership.

(ii) An increase in the unit cost decreases each firm’s best-response output.

(iii) An increase in each firm’s own share \( \sigma_{ii} \) increases her best-response output.

(iv) An increase in the other firm’s own share \( \sigma_{jj} \) increases each firm’s best-response output.

Eq.(20) represents that each firm’s optimal response output is decreasing(or increasing) in the rival’s if and only if the outputs of both firms are a strategic substitute (or complement) to each other. Eq.(21) shows that an increase in the marginal cost \( c_i \) raises the firm’s marginal value by lowering its own output. The above two results are consistent with the cross ownership case. Eq.(22) and (23) imply that an increase in the domestic shareholding rate owned by the domestic shareholders \( \sigma^S_{ii} \) and the foreign shareholding rate owned by the foreign shareholders \( \sigma^S_{jj} \) always raise the optimal response output of the domestic firm. The intuition behind can be explained as following. An increase in \( \sigma^S_{ii} \) or \( \sigma^S_{jj} \) raises the weight of the domestic firm’s marginal profit in the FOC condition (18). To maximize the firm’s value defined in (18), the marginal profit of the foreign firm should decrease and the domestic firm’s output should increase.
We assume that the equilibrium under cross shareholding is globally stable in the standard Cournot output adjustment process which requires

\[
\Delta^S_R \overset{\text{def}}{=} 1 - R^HS_x R^FS_x = \frac{(2 - \mu_H - \mu_F) + (1 - \mu_H\mu_F)(1 - E^S)}{[2 - (\theta_H^S + \mu_H\mu_F^S)E^S][2 - (\theta_F^S + \mu_F\theta_H^S)E^S]} = \frac{\sigma_{HH}^S \sigma_{FF}^S \Delta \mu}{[2 - (\theta_H^S + \mu_H\theta_F^S)E^S][2 - (\theta_F^S + \mu_F\theta_H^S)E^S]} \left( \Delta \mu - \frac{E^S - 2}{\sigma_{HH}^S \sigma_{FF}^S} \right) > 0.
\]

In view of Lemma 5 and Assumption 6, it is equivalent to assume:

\[
\Delta \mu - \frac{E^S - 2}{\sigma_{HH}^S \sigma_{FF}^S} = \frac{\sigma_{HH}^S + \sigma_{FF}^S}{\sigma_{HH}^S \sigma_{FF}^S} > 0.
\]

Hence we assume:

**Assumption 7.** For all relevant equilibria, \( \sigma_{HH}^S + \sigma_{FF}^S + 1 - E^S > 0 \).

4.2. Market Outcome under cross ownership vs. cross shareholding. Under international cross ownership, summing the first-order condition of (2) over both firms gives

\[
\sum_{k=H,F} (P(X^O) - c_k) + X^OP'(X^O) = 0.
\]

Note the total output is independent of the ownership structure. Meanwhile under international cross shareholding, summation of both firms’ FOC conditions of (18) yields

\[
\sum_{k=H,F} \sigma_{kk}^S (P(X^S) - c_k) + X^SP'(X^S) = 0
\]

where the total output is critically dependent on the shareholding structure. To analyze the cross shareholding effect on the total output, we define:

\[
\phi(X^S, \sigma^S, c) \overset{\text{def}}{=} \sum_{k=H,F} \sigma_{kk}^S (P(X^S) - c_k) + X^SP'(X^S)
\]

Using the implicit function theorem in the above equation leads to

\[
\frac{\partial X^S}{\partial c_k} = -\frac{\partial \phi}{\partial c_k} = \frac{\sigma_{kk}^S}{P'(X^S)(\sigma_{HH}^S + \sigma_{FF}^S + 1 - E^S)} < 0 \quad (k = H, F)
\]

\[
\frac{\partial X^S}{\partial \sigma_{kk}^S} = -\frac{\partial \phi}{\partial \sigma_{kk}^S} = -\frac{P(X^S) - c_k}{P'(X^S)(\sigma_{HH}^S + \sigma_{FF}^S + 1 - E^S)} > 0 \quad (k = H, F)
\]
where use was made of (25). It is implicitly shown that the total output increases with respect to the domestic shareholding rate $\sigma^S_{HH}$ under cross shareholding, i.e.,

$$X^S(c_H, c_F, 1, 1) > X^S(c_H, c_F, \sigma^S_{HH}, \sigma^S_{FF}) \quad \text{for } \sigma^S_{HH}, \sigma^S_{FF} < 1$$

whereas under cross ownership, the ownership structure has no effect on the total output. Such results can be summarized into the following Proposition and Figure.

**Proposition 2.** An increase in each firm’s own shareholding rate raises the total output under cross shareholding, whereas the total output remains constant regardless of ownership structure under cross ownership.

**Figure 4.** Total Output under cross ownership vs. cross-shareholding

### 4.3. Summary of Comparative Statics.

Given the cross shareholding structure as $\sigma^S = (\sigma^S_{HH}, \sigma^S_{FF})$, comparative statics results are quite different from those under cross ownership. The key point is that the change in the cross shareholding rates definitely affects the firm’s response output.

First, as with the effect of changes in the unit cost, differentiating (19) with respect to $c_i$ yields the following results.

\[
\frac{\partial x^S_i(\sigma^S, c_i)}{\partial c_i} = \frac{R^i_c(x^S_i(\sigma^S, c), \sigma^S, c_i)}{\Delta^S_{Ri}} < 0, \tag{28}
\]

\[
\frac{\partial x^S_j(\sigma^S, c_i)}{\partial c_i} = R^j_c(x^S_i(\sigma^S, c), \sigma^S, c_j) \frac{\partial x^S_i}{\partial c_i} > 0, \tag{29}
\]
where use was made of Assumption 7 and Lemma 6.

The equilibrium profit of each firm thus changes as below

\[
\frac{\partial \hat{\pi}_i^S}{\partial c_i} = \frac{\partial \hat{\pi}_i^S}{\partial x_i} + \frac{\partial \hat{\pi}_i^S}{\partial x_j} \frac{\partial x_j}{\partial c_i} + \frac{\partial \hat{\pi}_i^S}{\partial c_i} = (P(X^S) - c_i + x_i^S P'(X^S)) \frac{\partial x_i}{\partial c_i} + x_i^S P'(X^S) \frac{\partial x_i}{\partial c_i} - x_i^S
\]

(30)

\[
\frac{\partial \hat{\pi}_j^S}{\partial c_i} = \frac{\partial \hat{\pi}_j^S}{\partial x_j} + \frac{\partial \hat{\pi}_j^S}{\partial x_i} \frac{\partial x_i}{\partial c_i} = (P(X^S) - c_j + x_j^S P'(X^S)) \frac{\partial x_j}{\partial c_i} + x_j^S P'(X^S) \frac{\partial x_j}{\partial c_i} - x_j^S < 0
\]

(31)

where use was made of (18), (28), and (29).

Thus we have established

**Proposition 3.** Under cross shareholding, an increase in each firm’s unit cost (i) decreases her own equilibrium output and her equilibrium profit, but (ii) increases the other’s output and profit if both firm’s outputs are mutually strategic substitutes.

Second, we discuss the effects of changes in the shareholding structures. We have already discussed their effects on each firm’s reaction function (Lemma 6) and the total output (Proposition 2). An increase in either firm’s own share \(\sigma_{ii}\) increases each firm’s best-response output, leading to an increase in the total output. But unless either each firm’s output is a strategic complement to the other’s or the two firms are totally symmetric, it is generally ambiguous in which direction each firm’s equilibrium output changes.\(^{13}\)

#### 4.4. Optimal subsidy for welfare maximization

Then solve the resulting equilibrium subsidy at the first-stage. Under international cross shareholding, national welfare is measured by the domestic firm’s value minus the total subsidy payment by the government. Country i’s welfare function is given by

\(^{13}\)See the detail in Appendix B.
\[
W^i_S(s, \sigma^S) \overset{\text{def}}{=} \sigma_i^S \pi^i_S(\sigma^S, c^0 - s) + (1 - \sigma_j^S) \pi^{jS}(\sigma^S, c^0 - s) - s_i x^S_i(\sigma^S, c^0 - s)
\]

which has the same expression as international cross ownership. Welfare maximization also requires us to assume

**Assumption 8.** The welfare function of each country in cross shareholding is strictly concave in its own subsidy, i.e.,

\[
\frac{\partial^2 W^i_S(s, \sigma^S)}{\partial s_i^2} < 0 \quad (i = H, F)
\]

Solving the first order condition of the welfare function, after a little manipulation by using (17), it yields

\[
0 = \sigma_i^S x_i^S P'(X^S) \left( -\frac{\partial x_j^S}{\partial c_i} \right) - (1 - \sigma_i^S) x_i^S - s_i \left( -\frac{\partial x_j^S}{\partial c_i} \right) + (1 - \sigma_j^S) \frac{\partial \hat{\pi}^{iS}}{\partial x_j} \left( -\frac{\partial x_j^S}{\partial c_i} \right)
\]

As in the case of cross ownership, we may extract the additional incentive terms under cross shareholding as below

(32)

\[
I^i_S(\sigma, s) = - (1 - \sigma_i^S) x_i^S P'(X^S) \left( -\frac{\partial x_j^S}{\partial c_i} \right) - (1 - \sigma_i^S) x_i^S + (1 - \sigma_j^S) \frac{\partial \hat{\pi}^{jS}}{\partial x_j} \left( -\frac{\partial x_j^S}{\partial c_i} \right)
\]

which looks just the same under cross ownership at the first glance. However the strategic output decisions differ between the two shareholding structures as we have discussed so far. In fact, under cross shareholding case, (18) yields

\[
\frac{\partial \hat{\pi}^{iS}}{\partial x_j} = P(X^S) - c_j + x_j^S P'(X^S) = -\mu_j x_i^S P'(X^S)
\]

Its substitution into (32) yields

\[14\text{See (12).} \]
Compared with (12) under cross ownership, there come some remarks. First, the dividend suppression effect vanishes, for the effect is already taken into account by the home firm’s output decision in her value maximization; the foreign firm’s receipt of subsidy increases the home firm’s value though an increase in the dividend. Second, there remains the subsidy outflow effect intact, though its value may be different through a change in the equilibrium outputs. And lastly, the cross rent-shifting effect is magnified by the factor $1/\sigma_{ij}$. This multiplier effect is peculiar to cross shareholding.

Since the outputs as well as the comparative statics differ between the two regimes, it is hard in general to directly compare between the two additional incentive effects. However, it is possible when the demand function is linear, and we can establish the following result.\(^{15}\)

**Proposition 4.** When the demand function is linear, then the additional incentive for strategic export subsidy is stronger under cross shareholding than under cross ownership, given the mutual shareholding structure $\sigma = (\sigma_{HH}, \sigma_{FF})$ and the export subsidy profile $s = (s_H, s_F)$, i.e., $I^S_i (\sigma, s) > I^O_i (\sigma, s)$.

Let us explore the properties of the optimal subsidy further in detail. Its characterization is not possible in general. But we can undertake the more modest task, i.e., the conditions leading to positive subsidies. Let $s_i^S$ denote the optimal subsidy for country $i$ under the cross shareholding. In fact, at the marginal subsidy rate as $s_i = 0$, the first-order condition of national welfare function yields the following results.

\(^{15}\)See Appendix for the proof.
\[
\frac{\partial W^{iS}(\sigma_{ii}, \sigma_{jj})}{\partial s_i} \bigg|_{s_i=0} = x_i^S P'(X) \left( -\frac{\partial x_i^S}{\partial c_i} \right) + I_s^S \\
= -\sigma_{ii}^S x_i^S P'(X) \Delta_{\mu} \frac{\partial x_i^S}{\partial c_i} - (1 - \sigma_{ii}^S) x_i^S \\
= -\sigma_{ii}^S x_i^S P'(X) \Delta_{\mu} R_{xz}^S \frac{R_{z}^S}{\Delta_{\mu}^S} - (1 - \sigma_{ii}^S) x_i^S \\
= x_i^S \left( 1 + \mu_j \right) - (\theta_j^S + \mu_j \theta_i^S) E^S - \sigma_{jj}^S (1 - \sigma_{ii}^S) \left( \Delta_{\mu} - \frac{E^S - 2 \sigma_{ii}^S}{\sigma_{ii}^S \sigma_{jj}^S} \right) \\
= \frac{x_i^S}{\sigma_{jj}^S \left( \Delta_{\mu} - \frac{E^S - 2 \sigma_{ii}^S}{\sigma_{ii}^S \sigma_{jj}^S} \right)} \tilde{\Psi}^i (\sigma)
\]

which follows from (30), (31), (20), (21), (22), and \(\Psi\) is defined as

\[
\tilde{\Psi}^i (\sigma) \overset{\text{def}}{=} (1 + \mu_j) - (\theta_j^S + \mu_j \theta_i^S) E^S - \sigma_{jj}^S (1 - \sigma_{ii}^S) \left( \Delta_{\mu} - \frac{E^S - 2 \sigma_{ii}^S}{\sigma_{ii}^S \sigma_{jj}^S} \right)
\]

(34)

by using (25).

When the demand function is linear, i.e., \(E^S = 0\), the above equation becomes

\[
\tilde{\Psi}^i (\sigma) = \left( 1 + \frac{1 - \sigma_{ii}^S}{\sigma_{jj}^S} \right) - \left( \theta_j^S + \frac{1 - \sigma_{ii}^S}{\sigma_{jj}^S} \theta_i^S \right) E^S - \frac{1 - \sigma_{ii}^S}{\sigma_{ii}^S} (\sigma_{ii}^S + \sigma_{jj}^S + 1 - E^S)
\]

which shows that the government’s subsidy incentive depends only on the mutual shareholding structure \(\sigma\). It is always immediate to verify that \(\tilde{\Psi}^i \left( \frac{1}{2}, \frac{1}{2} \right) = 0\) holds, so that country \(i\)’s government has no incentive to subsidize the own firm.

Then what if the demand function is non-linear. Though it is hard to obtain an unambiguous result for general cases, it is for the case of the symmetric cross-shareholding structure as \(\sigma_{ii}^S = \sigma_{jj}^S = \sigma^S\). In fact, we find

\[
\tilde{\Psi}^i (\sigma^S, \sigma^S) = \frac{(1 - 2 \sigma^S)(\theta_j^S E^S - \sigma^S)}{(\sigma^S)^3},
\]

on
Thus it is straightforward to see (i) \( \hat{\Psi}_i \left( \frac{1}{2}, \frac{1}{2} \right) = 0 \) and (ii) \( \hat{\Psi}_i \left( \sigma^S, \sigma^S \right) > 0 \) if and only if \( \theta_j^S E^S - \sigma^S < 0 \) in view of \( \sigma^S > \frac{1}{2} \).

We may summarize the above results in the following proposition. Thus it is straightforward to establish

**Proposition 5.** (i) Each government’s incentive for export subsidies is independent of the cost conditions of the firms either (a) when the demand function is linear or (b) when each firm owns just a half of the other firm’s equity (i.e., \( \sigma_{HH}^S = \sigma_{FF}^S = \frac{1}{2} \)).

(ii) When each firm owns just a half of the other firm’s equity, then neither government has an incentive to subsidize the own firm.

(iii) When the percentage share of each firm’s holding the other firm’s equity (i.e., \( \sigma_{HH}^S = \sigma_{FF}^S \)), then given \( \sigma^S > \frac{1}{2} \), country \( i \) has an incentive to subsidize the own firm if and only if \( E^S < \sigma_{ij} \).

Now we want to specify exactly the region in which each government subdize the own firm for the case of linear demand. In view of (34), such a pair of equity-holding shares should satisfy \( \hat{\Psi}_i \leq 0 \) for country \( i \), i.e., \( \Psi_i \left( \sigma_{ii}^S, \sigma_{jj}^S \right) = (\sigma_{ii}^S - \sigma_{jj}^S) + \sigma_{ii}^S \sigma_{jj}^S (\sigma_{ii}^S + \sigma_{jj}^S - 1) \geq 0 \) where \( \Psi_i \left( \sigma_{ii}^S, \sigma_{jj}^S \right) = \sigma_{ii}^S \sigma_{jj}^S \cdot \hat{\Psi}_i \left( \sigma_{ii}^S, \sigma_{jj}^S \right) \). The associated curves are shown in Figure 5.

To combine the results under cross-ownership with cross-shareholding, rewrite (14) and (??) as below.

\[
\Psi_i^O \left( \sigma_{ii}^O, \sigma_{jj}^O \right) = 4\theta_i^O \sigma_{ii}^O + 2\theta_i^O \sigma_{jj}^O - (3 - \theta_i^O) \\
\Psi_i^S \left( \sigma_{ii}^S, \sigma_{jj}^S \right) = (\sigma_{ii}^S - \sigma_{jj}^S) + \sigma_{ii}^S \sigma_{jj}^S (\sigma_{ii}^S + \sigma_{jj}^S - 1)
\]

When the unit cost of each firm is symmetric as \( c_i = c_j \), we have the following figure.

Figure 6 obviously shows that under cross shareholding both countries have more incentive to subsidize its own firm than under cross ownership.\(^{16}\)

### 5. World Welfare

The welfare function of each country under cross ownership vs. cross shareholdings can be expressed as below.

\[
W^i^O \left( \sigma^O, s^O, c^0 \right) = \sigma_{ii}^O \hat{\pi}^i^O (c) + (1 - \sigma_{jj}^O) \hat{\pi}^j^O (c) - s_i^O x_i^O (c) = V^i^O \left( \sigma^O, c \right) - s_i^O x_i^O (c) \\
W^i^S \left( \sigma^S, s^S, c^0 \right) = \sigma_{ii}^S \hat{\pi}^i^S (\sigma^S, c) + (1 - \sigma_{jj}^S) \hat{\pi}^j^S (\sigma^S, c) - s_i^S x_i^S (\sigma^S, c) = V^i^S \left( \sigma^S, c \right) - s_i^S x_i^S (c)
\]

\(^{16}\)See Appendix C for more rigorous proof in the case of linear demand function.
The welfare function can be written as the sum of the firm value and subsidy payment. In the symmetric cost structure as $c_i^0 = c_j^0$, the subsidy payment will the same under both cases given $s_i^O = s_j^S$. To compare the national welfare under both cases, we should only compare the firm value $V^{io}(\sigma)$ vs. $V^{is}(\sigma)$ in the same cross ratio as $\sigma^O = \sigma^S = \sigma$.

As for the symmetric cost structure, we have the following results.

$$\frac{\partial V^i}{\partial X} = \frac{\partial V^i}{\partial x_i} + \frac{\partial V^i}{\partial x_j}$$

Under cross shareholding
\[ \frac{\partial V^{iS}}{\partial X} = \frac{\partial V^{iS}}{\partial x_j} = \sigma_{jj} \frac{\partial \hat{\pi}^{jS}}{\partial x_j} + (1 - \sigma_{ii}) \frac{\partial \hat{\pi}^{iS}}{\partial x_j} = \]

\[ = \frac{\partial V^{iS}}{\partial x_j} + \left( \frac{\partial \hat{\pi}^{i}}{\partial x_j} + \frac{\partial \hat{\pi}^{j}}{\partial x_j} \right) \]

\[ = x_i P' + (p - c_j) + x_j P' \]

\[ = (1 - \mu_j) x_i P' < 0 \]

where use was made of the FOC condition of (18). Note in the equilibrium, the total output \( X^O > X^S \) is proved in the preceding section. It leads to \( V^{iS}(\sigma) > V^{iO}(\sigma) \), then \( W^{iS}(\sigma) > W^{iO}(\sigma) \).

As no production in the importing country, its national welfare function can be expressed only by the consumer surplus as following.

\[ W^{iLM}(X) = \int_0^X p(X)dX - p(X)X \]

Note the inverse demand function is decreasing with the total output, then \( P(X^O) < P(X^S) \). The nation welfare for the importing country yields the following result.
Combine the national welfare of the two exporting countries and one importing country as the world welfare function.

\[
W^{IMO}(X) > W^{IMS}(X)
\]

In the symmetric cost structure, the world welfare is strictly increasing with the total output as \( \frac{\partial W^T}{\partial X} > 0 \). It is evident to show the important result as below.

\[
W^{TO}(\sigma) > W^{TS}(\sigma)
\]

**Proposition 6.** In the symmetric cost structure, the world welfare under cross ownership will be higher than cross shareholding given the government of two exporting countries gives the same subsidy rate.

The intuition behind the above proposition is that from the point of the world welfare maximization, the cross ownership is better than cross shareholding.

6. **Concluding Remarks**

**Appendix A. Appendix**

To clarify how the cross ownership structure governs the equilibrium export subsidy profile, let us focus our attention to linear demand cases and see what factors induce the government to impose export subsidies. At the marginal subsidy rate when \( s_i = 0 \), the first-order condition of (11) can be written as
\[
\frac{\partial W^{iO}(\sigma_{ii}^O, \sigma_{jj}^O)}{\partial s_i} \bigg|_{s_i=0} = -\sigma_{ii}^O \left( x_i^O P'(X^O) \frac{\partial x_i^O}{\partial c_i} - x_i^O \right) - (1 - \sigma_{jj}^O) x_j^O P'(X^O) \frac{\partial x_i^O}{\partial c_i} - x_i^O
\]
\[
= (\sigma_{ii}^O - 1) x_i^O + \sigma_{ii}^O x_i^O \frac{1 - \theta_j^O E^O}{3 - E^O} - (1 - \sigma_{jj}^O) x_j^O \frac{2 - \theta_j^O E^O}{3 - E^O}
\]
\[
= - \frac{X^O}{3 - E^O} \left( \theta_i^O (2 - \theta_j^O E^O)(1 - \sigma_{jj}^O) + \theta_i^O (3 - E^O)(1 - \sigma_{ii}^O) - \theta_i^O (1 - \theta_j^O E^O) \sigma_{ii}^O \right)
\]

where use was made of (7), (8), (9) and (10). Obviously when each firm’s output is a strategic complement to the other’s, it is optimal for the government to tax its export given Assumption 2 and 4. However for the case of strategic substitution, it is generally ambiguous to determine.

Another interesting case for discussion is that of symmetric cross ownership structure as \( \sigma^O = (\frac{1}{2}, \frac{1}{2}) \), the above result can be simplified to

\[
\frac{\partial W^{iO}(\frac{1}{2}, \frac{1}{2})}{\partial s_i} \bigg|_{s_i=0} = - \frac{X^O}{2(3 - E^O)} [\theta_i^O (2 - \theta_i^O E^O) + \theta_j^O (2 - \theta_j^O E^O)] < 0
\]

which is definitely negative, making an export tax optimal. Intuitively, it is similar to the case when both firms locate in one country. Since strategic subsidization makes the competition between firms more fiercely and lowers the national welfare, each government has an incentive to tax its exports.

In the absence of cross ownership as \( \sigma^O = (1, 1) \), it becomes the standard subsidy policy in Brander and Spencer[2], which yields

\[
\frac{\partial W^{iO}(1, 1)}{\partial s_i} \bigg|_{s_i=0} = \frac{x_i^O}{3 - E^O} (1 - \theta_j^O E^O) \begin{cases} > 0 & \text{Strategic Substitution} \\ < 0 & \text{Strategic Complementary.} \end{cases}
\]

Each government has a unilateral incentive to subsidize or tax its exports in the case of strategic substitution or complement. A linear model with inverse demand function \( p = a - X \) yields (14) in the text.
APPENDIX B. APPENDIX

Differentiating (19) with respect to $\sigma^S_{ij}$ yields

$$\frac{\partial x^S_i(\sigma^S, c)}{\partial \sigma^S_{ij}} = \frac{R^S_{\sigma_{ij}} + R^S_{i} R^S_{ij}}{\Delta \sigma_{ij}^S}$$

$$= \frac{- \left[ (P(X^S) - c_i + x^S_i P'(X^S)) \left[ 2 - (\theta^H_{ij} + \mu^S_{ij} E^S) \right] \sigma^S_{ij} P'(X^S) \right]}{\sigma^S_{ij} P'(X^S)} + \frac{x^S_i \left[ (1 + \mu_i) - (\theta^H_{ij} + \mu^S_{ij} E^S) \right]^2}{\sigma^S_{ij} P'(X^S)}$$

Then we further note that in the case of linear demand function as $p = a - (x_H + x_F)$, each firm’s equilibrium output is given by

$$\partial x^H_i \partial \sigma^S_{ij} = \cdots$$

which is indeterminate.

APPENDIX C. APPENDIX

Note the relation $\sigma_{ii} = \frac{1-\mu^S_{i}}{\Delta x}$. Then put it into the first-order conditions for welfare maximization in both regimes, and obtain

$$\frac{\partial W^{io}}{\partial s_i} \bigg| \Delta \mu = x^O_i \left( 1 - \mu_j \frac{\partial \hat{x}^{io}}{\partial c_i} - \mu_j (1 - \mu_i) \right) + \mu_i (1 - \mu_j) x^O_j \frac{\partial \hat{x}^{io}}{\partial c_i}$$

$$\frac{\partial W^{is}}{\partial s_i} \bigg| \Delta \mu = = x^S_i \left( 1 - \mu_j \frac{\partial \hat{x}^{is}}{\partial c_i} - \mu_j (1 - \mu_i) \right)$$

Then we further note that in the case of linear demand function as $p = a - (x_H + x_F)$, each firm’s equilibrium output is given by
\[ x_i^O = \frac{2(a - c_i) - (a - c_j)}{3} \]
\[ x_i^S = \frac{2(a - c_i) - (1 + \mu_i)(a - c_j)}{3 - \mu_i - \mu_j - \mu_i\mu_j}. \]

In view of \( \Delta \mu > 0 \), to assure \( \frac{\partial W^{iO}}{\partial s_i} \bigg|_{s_i=0} \geq 0 \)

\[ x_i^O \left(1 - \mu_j\right) \frac{\partial \hat{\mathbf{x}}^{jO}}{\partial c_i} - \mu_j(1 - \mu_i) \geq -\mu_i(1 - \mu_j)x_j^O \frac{\partial \hat{\mathbf{x}}^{jO}}{\partial c_i} > 0 \quad \implies \quad \mu_j(1 - \mu_i) < (1 - \mu_j) \frac{\partial \hat{\mathbf{x}}^{jO}}{\partial c_i} \]

should be satisfied. Substituting the above result into \( \frac{\partial W^{iS}}{\partial s_i} \bigg|_{s_i=0} \) yields the following results.

\[ \frac{\partial W^{iS}}{\partial s_i} \bigg|_{s_i=0} \Delta \mu = x_i^S \left(1 - \mu_j\right) \Delta \mu \frac{\partial \hat{\mathbf{x}}^{jS}}{\partial c_i} - \mu_j(1 - \mu_i) \]
\[ > x_i^S \left(1 - \mu_j\right) \Delta \mu \frac{\partial \hat{\mathbf{x}}^{jS}}{\partial c_i} - (1 - \mu_j) \frac{\partial \hat{\mathbf{x}}^{jO}}{\partial c_i} \]
\[ = x_i^S (1 - \mu_j) \left( \Delta \mu \frac{\partial \hat{\mathbf{x}}^{jS}}{\partial c_i} - \frac{\partial \hat{\mathbf{x}}^{jO}}{\partial c_i} \right) \]
\[ = x_i^S (1 - \mu_j) \left( \frac{(1 - \mu_i \mu_j)(1 + \mu_j)}{3 - \mu_i - \mu_j - \mu_i \mu_j} - \frac{1}{3} \right) \]
\[ = x_i^S (1 - \mu_j) \frac{F^i(\mu_i, \mu_j)}{3(3 - \mu_i - \mu_j - \mu_i \mu_j)} \]

where \( F^i(\mu_i, \mu_j) := (1 - \mu_i \mu_j)(2 + 3\mu_j) - 2 + \mu_i + \mu_j \). We can prove \( F^i(\mu_i, \mu_j) \geq 0 \) for the relevant values of \( \mu_i, \mu_j \) as following.

We first note \( \mu_i := \frac{1 - \sigma_{ij}}{\sigma_{ii}} \in (0, 1) \) when \( \sigma_{ik} (k = i, j) \) runs over \((\frac{1}{2}, 1)\). Second given \( \mu_j \in (0, 1) \), \( F(\mu_i, \mu_j) \) is strictly concave in \( \mu_i \), for we have

\[ \frac{\partial F(\mu_i, \mu_j)}{\partial \mu_j} = -2\mu_i - 6\mu_i \mu_j + 4, \]
\[ \frac{\partial^2 F(\mu_i, \mu_j)}{\partial \mu_j^2} = -6\mu_i < 0. \]

Thus if \( F(\mu_i, \mu_j) = \min \{ F(\mu_i, 0), F(\mu_i, 1) \} = \min \{ \mu_i, 4(1 - \mu_i) \} \geq 0 \) must hold. This establishes our desired result as:
\[
\frac{\partial W^{IO}}{\partial s_i} \bigg|_{s_i=0} \geq 0 \quad \implies \quad \frac{\partial W^{IS}}{\partial s_i} \bigg|_{s_i=0} \geq 0
\]

which conveys the message that under cross shareholding, the government has more incentive to give the subsidy to their own exports than under cross ownership.

REFERENCES


